

# QUANTUM WALKS

**Bereneice Sephton, Angela Dudley and Andrew Forbes explain the importance of random motion.**

Consider the path of tiny dust particles as they float in still air or the motion of those small specs of matter just visible in your glass of water. These are examples of random motion. It is an integral part of how the world operates and facilitates many of the fundamental processes that maintain the Earth, such as heat transport in materials and the transport of gases like oxygen, which are crucial to survival. It can even go so far as to describe the way animals forage and aspects of finances.

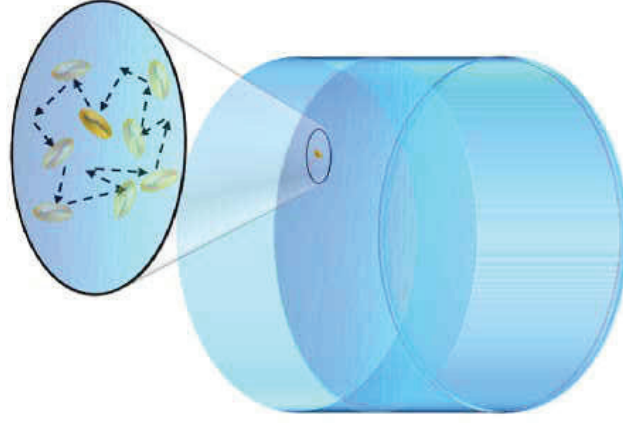
## RANDOM MOTION

So, what exactly is random motion? The answer can be dated back to 1827 when a botanist, Robert Brown, noted the movement of pollen floating on water (Fig. 1). He saw that the tiny bits of pollen moved in many random directions, but did not actually go anywhere – they had no overall displacement. This tied in with the study of atoms and this movement was eventually shown to come from the random motion of fast-moving molecules that hit the pollen grains from all sides. However, different sides of the pollen experienced slightly greater forces than others, which resulted in the random movement of the pollen itself. Soon after this, in 1905, Einstein provided the mathematical foundation for this phenomenon, consolidating random motion as a foundation in many scientific studies today. Understanding and being able to predict such motion lies in our ability to simulate it. However, how do you do this when the basic feature of something that is random is that there is no definable pattern and thus should not be predictable? The answer lies in the concept of the random walk.

## RANDOM WALKS

Consider for a moment the idea of using the flip of a coin to make a decision. An example would be – shall I move left or right? To do this, you make each side of the coin indicate a direction, for example, heads to right and tails to left. By flipping the coin and moving in that direction, you are moving randomly. If you flip the coin again and move according to the outcome, another random movement is carried out. If you do this over and over again, you are creating random motion, or more specifically a random walk (see Figure 3(a) in Box 3). It follows that by assigning particular values to the available options and then randomly generating those assigned values, ‘randomness’ can be simulated by simply acting upon the randomly generated values. Associated outcomes can then be predicted.

The result of such a simulation is a Gaussian distribution (see Box 1) where the most probable outcome, and thus the option that will happen most often, is found at the starting position. This is exactly what Robert Brown noted for the pollen. Being able to predict random motion has allowed scientists



**Figure 1. Robert Brown noted the stochastic, random movements of pollen floating on water, which resulted in the concept of random motion. The black arrows show the random path taken by the pollen grain where the overall path results in no net displacement.**



**Students aboard SA Agulhas II throw a message-in-a-bottle into the South Atlantic. This activity is a mini experiment to test surface currents. A student on SEAmester I was eventually contacted by a man in Brazil, who found her message-in-a-bottle nine months after she threw it into the Agulhas Current.** Morgan Trimble



**The SA Agulhas II's crew has successfully hauled in a CPIPE that has been collecting data at the sea floor for four years.** Morgan Trimble

changes. Currently, researchers are measuring a baseline for the meridional overturning circulation and its natural variability so that they will be able to detect possible alterations in the future. The goal is to detect changes in the chemical and physical properties of the circulation and investigate how these changes are influenced by and in turn influence global climate. SEAmester is funded by the Department of Science and Technology. The Department of Environmental Affairs provides access to the SA Agulhas II.

Dr Morgan Trimble is an ecologist and science writer and lectured scientific communication aboard this year's SEAmester voyage. Katherine Hutchinson, a PhD student at UCT affiliated with the South African Environmental Observations Network (SAEON), helped report this article and lectured ocean dynamics on SEAmester.

**LANGUAGE TRANSLATION**

**Science of Sea - ubuchwepheshe obusolwandle**

Throw a message in a bottle into the sea off the coast of South Africa, and there's a chance it will wash up on the shores of Brazil in less than a year! The dynamics of water in the oceans are critical for controlling Earth's weather and climate and crucial to sea life. Lines of monitoring equipment moored to the sea bed are used to monitor everything about the ocean and how it changes.

► faka umlayezo obuhleleni, uwufake olwandle olusogwini leningizimu ne afrika, kunamathaba okuthi llingagcina seligamuka ogwini lase Brazil kungakaphelel unyaka! Ukushinsha kwamanzu olwandle kubalulekile ukulawula isimo sezulu ezweni kamti futhi nakusukho komke okuphila olwandle. Izintambo zemishini yokusapha ezifakwe ngaphansi kolwandle sisetshenziswa ukubhekisa yonke into emayelana nolwandle nokushinsha kwalo.

**CURRICULUM CORNER**

**GEOGRAPHY GRADE 10-11**

The world's oceans

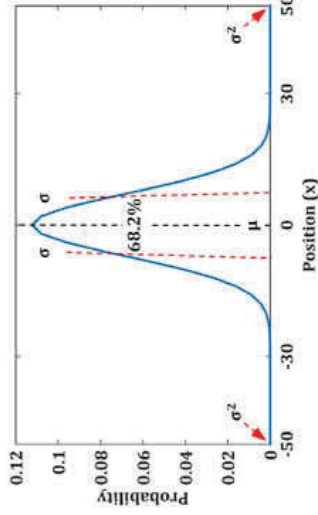
**LIFE SCIENCES GRADE 10-12**

Human impact on the environment

data, ping the water column and recording its temperature and the velocities of its currents. The data are stored in the devices, patiently waiting for scientists to call the devices back to the surface and download this precious information. Our job on the SEAmester 2017 cruise was to do just this. The first step in recovering an instrument is to navigate to its known coordinates. Then, to call the devices to the surface, scientists use a hydrophone – a specialised underwater speaker that transmits a unique code embedded in sound waves. When the CPIEs and ADCPs hear their special instructions, they send an electric current through a wire attaching the main body of the instrument to its anchor, which holds it to the sea floor. The electric current moving through the wire exposed to salty seawater causes immediate corrosion. Within minutes, the wire breaks, releasing the instrument to float to the surface. From the ship, observers spot the instrument, the crew uses a hook to catch and haul in the device, and the scientists set about excitedly downloading four years of data. Scientists then clean up and replace the batteries of the instruments before sending them back down to the sea floor to resume data-collection duties. The value of the SAMBA line lies in the power of long-term observations to expose subtle environmental

**BOX 1: PROBABILITY DISTRIBUTION**

This shows the Gaussian curve for a random walk with 50 steps taken. One standard deviation ( $\sigma$ ) from the mean value ( $\mu$ ) for a data set describes the bounding value for which 68.2% of all the data points will be found (34.1% will be found on either side of the mean value). A Gaussian probability distribution (type of bell curve) is a curve



to use the theory in modelling processes such as particle movements, how certain processes take place in your DNA and even in recommendations on who to follow on Twitter.

**RANDOM SPREADING**

The rate at which a probability distribution spreads (how wide the shape becomes) with the number of steps taken (coin flips) is of particular interest when characterising random walks, because this is related to how fast an answer (equivalent to a particular position) can be found in computing. Therefore, the variance ( $\sigma^2$ ) is considered, which may be shown to vary linearly with the number of steps ( $n$ ) taken in the characteristic random walk probability distribution:  $\sigma^2 \propto n$

**THE RECIPE FOR RANDOM**

The recipe of a random walk may then be generalised into a walker (Bob), a random value generator (coin), a position space (road intersections), where the random values are assigned to particular directions and a propagator that moves the walker in the randomly generated direction (walking). By adding more directions ( $n$ ) and another random value generator (e.g. dice) with a number of options that match the number of directions, Bob can go on an  $n$ -dimensional walk.

**WALKING IN THE QUANTUM WORLD**

The random walk has been largely successful in the world of classical physics, so what is the effect of taking the random walk to the world of quantum mechanics (see Box 2)? That is, what can be accomplished if the principle behind the working of the random walk is extended to work under the principles of the quantum world? To take the random walk into the quantum world, you have to use the phenomenon of quantum superposition. Superposition essentially says that something can occupy more than one position or property at the same time – put scientifically that ‘something’ can exist as a linear superposition of more

than one basis state. This means that the coin in the random walk can no longer be either heads or tails in the quantum equivalent, but is rather a superposition of heads and tails. In other words, the coin is simultaneously in the state of landing on heads *and* tails. We can describe this with the expression given in Figure 2. More specifically, complex probability amplitudes (see Box 3) are used – how much each property or position is favoured is determined by the relative value of the associated probability.



$Coin = a[\text{Heads}] + b[\text{Tails}]$

Figure 2. In the quantum world, the coin is in a state of being both heads and tails (superposition). ‘How much’ it is in either state is given as  $a$  for the heads state and  $b$  for the tails state.

**BOX 3: PROBABILITY AND PROBABILITY AMPLITUDES**

Probability in the classical world refers to the option of something occurring

$P_{\text{classical}} = A^*A$

or, conversely, the frequency at which it will happen should you do it a certain number of times. It follows that there can be absolute certainty (100% = 1), no possibility (0% = 0) or something in between, such as 50% = 0.5. For example, in the random walk, the probabilities split at each intersection so  $1 \rightarrow 0.5 \rightarrow 0.25$  and when the intersections overlap, we add the contributing probabilities (see Fig 3(a)).

you add together some of the values, they can become less or cancel out. So if you add up the amplitudes before converting back to classical probability, the answers are completely different – for example, as in the case of a quantum walk, where the probability amplitude for every left turn (heads) has an  $i$  (see Figure 3(b)).

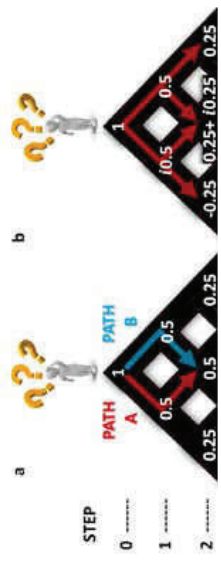


Figure 3: (a) Classical and (b) quantum probabilities for Bob’s coin walk. Bob flips the coin to decide which intersection to take where tails indicates left and heads, right. Consecutive repetition of such decisions lead Bob on a random walk. The blue and red paths in (a) show two of the four possible routes Bob can take, both end at the same position, making it the most probable (0.5). Conversely, for the quantum walk, Bob flipping a coin causes him to occupy and interfere with himself at all the possible positions at the same time.

such that there are ‘two movements’ to 0. It follows that the position at 0 is more strongly favoured than that of  $\pm 2$ , resulting in constructive interference of the probabilities associated with these states. Bob will thus be more likely to be found at 0 when looking for him. Because these are probability amplitudes, the ‘overlapping states’ may also result in destructive interference. Subsequently, a reduced probability of finding Bob occurs at certain positions by simply adding more of quantum Bob. Bob is now interfering with himself: reinforcing and cancelling himself out over several

positions at the same time. It is this interference that forms a significant difference between the classical random walk and quantum random walk.

**QUANTUM SPREADING AND WEIRDNESS**

Also, when we look at how fast quantum spreading can occur and thus how fast we can get our answer, the game of computing changes. In contrast to the classical case, where the variance has a linear limit, the quantum walk has a variance where:  $\sigma^2 \propto n^2$

**BOX 2: CLASSICAL AND QUANTUM WORLDS**

The classical world, in physics, refers to the experiences around us every day – everything happening on a macroscopic scale. Here the rules that can be expected: a ball hitting a wall will bounce back and you can only be in one place at a time. The quantum world is a little stranger. Here the rules are different and often opposite to what you would ordinarily expect. For example, the little transistors in your phone storing all your information in terms of 1s and 0s work by electrons ‘tunnelling’ through the bounding walls, i.e. the ball doesn’t always bounce back. When you aren’t looking, the question for particles isn’t ‘where is it?’ but rather ‘where isn’t it?’



Image of a transistor. [https://realizer.com/wiki/History\\_of\\_Computing/hardware](https://realizer.com/wiki/History_of_Computing/hardware)

These rules are for a much smaller scale though – the atomic scale. Consider the full stop at the end of this sentence. You would need to shrink it about  $10^8$  times to approximate the size of a carbon atom, so you

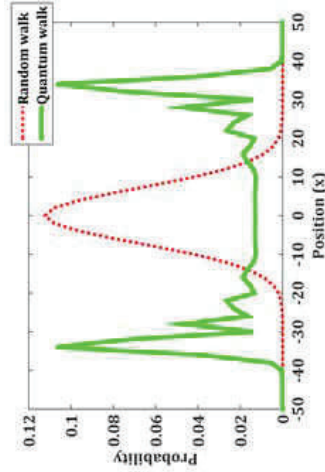


Figure 4: The quantum walk probability distribution (green line) is distinctly different to that of the Gaussian (dotted red line) occurring in the classical case where the destructive interference plays a large role. Here the probability of finding the walker is closest to the ends of the distribution, with minimal probability at the original position (position 0).

which means the answer/position can be found quadratically faster.

All this peculiarity takes the form of a largely surprising result, wrapping up the strangeness associated with these walks. If you follow the interference in a quantum walk, what you end up with is opposite to what you expect – almost an inverse of what we see for the classical case. Destructive interference occurs at the centre, which seems counter-intuitive to what you see in the random walk. In fact, the greatest probability of finding Bob is close to the ends of the distribution and the larger the number of steps taken, the smaller this probability in the middle becomes (see Fig. 4).

The quantum walk has fast become a field of great interest to scientists over the last 20 years as it holds the potential for unlocking many scientific and technological advances. Derived algorithms already show promise in classical computers. When physically implemented, though, the quantum walk can form the basis of quantum computers. Here the quadratic speedup will direct potential to crack the bit-encryptions used for many security systems could become something done in days, rather than over an infinitely long time. Many complex mathematical problems will be easy to solve. Nature does its own quantum walk when plants photosynthesise, so generating our own quantum walk will help in understanding one of nature's energy transportation systems. It is of little wonder that researchers

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Andrew Forbes is a Distinguished Professor within the School of Physics, University of the Witwatersrand, where he leads a group working on Structured Light ([www.structredlight.org](http://www.structredlight.org)). Dr Angela Dudley is a senior researcher at the CSIR and has an honorary appointment at the University of the Witwatersrand. She works on laser beam shaping and its applications. Berenice Sephton is completing her Masters degree at the University of the Witwatersrand, on a studentship from the CSIR, with a focus on quantum walks.

## LANGUAGE TRANSLATION

### Quantum walks- ukuhamba okubalulekile/okukhulu

Consider the path of tiny dust particles as they float in still air or the motion of those small specs of matter just visible in your glass of water. These are examples of random motion. It is an integral part of how the world operates and facilitates many of the fundamental processes that maintain the Earth, such as heat transport in materials and the transport of gases like oxygen, which are crucial to survival. Cabanga umgudu wezinhlavya ezincane zezintuli zindiza emoyeni, noma ukunyakaza kwakwezinhlayiya ezincane engizuzi yakho manzi. Lokhu kukumzekelo wokunyakaza okuzenzekelayo. I.e. indlela umhlaba osebenza ngayo, futhi kwenzeka izinto ezibalulekile emhlabeni zenzeke ukuze umhlaba uhlele uyiywo. Njengokuhamba kokushisa okuhamba ngokuthintana kwenzito noma ukuhamba komoya esivuphetumulayo, okubalulekile ukuthi kube nempilo.

## CURRICULUM CORNER

- PHYSICAL SCIENCES GRADE 10-11
- Kinetic molecular theory- Brownian motion
- MATHEMATICAL LITERACY GRADE 10-12
- Probability

# Hubble's Messier Catalogue

Although there are as many as 100 billion comets in the outer regions of the solar system, prior to 1995 only around 900 had ever been discovered. This is because most comets are too dim to be detected without the proper astronomical equipment. Occasionally, however, a comet will sweep past the sun that is bright enough to be seen during the daytime with the naked eye.

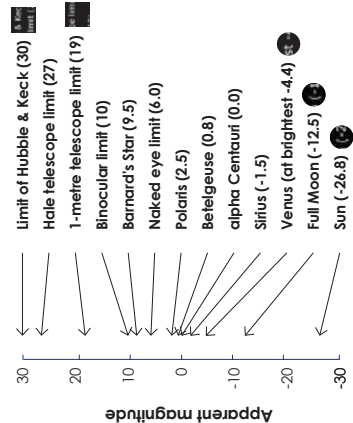
## APPARENT AND ABSOLUTE MAGNITUDE

Some stars appear very bright but are actually fainter stars that lie closer to us. Similarly, we can see stars that appear to be faint, but are intrinsically very bright ones lying far away from Earth. The Greek astronomer Hipparchus was the first to categorise stars visible to the naked eye according to their brightness. Around 120 BC, he invented six different brightness classes, called magnitudes, where the brightest stars were magnitude 1 and the faintest were categorised as magnitude 6. Today, astronomers use a revised version of Hipparchus's magnitude scheme called 'apparent magnitudes', as well as 'absolute magnitudes' to compare different stars.

### Apparent magnitude

The power radiated by a star is known as its luminosity. However, the apparent magnitude,  $m$ , is the power received by an observer on Earth. Since we now can see very faint stars using telescopes, the scale extends beyond the magnitude 6 that Hipparchus marked down as the faintest on his scale.

As you can see, the magnitude numbers are bigger for faint stars, and magnitudes are negative for very bright stars. Since the scale is logarithmic, a magnitude 1 star is 100 times brighter than a magnitude 6 star, i.e. the difference between each step on the scale is equal to a decrease in brightness of  $2.512$  and  $(2.512)^5 = 100$ .



### Absolute magnitude

Comparing apparent magnitudes is a useful reference for astronomers, and these often appear next to stars on star maps. Apparent magnitude, however, does not tell us about the intrinsic properties of the star, so it is necessary to use the concept of absolute magnitude.

The absolute magnitude,  $M$ , of a star is defined as what the apparent magnitude of that star would be if it were placed exactly 10 parsecs away from the Sun. Most stars are much further away than this, so the absolute magnitude of stars is usually brighter than their apparent magnitudes.

To calculate the absolute magnitude for stars, we use the following equation:

$$M = m - 5 \log (D/10)$$

The value  $m-M$  is known as the distance modulus and can be used to determine the distance to an object, often using the following equivalent form of the equation:

$$D = 10^{(m-M+5)/5}$$

Star (Bayer)	Star (Proper)	Parallax (arcseconds)	Apparent mag. (m)	Absolute mag. (M)
$\alpha$ Canis Majoris	Sirius	0.37921	-1.44	1.45
$\alpha$ Carinae	Canopus	0.01043	-0.62	-5.53
$\alpha$ Boötis	Arcturus	0.08885	-0.05	-0.31
$\alpha 1$ Centauri	Rigel Kent	0.74212	-0.01	4.34
$\alpha$ Lyrae	Vega	0.12893	0.03	0.58
$\alpha$ Aurigae	Capella	0.07729	0.08	-0.48
$\beta$ Orionis	Rigel	0.00422	0.18	-6.69
$\alpha$ Canis Minoris	Procyon	0.28593	0.40	2.68
$\alpha$ Orionis	Betelgeuse	0.00763	0.45	-5.14
$\alpha$ Eridani	Achernar	0.02248	0.45	-2.77

Apparent and Absolute Magnitudes for the ten brightest stars on the night sky.