Quantum communication
and other quantum information technologies

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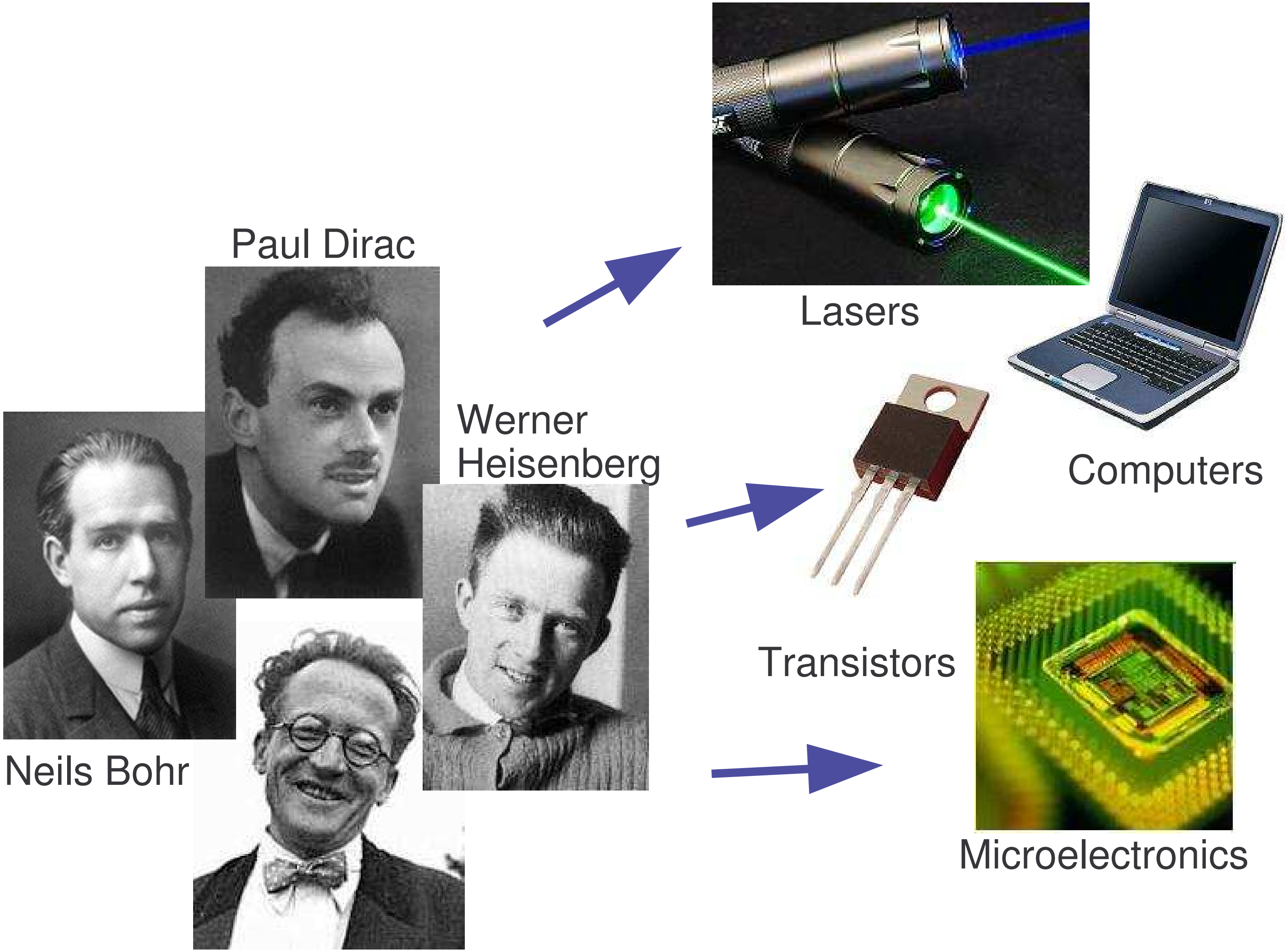
our future through science

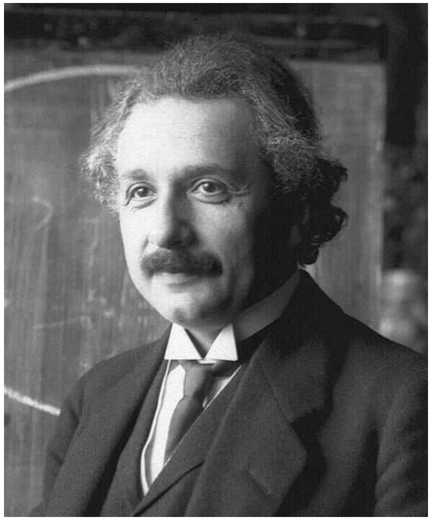
Contents

- ▷ Over view of quantum mechanics
- ▷ Quantum communication
  - — Quantum state preparation
  - — Quantum teleportation
- ▷ Decay of entanglement in turbulence
  - — Theory
  - — Numerical simulations
  - — Experimental results

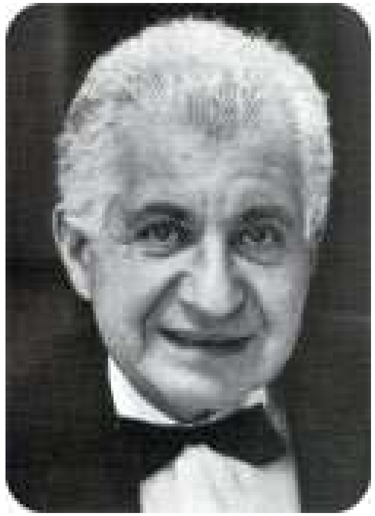
Quantum mechanics



Einstein-Podolsky-Rosen

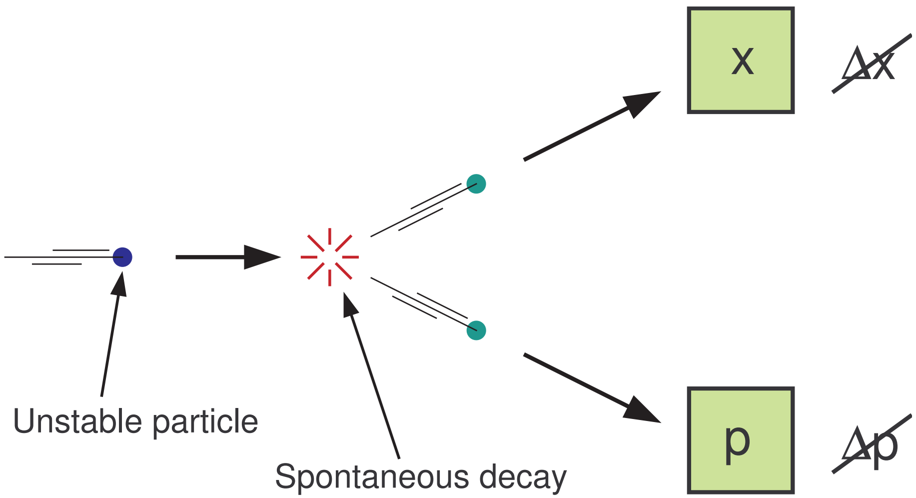


Albert Einstein



Boris Podolsky

Nathan Rosen



Quantum mechanics: measurements on one particle <u>dictate</u> the state of the other particle.

Parametric down conversion

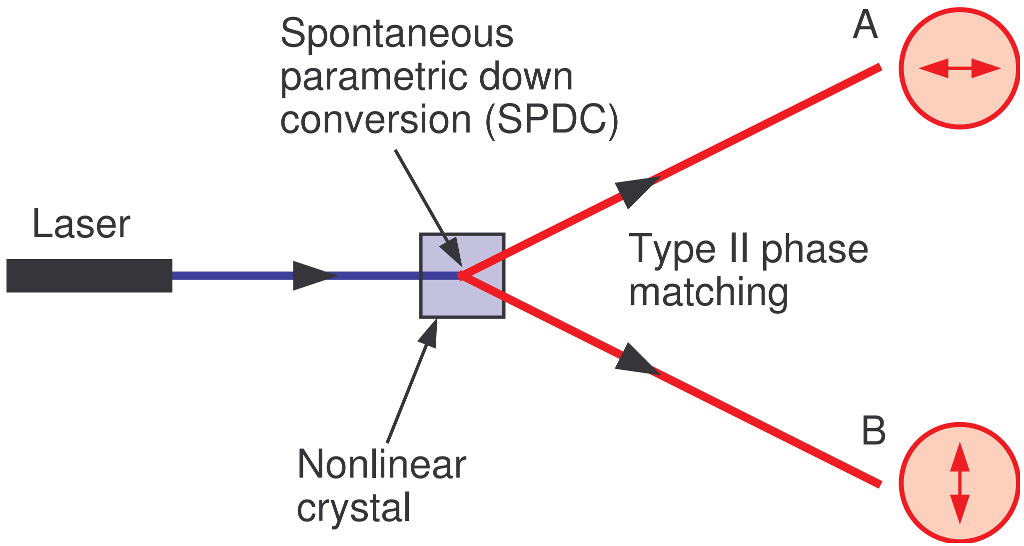
One incoming photon $\rightarrow$ Two outgoing photons

Energy conservation:

$\omega_{pump} = \omega_{signal} + \omega_{idler}$

Momentum conservation:

$\mathbf{k}_{pump} = \mathbf{k}_{signal} + \mathbf{k}_{idler}$



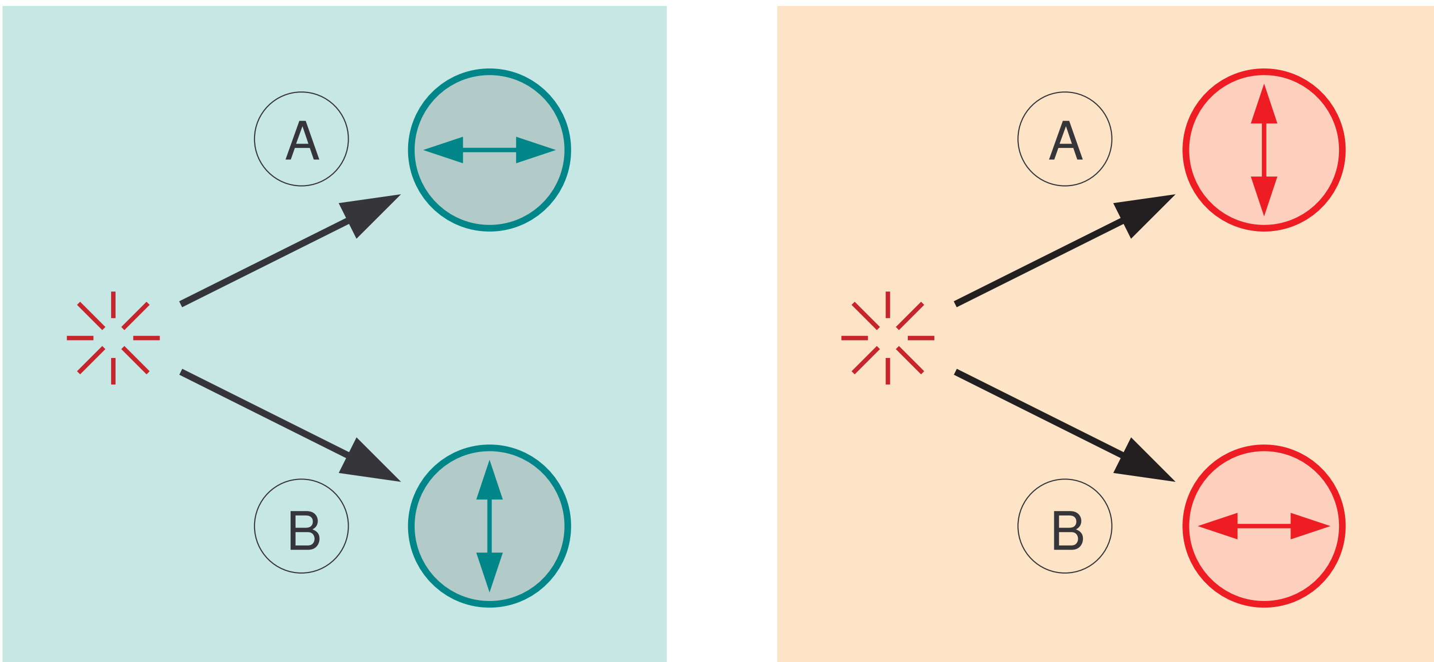
Type II phase matching $\Rightarrow$ photons have perpendicular polarization: $\theta_B = \theta_A - \pi/2$

However, each beam on its own is unpolarized
— contains all states of polarization.

Multiple realities

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |H\rangle_A |V\rangle_B - \frac{1}{\sqrt{2}} |V\rangle_A |H\rangle_B$$

Reality #1 — Reality #2

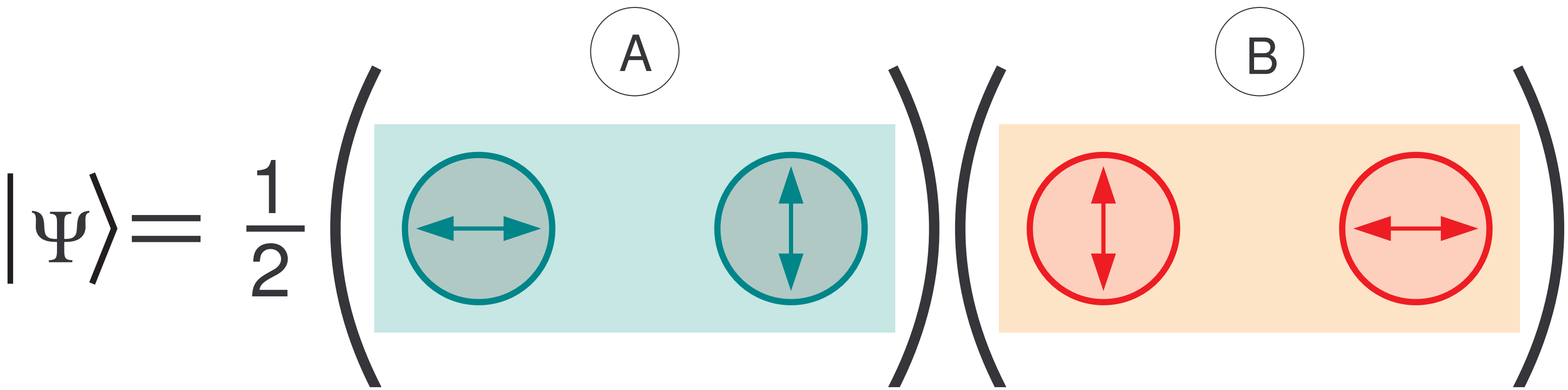


Separability

$$|\Psi\rangle = \frac{1}{2}|H\rangle_A|V\rangle_B - \frac{1}{2}|H\rangle_A|H\rangle_B + \frac{1}{2}|V\rangle_A|V\rangle_B - \frac{1}{2}|V\rangle_A|H\rangle_B$$

... can be factored (separated)

$$|\Psi\rangle = \frac{1}{2}\left(|H\rangle_A + |V\rangle_A\right)\left(|H\rangle_B - |V\rangle_B\right)$$

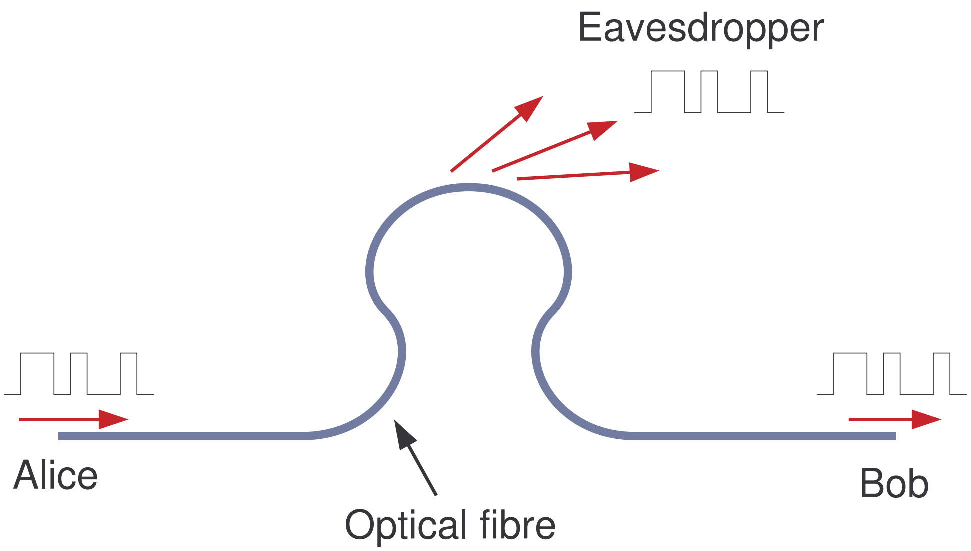


Separability $\Rightarrow$ Not entangled

Quantum communication

What does quantum communication have that classical communication doesn't? $\rightarrow$ Fundamental security!

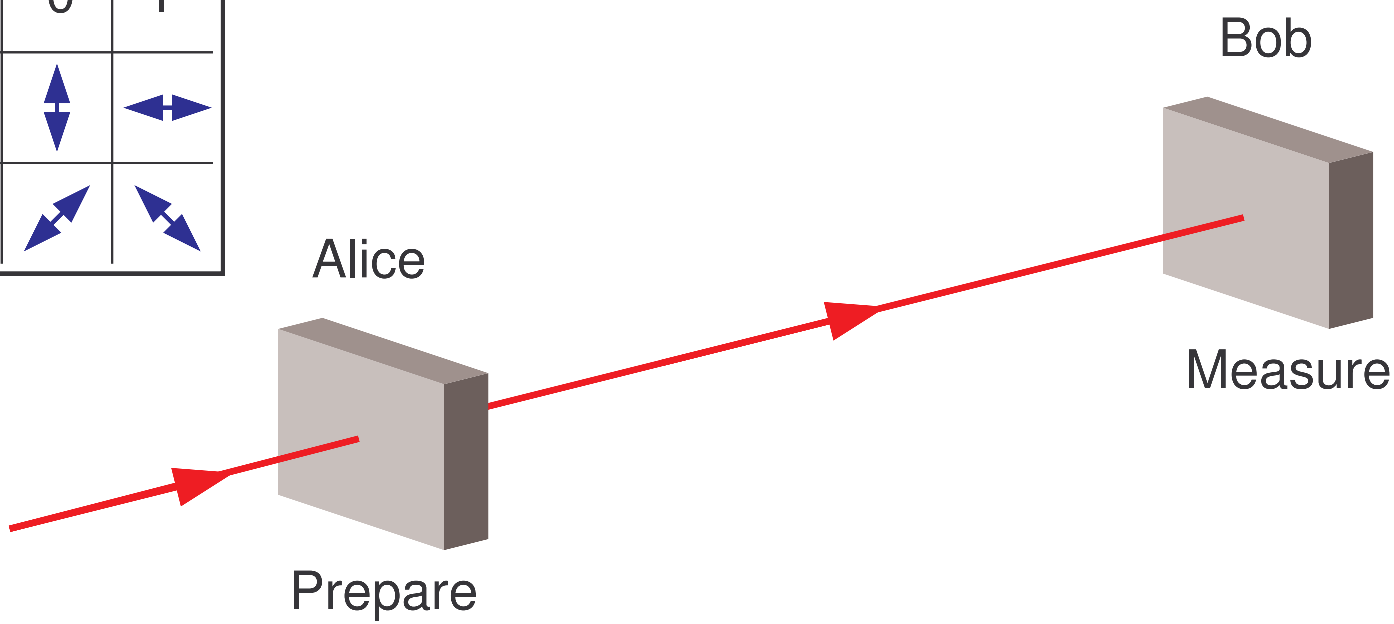
One cannot copy a quantum state $\Rightarrow$ one cannot eavesdrop without sender/receiver knowing



$\rightarrow$ Quantum protokol (Quantum Key Distribution — QKD) to produce an encryption key that is fundamentally secure

Quantum Key Distribution — BB84

Basis	0	1
+	↕	↔
×	⤢	⤡



Alice	+	+	×	+	×	+	+	×	×	+	×	+
	↔	↕	⤢	↕	⤡	↔	↕	⤢	⤡	↔	⤢	↕
Bob	+	×	×	+	×	×	+	×	+	+	×	×
	↔	⤡	⤢	↕	⤡	⤡	↕	⤢	↕	↔	⤢	⤡
Key	1		0	0	1		0	0		1	0	

Polarization vs modes

Can always specify polarization with two polarization states:

$$|\Psi\rangle = C_H|H\rangle + C_V|V\rangle = C_L|L\rangle + C_R|R\rangle$$

Polarization $\Rightarrow$ 2-dimensional Hilbert space

$\Rightarrow$ each photon can carry one qubit of information

For more information per photon (larger channel capacity)
$\Rightarrow$ need larger Hilbert space

Transverse spatial modes have an infinite dimensional Hilbert space

Laguerre-Gaussian modes

General solutions of the paraxial wave equation in normalized polar coordinates:

$$M_{p\ell}^{\mathrm{LG}}(r,\phi,t) = N \frac{r^{|\ell|}\exp(\mathrm{i}\ell\phi)(1+\mathrm{i}t)^p}{(1-\mathrm{i}t)^{p+|\ell|+1}} \mathrm{L}_p^{|\ell|}\left(\frac{2r^2}{1+t^2}\right)\exp\left(\frac{-r^2}{1-\mathrm{i}t}\right)$$

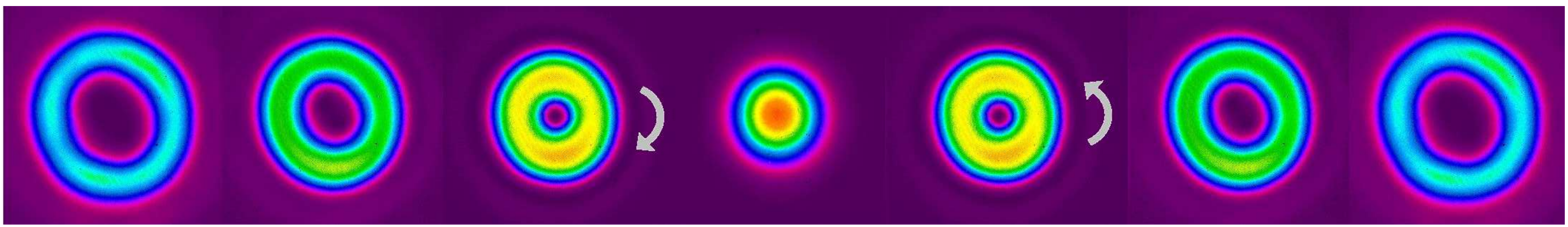
$x = rw_0\cos\phi$, $y = rw_0\sin\phi$, $z = z_R t$ ($z_R = \pi w_0^2/\lambda$)

$\mathrm{L}_p^{|\ell|}$ — associate Laguerre polynomials

p — radial mode index (non-negative integer)

ℓ — azimuthal index (signed integer)

N — normalization constant



Bessel-Gaussian modes

In normalized polar coordinates:

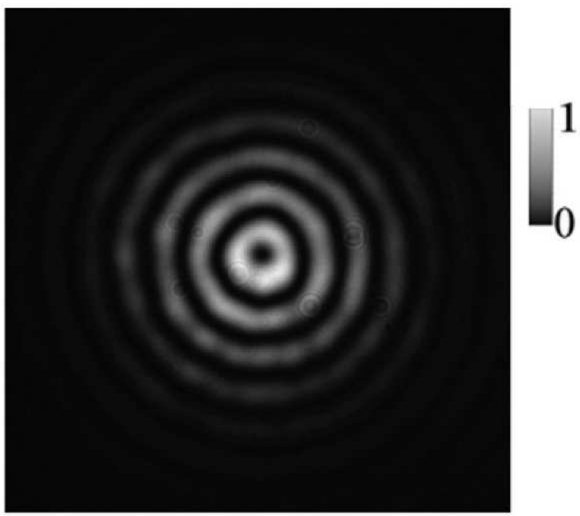
$$M_\ell^{\mathrm{BG}}(r,\phi,t;\chi) = \sqrt{\frac{2}{\pi}}\mathrm{J}_\ell\left(\frac{\sigma r}{1-\mathrm{i}t}\right)\exp\left(\frac{\mathrm{i}\sigma^2 t - 4r^2}{4(1-\mathrm{i}t)}\right)\exp(\mathrm{i}\ell\phi - \mathrm{i}zk_z)$$

$x = rw_0\cos\phi$, $y = rw_0\sin\phi$, $z = z_R t$ ($z_R = \pi w_0^2/\lambda$)

J_ℓ — Bessel function

$\sigma = w_0 k_r$ — normalized radial scale parameter

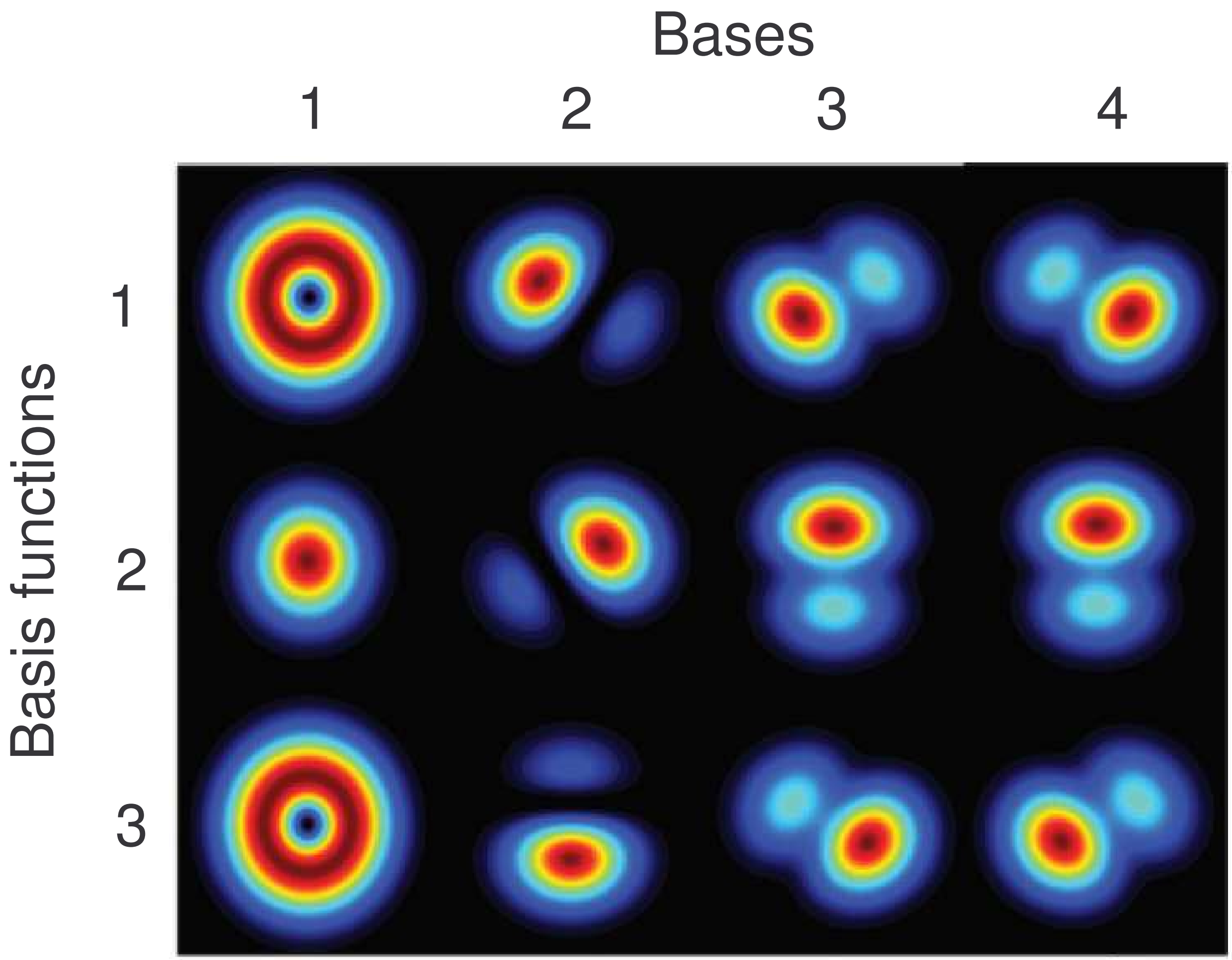
k_r, k_z — radial and longitudinal wavenumber



QKD in higher dimensions

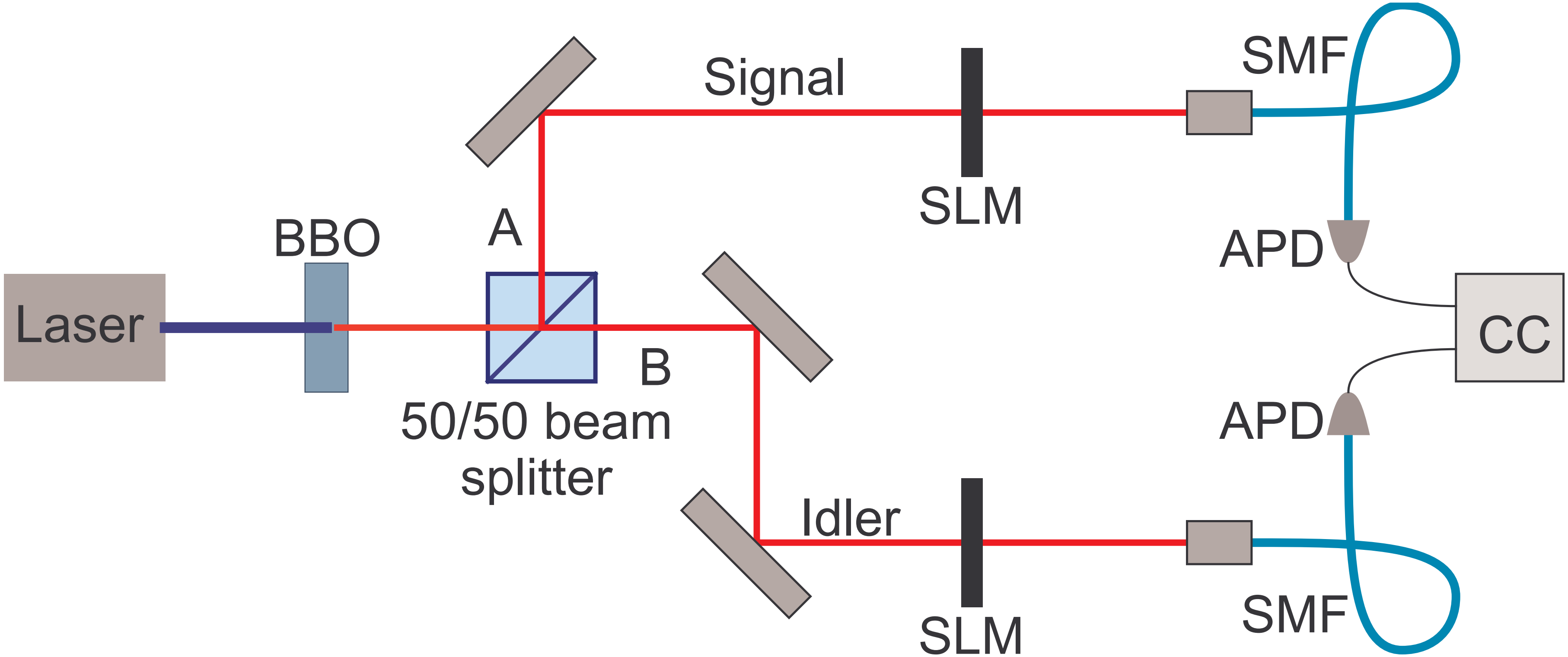
Need mutually unbiased bases in higer dimensions

$$|\langle \phi_{a,n} | \phi_{b,m} \rangle|^2 = \frac{1}{d} \quad \text{for} \quad a \neq b$$



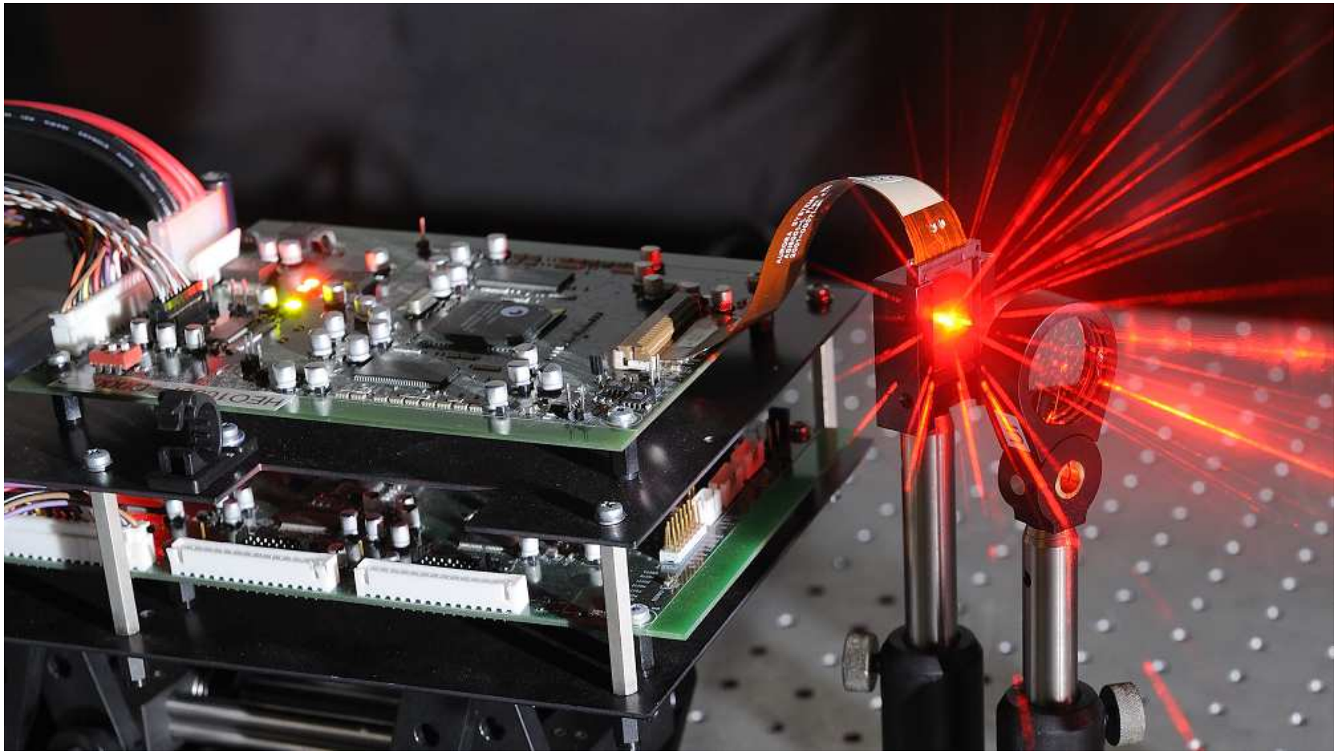
Quantum state preparation

Experimental setup to prepare and measure entangled photon states:



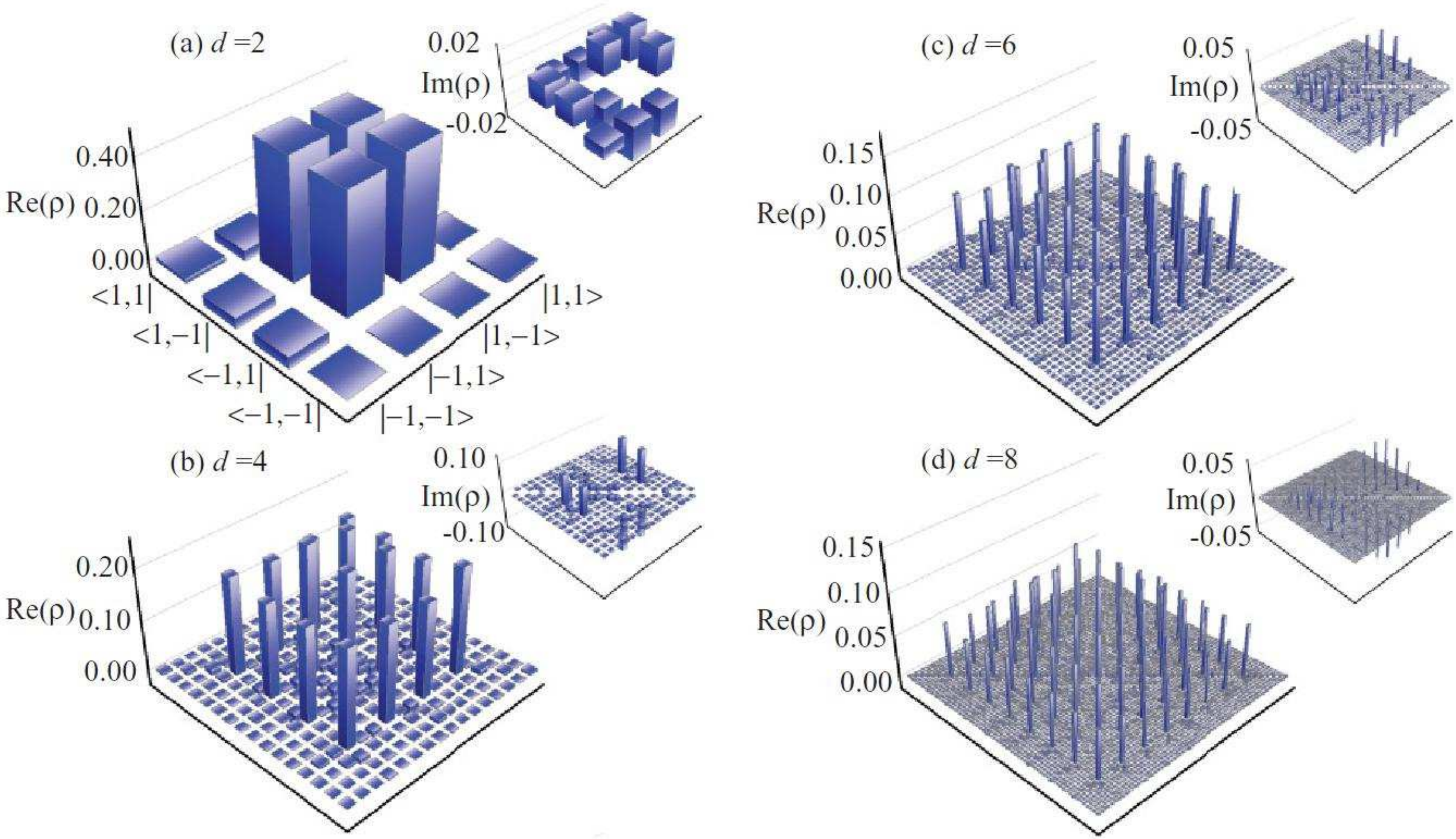
Spatial light modulators

Pure phase modulation or complex amplitude modulation



Quantum state tomography

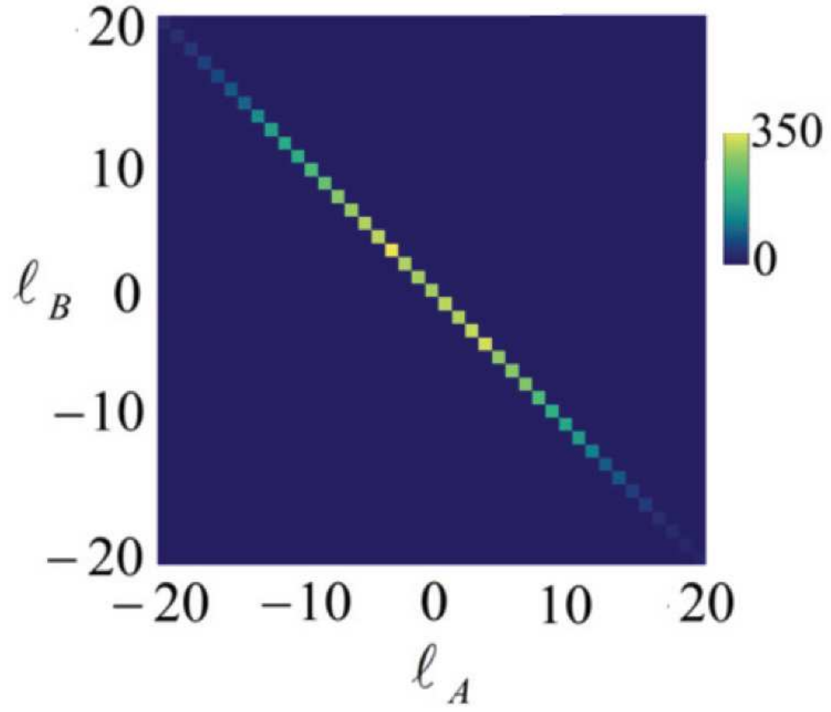
To reconstruct density matrix $\rho_{mn} = \langle \phi_m | \rho | \phi_n \rangle$
for density operator $\rho = \sum_n P_n |\psi_n\rangle\langle\psi_n|$

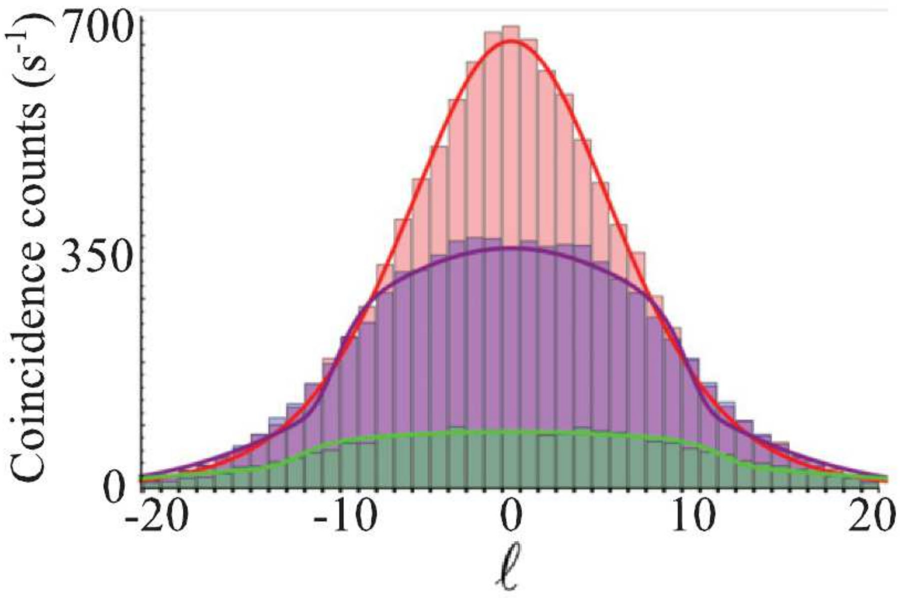


Spiral bandwidth

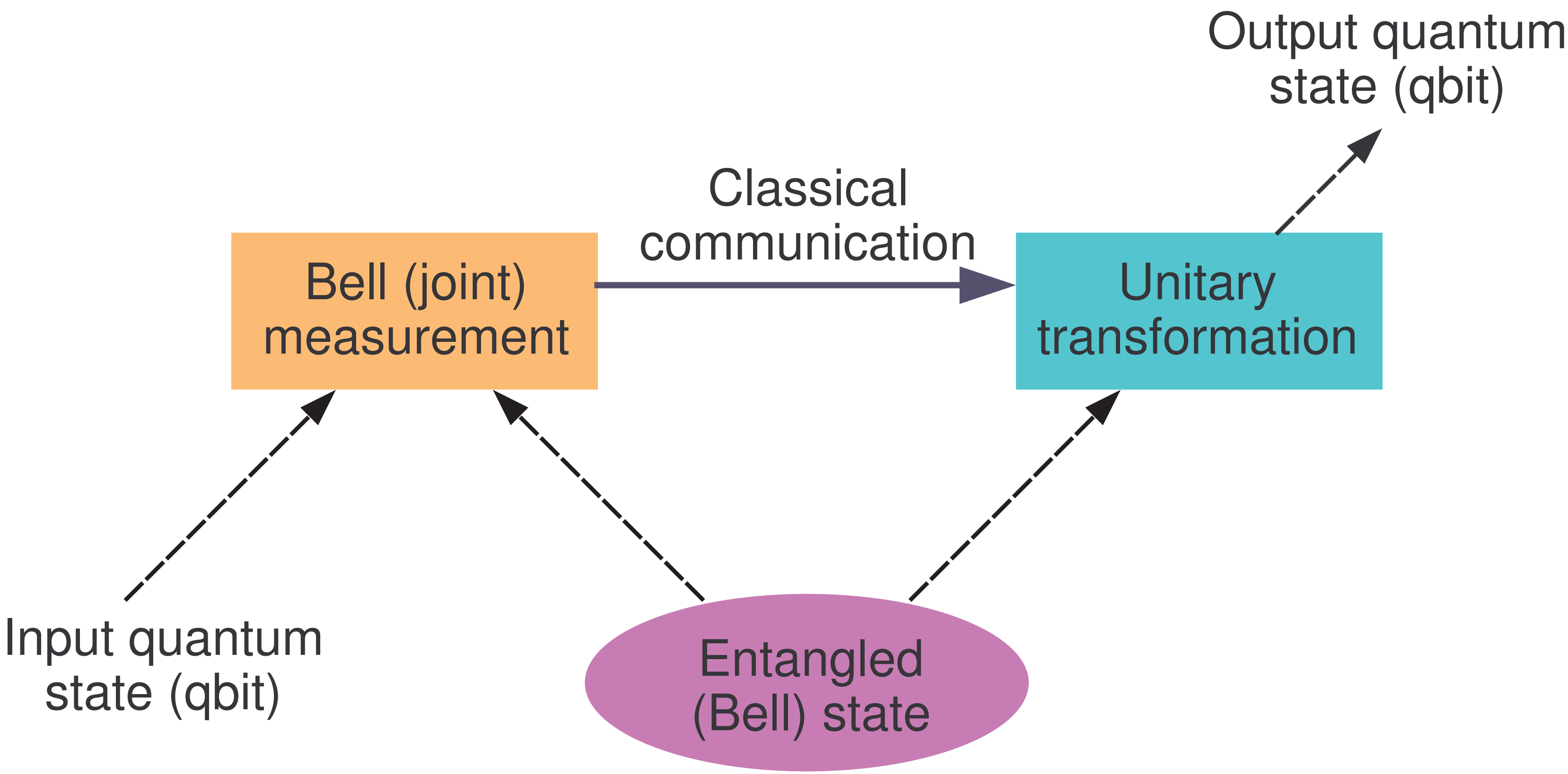
Orbital angular momentum (OAM) ($\propto$ azimuthal index) is conserved in SPDC $\Rightarrow$ OAM entanglement

High dimensional entanglement $\rightarrow$ broad OAM spectrum

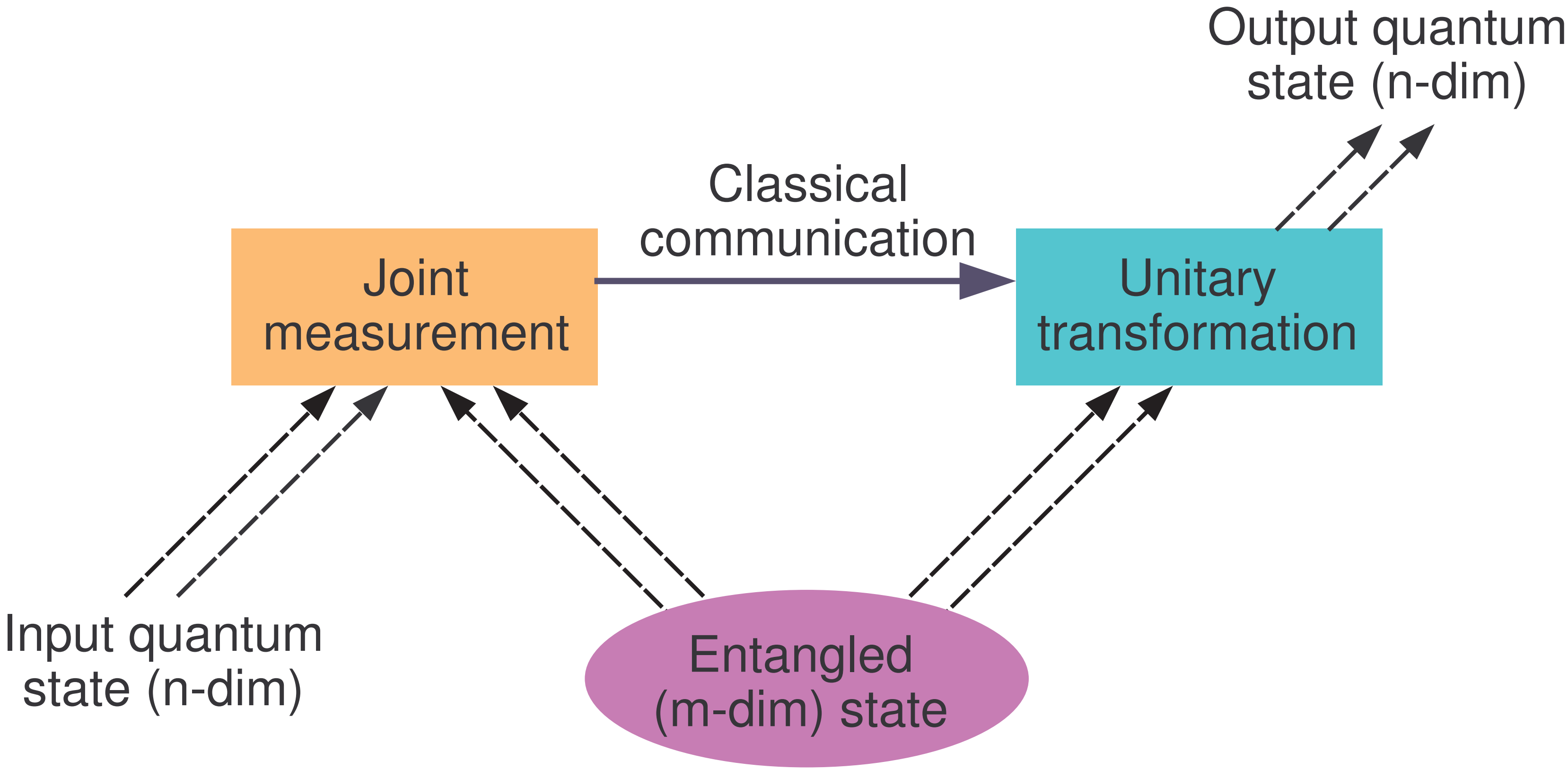




Quantum teleportation (2-dim)



Quantum teleportation (n-dim)

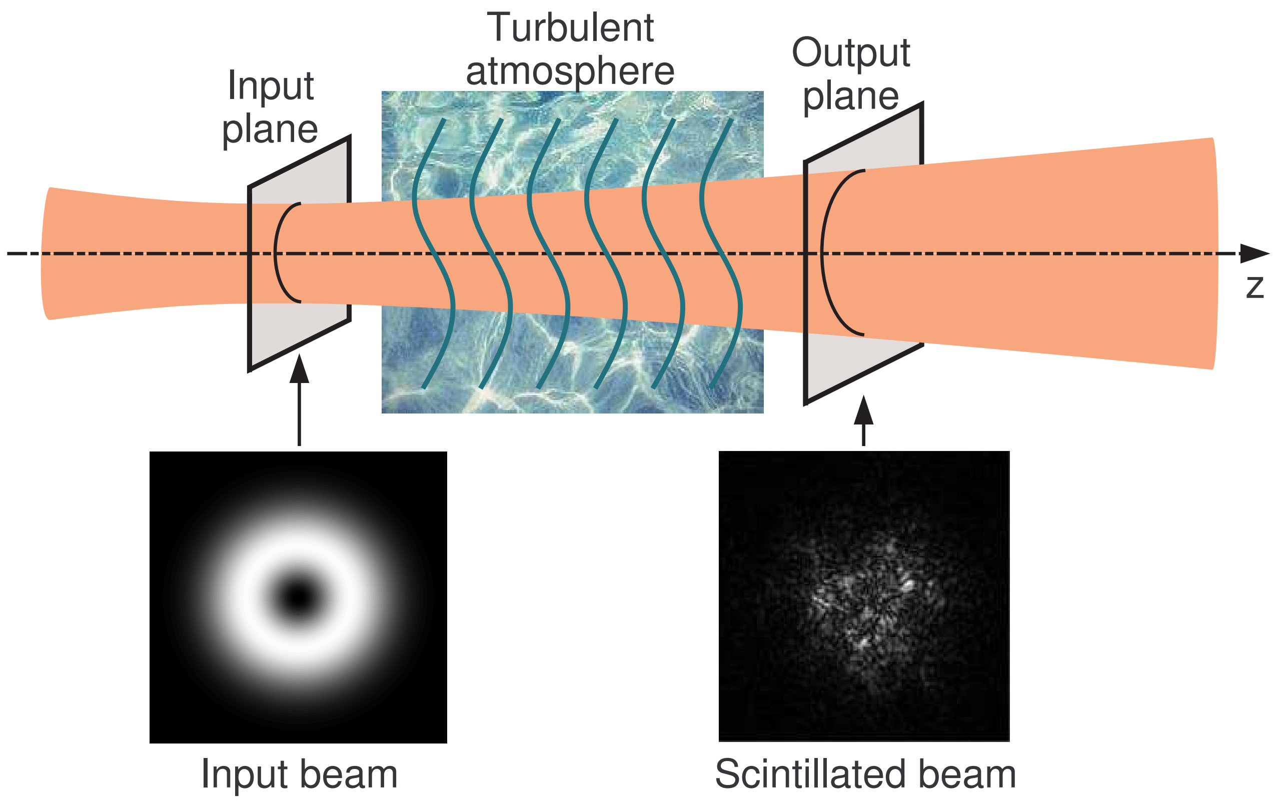


Turbulence vs Scintillation

Turbulence: velocity distribution in fluid

Refractive index: $n(\mathbf{r}) = 1 + \delta n(\mathbf{r})$

Scintillation: what happens to light in turbulence
Random phase modulations + diffraction.



Kolmogorov model

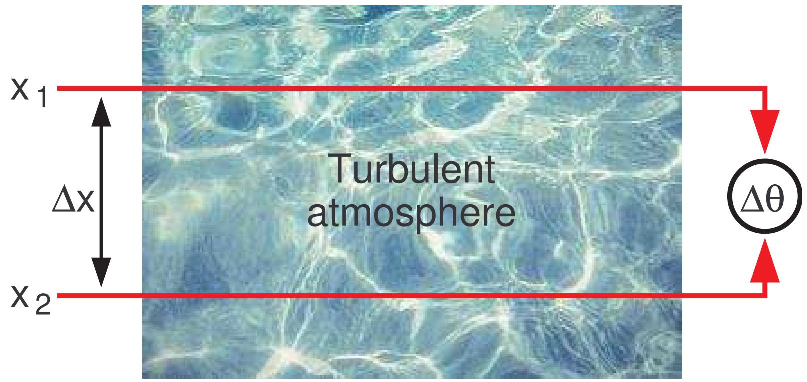
Refractive index structure function: [a]

$$D_n = \langle [\delta n(\mathbf{r}_1) - \delta n(\mathbf{r}_2)]^2 \rangle = C_n^2 (|\mathbf{r}_1 - \mathbf{r}_2|)^{2/3}$$

C_n^2 — Refractive index structure constant

Power spectral density:

$$\Phi_n(\mathbf{k}) = 0.033 C_n^2 |\mathbf{k}|^{-11/3}$$



Phase structure function:

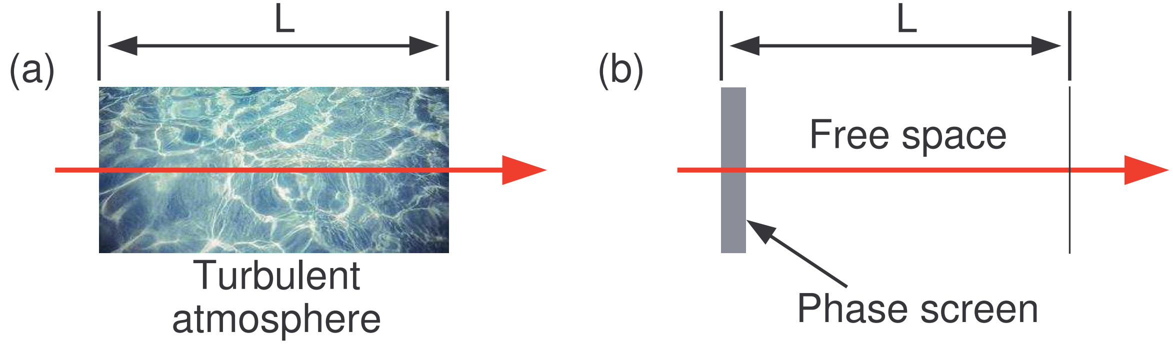
$$D_\theta(d) = \langle [\theta(x_1, y_1) - \theta(x_2, y_2)]^2 \rangle = 6.88 \left(\frac{d}{r_0}\right)^{5/3}$$

where $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
and Fried parameter (distance): $r_0 = 0.185 \left(\frac{\lambda^2}{C_n^2 z}\right)^{3/5}$

[a] LC Andrews and RL Phillips, *Laser beam propagation through random media*, 2nd ed. SPIE Press (2005)

Single phase screen

Assuming weak scintillation (only affects the phase)[a]



Use single phase screen with single parameter
(r_0 — Fried parameter):

$$\rho_{mn}(z) = \iint E_m^*(\mathbf{r}_1) E_n(\mathbf{r}_2) \psi(\mathbf{r}_1) \psi^*(\mathbf{r}_2) \times \exp\left[-\frac{1}{2} D_\theta\left(|\mathbf{r}_1 - \mathbf{r}_2|\right)\right] \mathrm{d}^2 r_1 \, \mathrm{d}^2 r_2$$

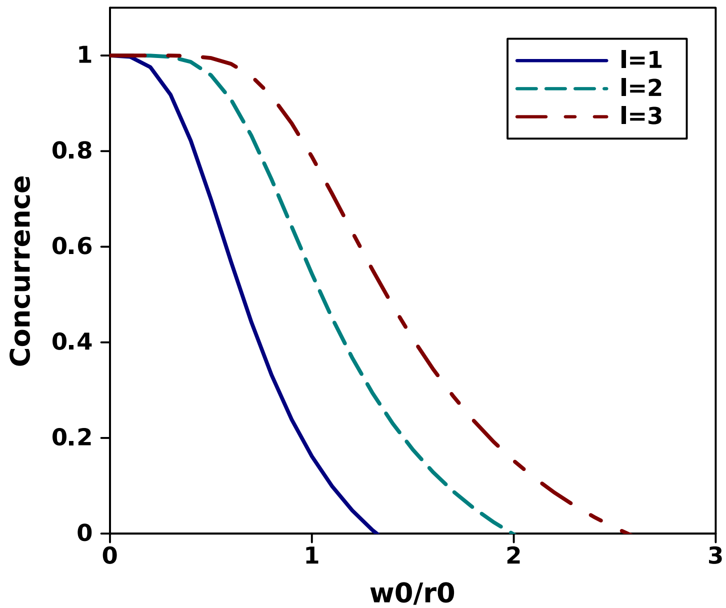
Function of only w_0/r_0 (contains all parameters including z)
Evaluated at $z = 0$

[a] C. Paterson, Phys. Rev. Lett., **94**, 153901 (2005)

Entanglement decay

Decay of qubit OAM entanglement (concurrence $\mathcal{C}$) in turbulence[a] use quadratic structure function approximation:

$$D \sim \left(\frac{x}{r_0}\right)^{5/3} \to \left(\frac{x}{r_0}\right)^{2}$$



Observations:

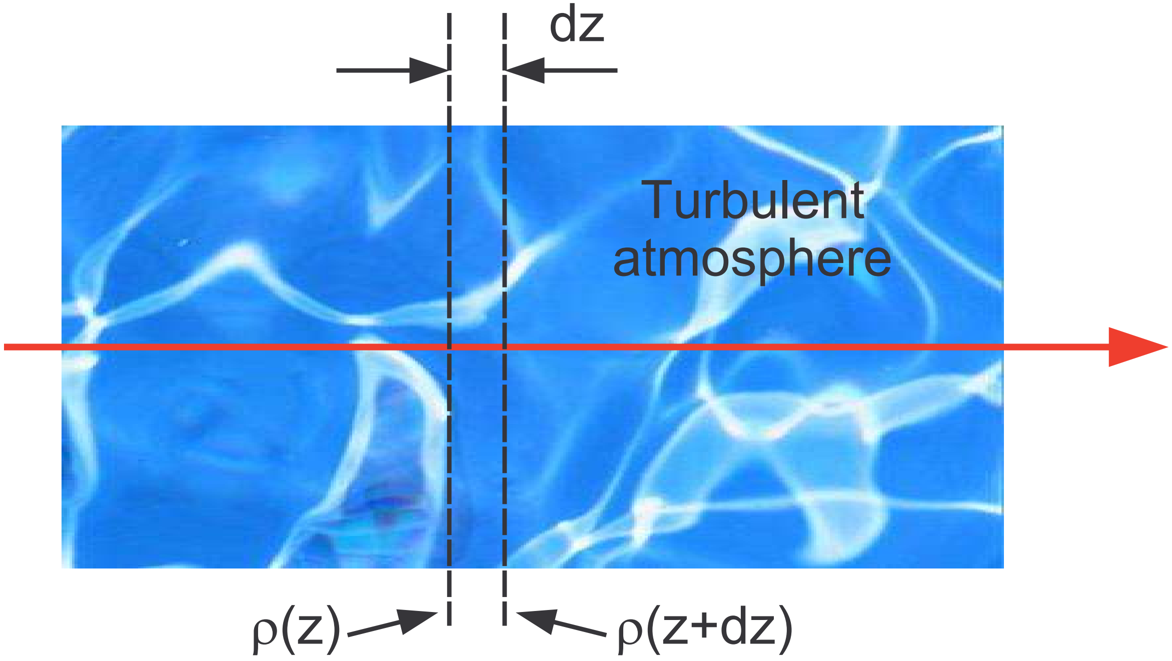
- ▷ Concurrence decays as function of w_0/r_0 only
- ▷ Decays to zero (sudden death) at $w_0/r_0 \approx 1$
- ▷ Last longer for larger azimuthal indices

[a] B.J. Smith and M.G. Raymer, Phys. Rev. A, **74**, 062104 (2006)

Infinitesimal propagation

Propagate over infinitesimal distance

Instead of going from 0 to z in 1 step,
we proceed in many small steps of dz



Infinitesimal propagation equation

Infinitesimal propagator equation (IPE):[a]

$$\partial_z \rho_{mnpq} = i \left(\mathcal{P}_{mx}\rho_{xnpq} - \rho_{mxpq}\mathcal{P}_{xn} + \mathcal{P}_{px}\rho_{mnxq} - \rho_{mnpx}\mathcal{P}_{xq} \right) + \Lambda_{mnxy}\rho_{xypq} + \Lambda_{pqxy}\rho_{mnxy} - 2\Lambda_T \rho_{mnpq}$$

$$\rho = \sum_{m,n} |m\rangle|p\rangle \ \rho_{mnpq} \ \langle n|\langle q|$$

$$\mathcal{P}_{mp}(z) = \frac{1}{2k} \int |\mathbf{a}|^2 G_m^*(\mathbf{a}, z) G_p(\mathbf{a}, z) \ \mathrm{d}^2 a$$

$$\Lambda_{mnpq} = k^2 \int W_{mp}^*(\mathbf{a}, z) W_{nq}(\mathbf{a}, z) \Phi_0(\mathbf{a}, 0) \ \mathrm{d}^2 a$$

$$W_{mn}(\mathbf{a}, z) = \int G_m(\mathbf{a}' + \mathbf{a}, z) G_n^*(\mathbf{a}', z) \ \mathrm{d}^2 a'$$

$$\Lambda_T = k^2 \int \Phi_0(\mathbf{a}, 0) \ \mathrm{d}^2 a$$

Properties of the IPE

- ▷ Derived in Fourier domain
  Based on power spectral density: $\Phi_n(\mathbf{k})$
- ▷ The resulting density matrix is hermitian
  Follows from identity: $\Lambda_{mnpq} = \Lambda^*_{nmqp}$
- ▷ Expressible as Master equation in Lindblad form
  (However z-derivative and not time-derivative)
  $\Rightarrow$ valid density matrix
- ▷ Transverse spatial modes
  $\rightarrow$ infinite dimensional Hilbert space
  $\Rightarrow$ IPE is an infinite set of coupled differential equations
- ▷ To solve them one needs to truncate the set
  $\Rightarrow$ truncated IPE is not trace preserving: $\mathrm{tr}\{\rho\} \leq 1$

Example: symmetric qubits

For initial state: $(|\ell,-\ell\rangle - |-\ell,\ell\rangle)/\sqrt{2}$, density matrix:

$$\rho_{mnpq} = \frac{T}{4}\begin{bmatrix} 1-R^2 & 0 & 0 & 0 \\ 0 & 1+R^2 & -2R & 0 \\ 0 & -2R & 1+R^2 & 0 \\ 0 & 0 & 0 & 1-R^2 \end{bmatrix}$$

where $$T = \exp\left[-\frac{127}{36}Z(t)\right] \qquad R = \exp\left[-\frac{5}{72}Z(t)\right]$$

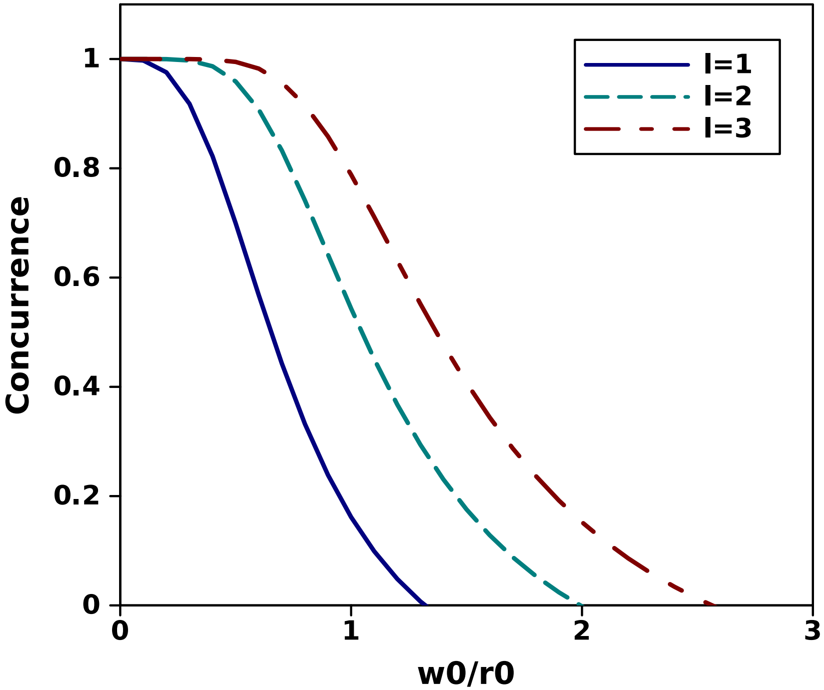
t-dependence: $$Z(t) \equiv \sigma \int_0^t (1+\tau^2)^{5/6}\, \mathrm{d}\tau \approx \sigma t$$

where $$\sigma = \frac{\pi^{3/2} C_n^2 w_0^{11/3}}{6\Gamma(2/3)\lambda^3}$$

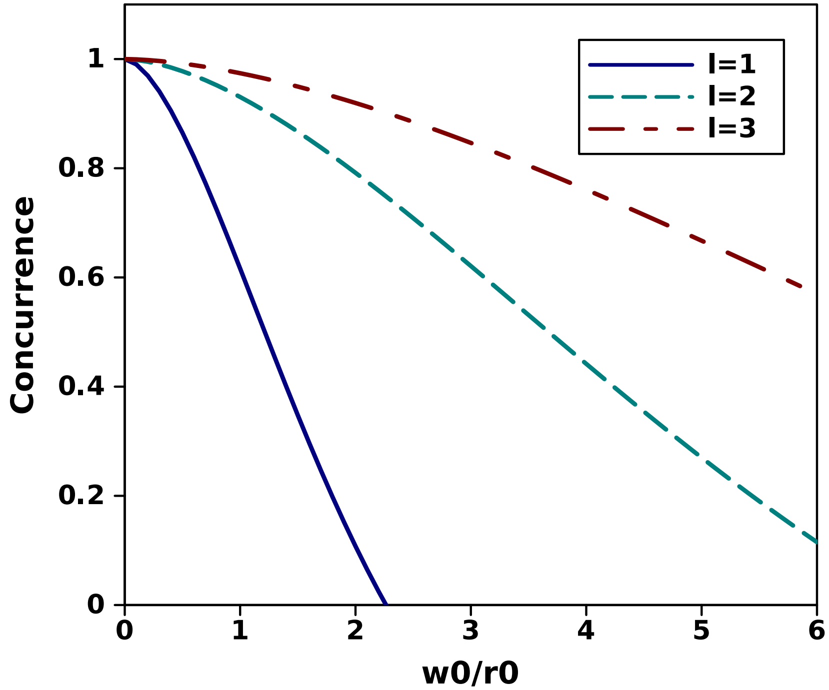
Concurrence: $$\mathcal{C} = \frac{1}{2}(R^2 + 2R - 1)$$

Comparison of results

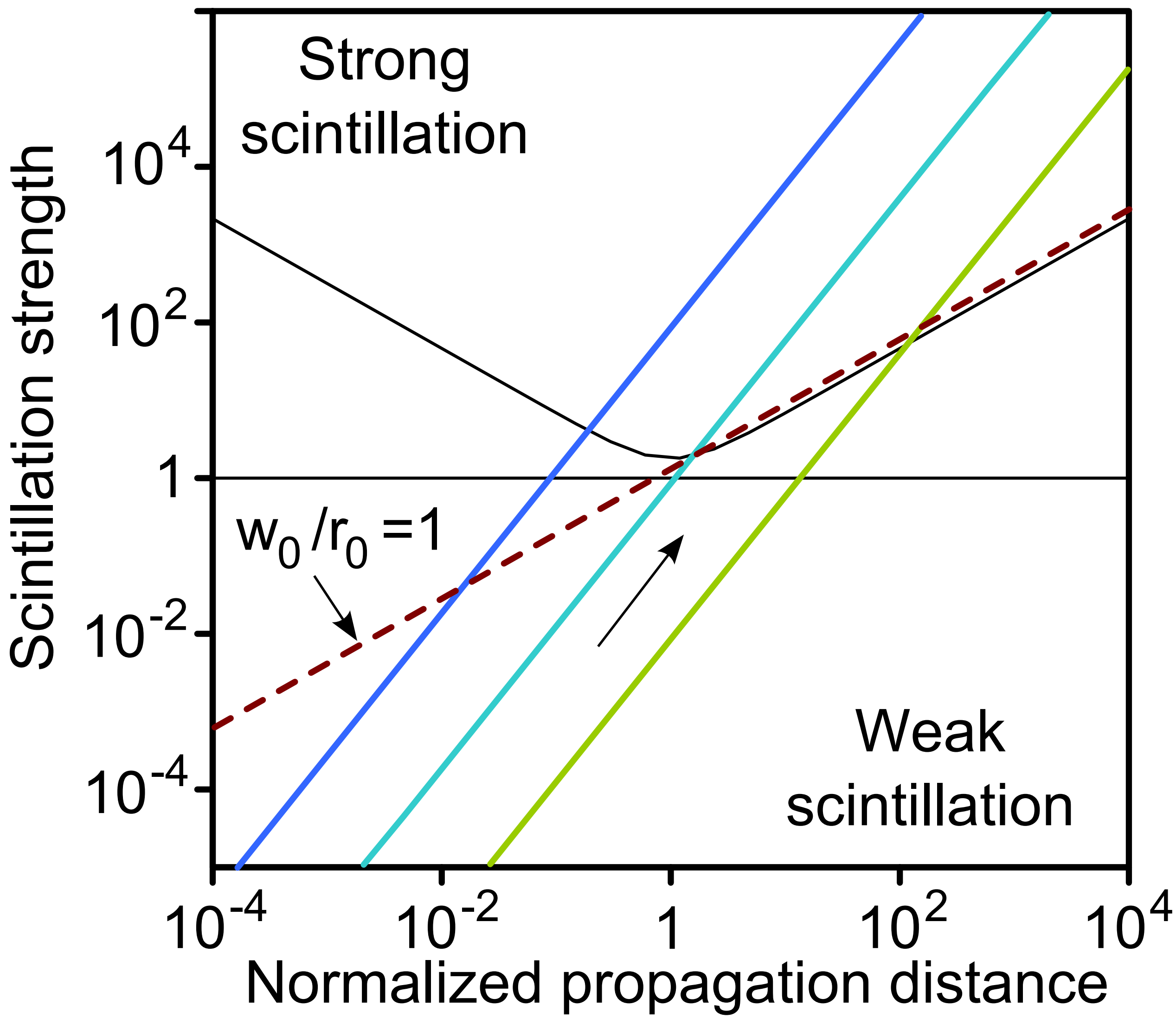
Single phase screen



IPE



Do we need the IPE?



Numerical simulations

General procedure:

- ▷ Prepare input state
- ▷ Split-step method:
  - — Multiply mode by random phase function
  - — Propagate through free-space (without turbulent)
- ▷ Extract density matrix
- ▷ Compute concurrence

Input state

Bell state: $$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\ell\rangle_A\,|-\ell\rangle_B + |-\ell\rangle_A\,|\ell\rangle_B\right)$$

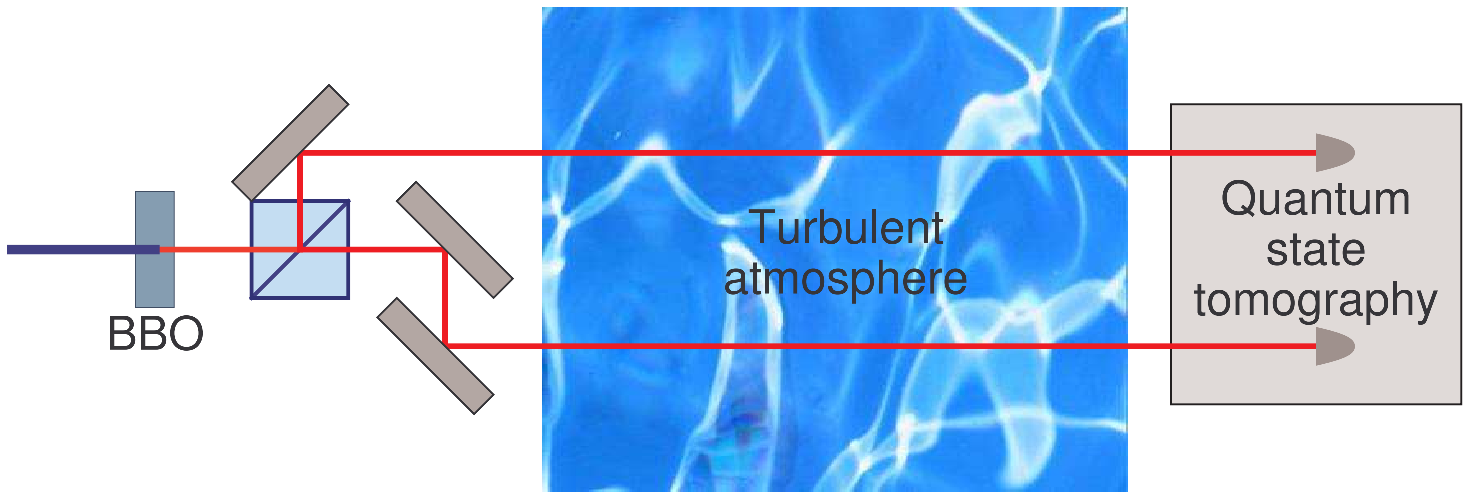
4 modes separately propagated through turbulence

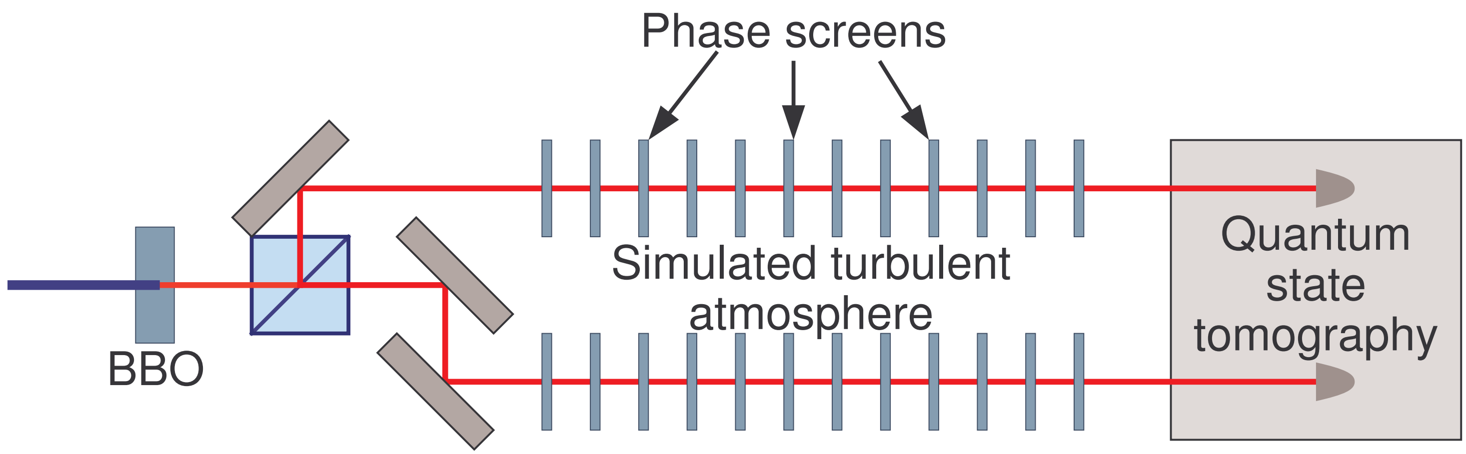
$|\ell\rangle$, $|-\ell\rangle$ — LG modes at the waist ($z = 0$) with $p = 0$

For $\ell = 1$: $$U_{01}^{(LG)} = 2\sqrt{\pi}r\exp(\mathrm{i}\phi)L_0^1(2r^2)\exp(-r^2)$$

r — normalized radial coordinate
ϕ — azimuthal angle
$L_0^1(\cdot)$ — associated Laguerre polynomial

Split-step method



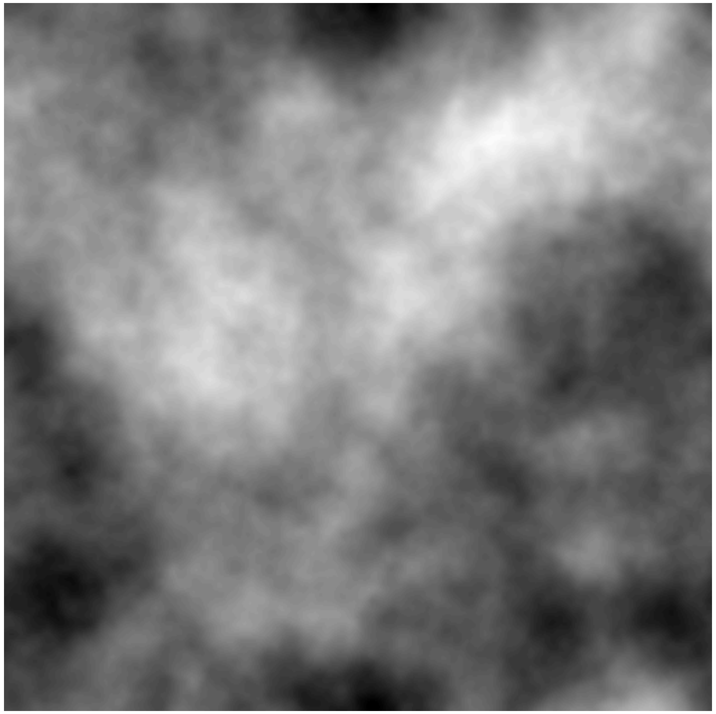


Random phase screens

Phase function: $$\theta(x,y) = \frac{k_0}{\Delta_k}\sqrt{\frac{\Delta z}{2\pi}}\mathcal{F}^{-1}\left\{\tilde{\chi}_n(\mathbf{K})\left[\Phi_0(\mathbf{K},0)\right]^{1/2}\right\}$$

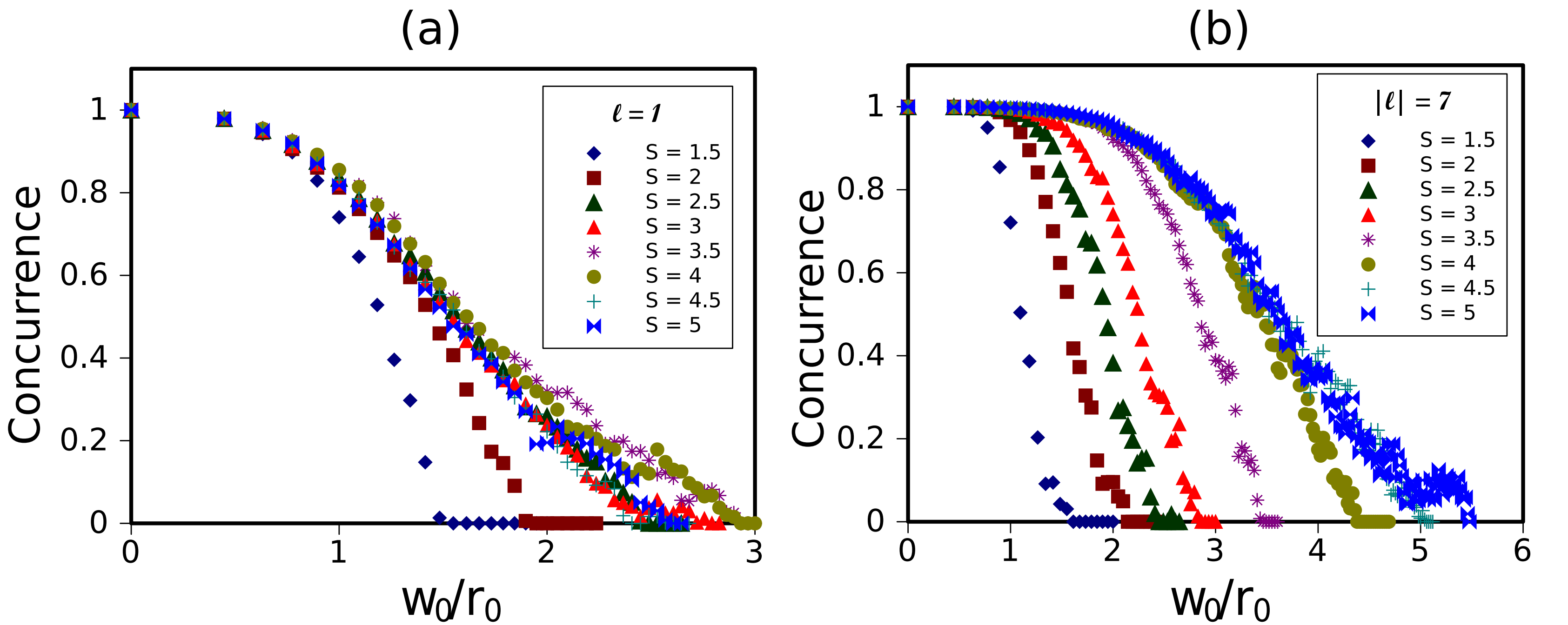
$\tilde{\chi}_n(\mathbf{K})$ — random complex spectral function:

- delta correlated: $\langle\tilde{\chi}(\mathbf{K}_1)\tilde{\chi}^*(\mathbf{K}_2)\rangle = (2\pi\Delta_k)^2\,\delta_2(\mathbf{K}_1 - \mathbf{K}_2)$
- normally distributed
- zero mean
- Δ_k sample spacing in frequency domain ($\sim$ 1/outer scale)

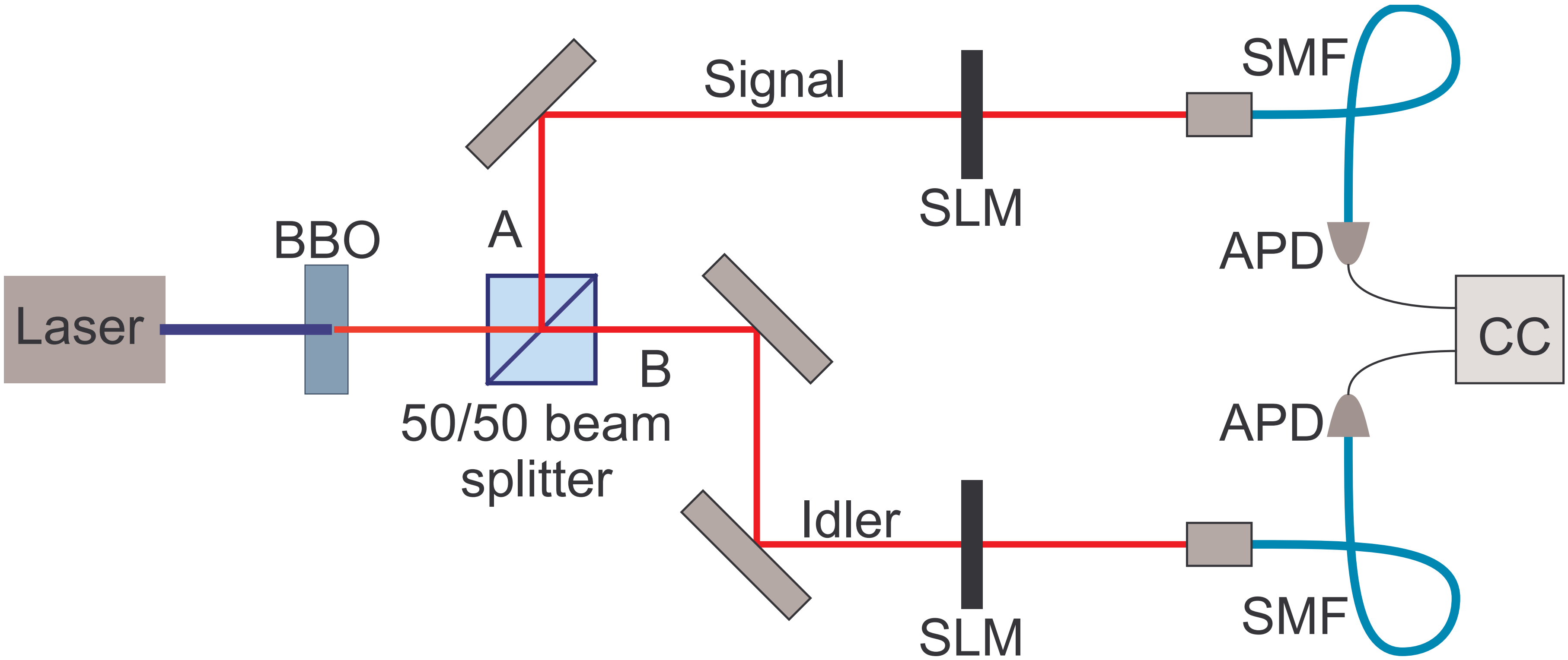


Numerical of results

$$S = \log_{10}\left(\frac{\pi^3 C_n^2 w_0^{11/3}}{\lambda^3}\right)$$

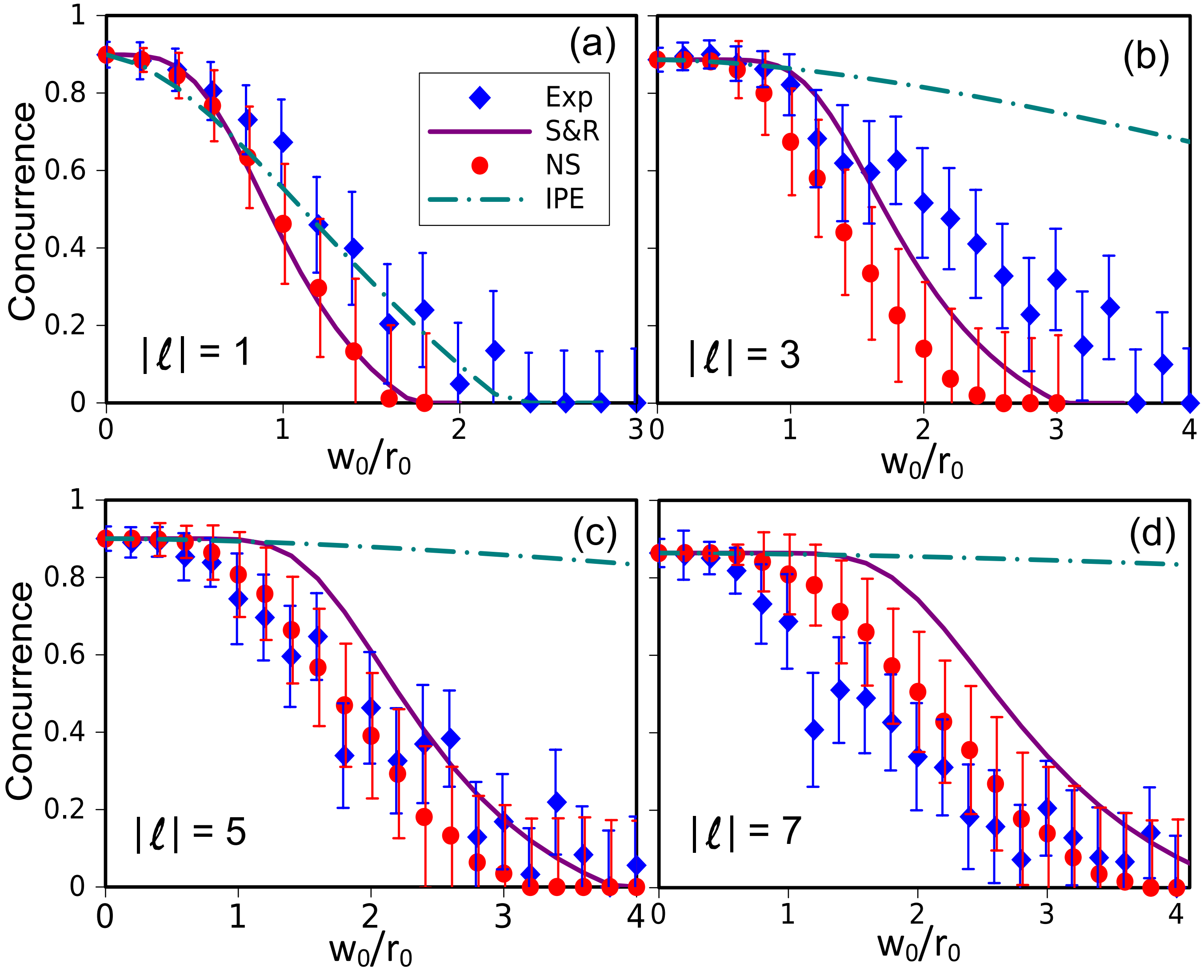


Experimental setup



Comparison of results

Qubit (Bell state) — both photons through turbulence:



Decay distance

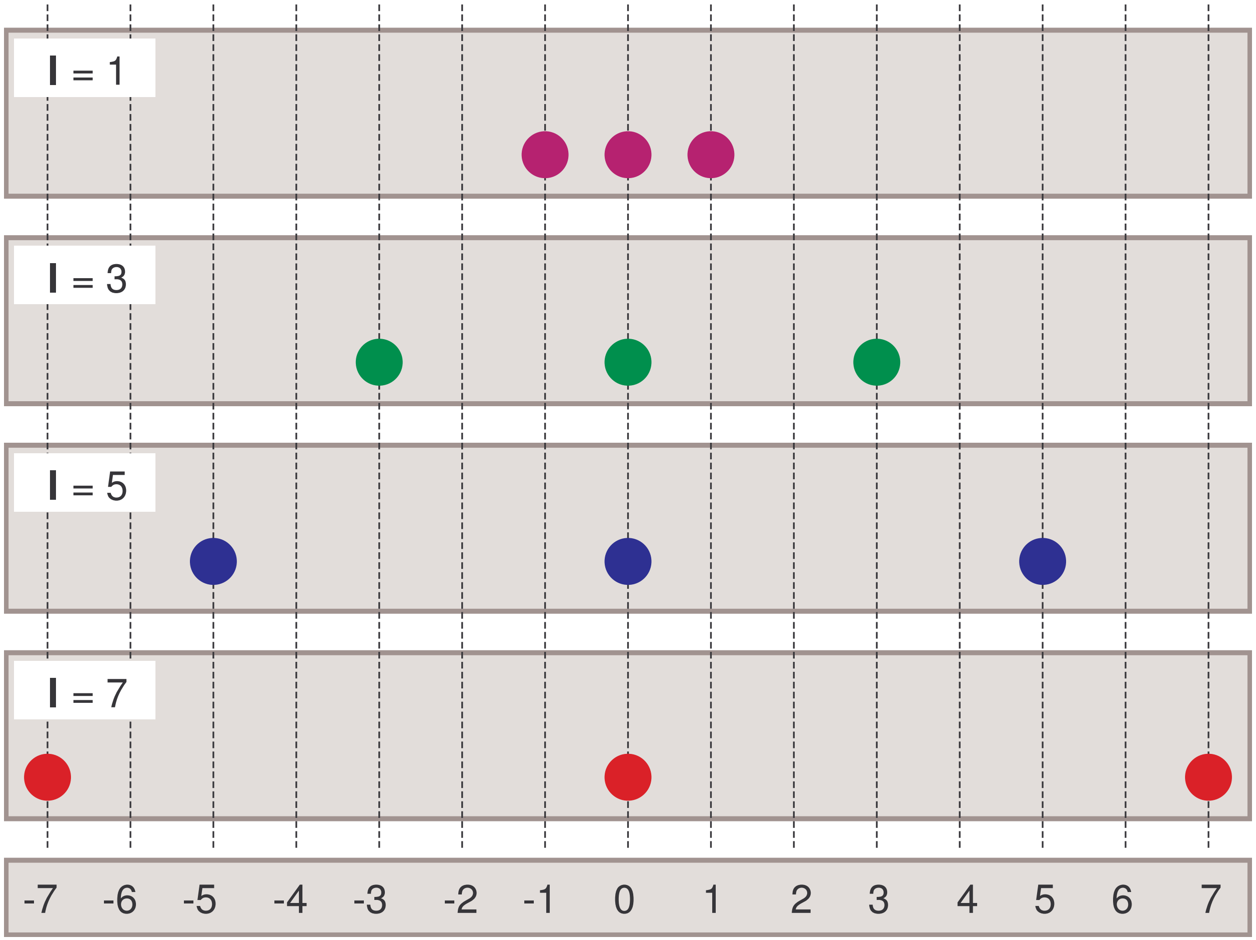
Distance scale for entanglement decay:

$$L_{\text{dec}}(\ell) = \frac{0.06\lambda^2 \ell^{5/6}}{w_0^{5/3} C_n^2}$$

For $w_0 = 10$ cm, $\lambda = 1550$ nm and $C_n^2 = 10^{-15}$ m$^{-2/3}$:

ℓ	L_{dec}
1	6.7 km
3	16.7 km
5	25.6 km
7	33.7 km

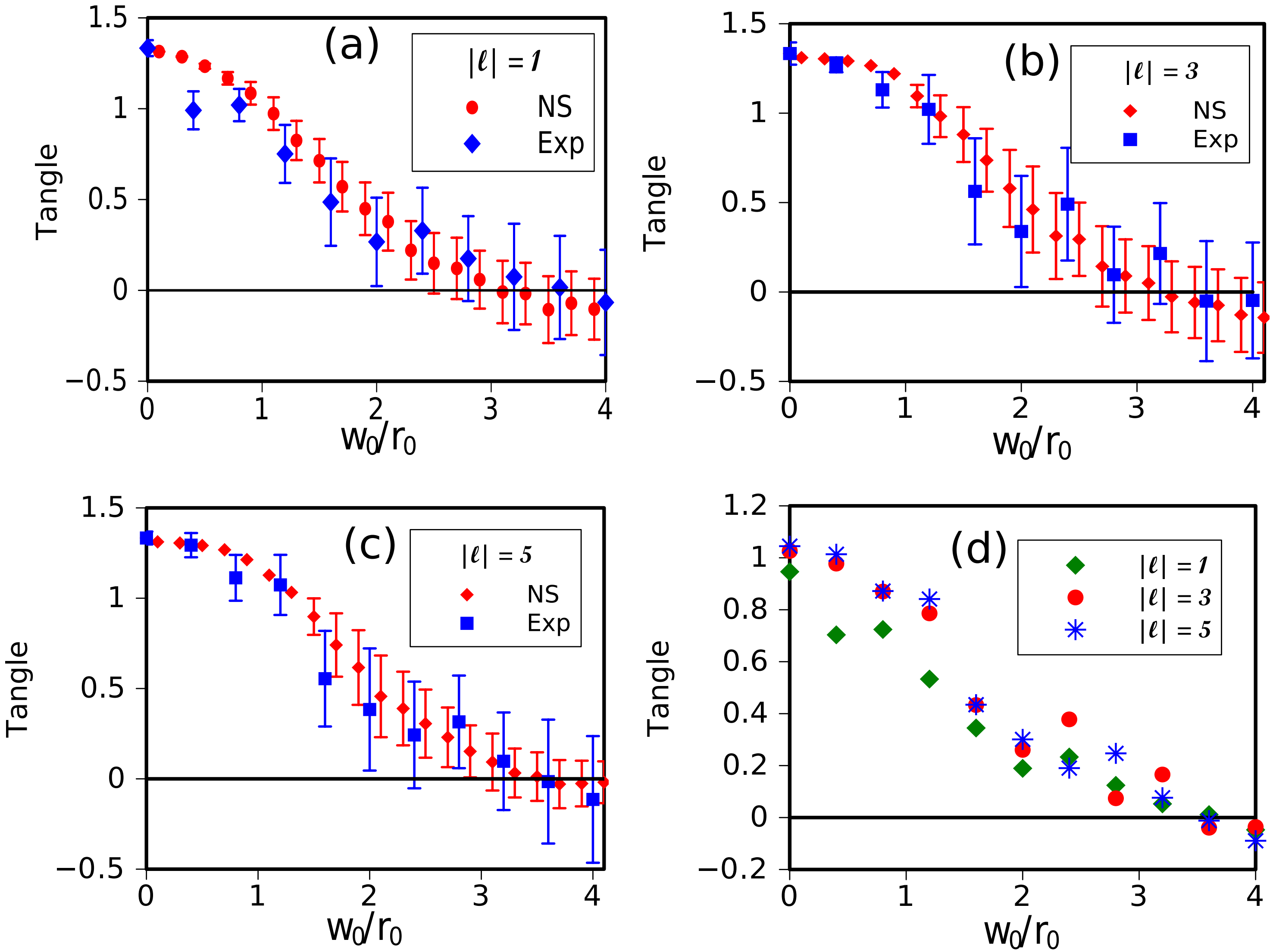
Higher dimensional states



Tangle (Lower bound for entanglement):

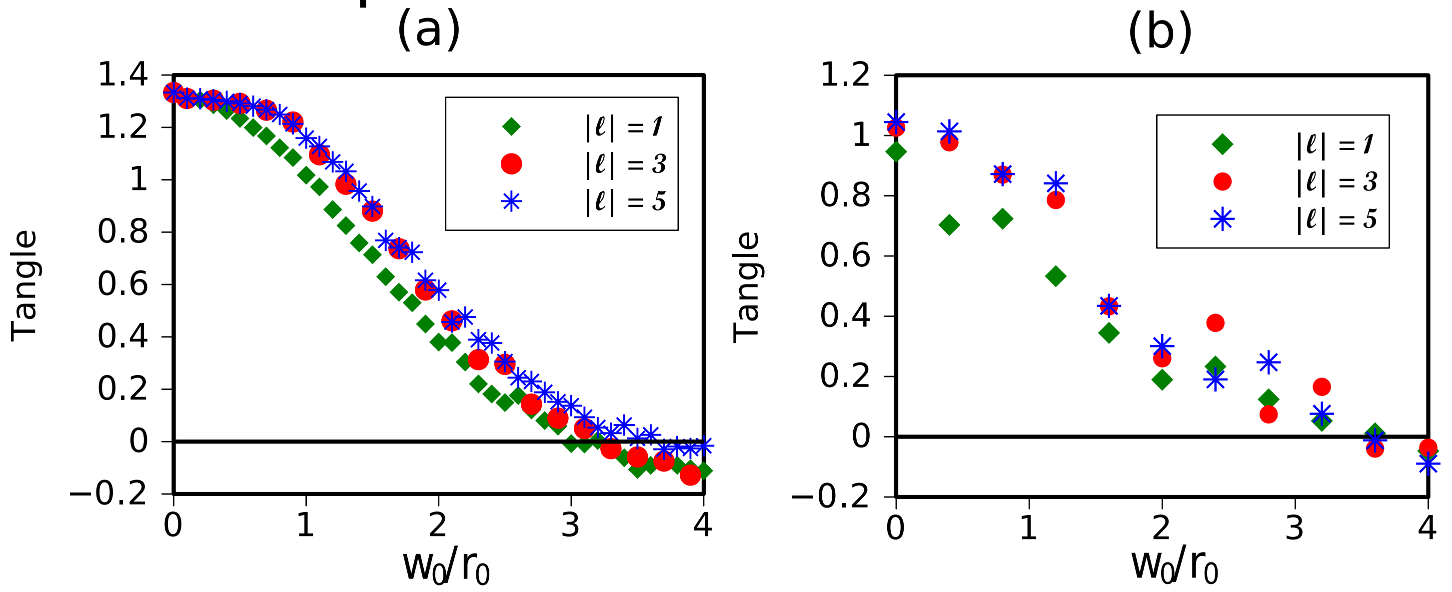
$$\tau\{\rho\} = 2\mathrm{tr}\{\rho^2\} - 2\mathrm{tr}\{\rho_R^2\} \qquad \max(\tau) = \frac{2(d-1)}{d}$$

Experimental results

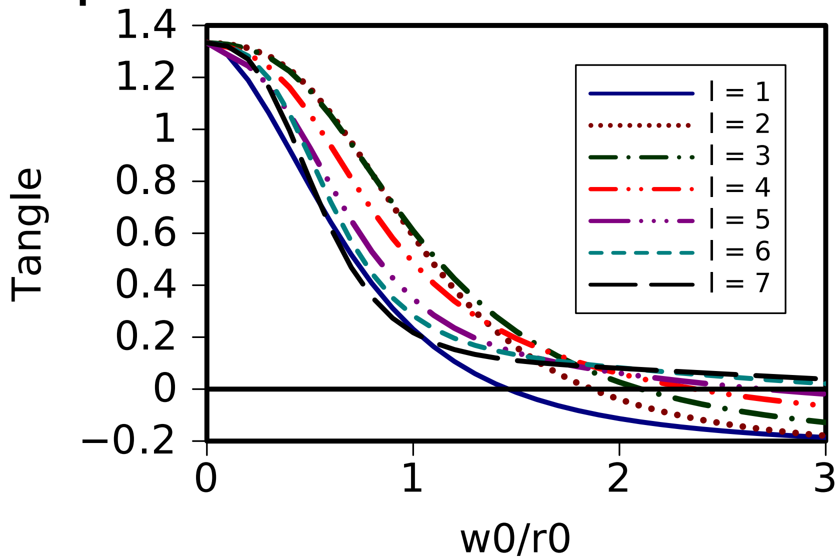


Comparison of results

Numerical and experimental data:



Theoretical single phase screen calculations:



Conclusions

- Quantum communication, which enables fundamentally secure communication, is a new technology that is actively being develop by international research groups
- Quantum communication requires various other technologies:
  - Quantum state preparation
  - Quantum teleportation
  - etc.
- Free-space quantum communication suffers decay of entanglement due to turbulence in the atmosphere
- By studying the effect of scintillation on entanglement we can determine design constraint for free-space quantum communication