How optics resolved the Einstein-Podolsky-Rosen paradox

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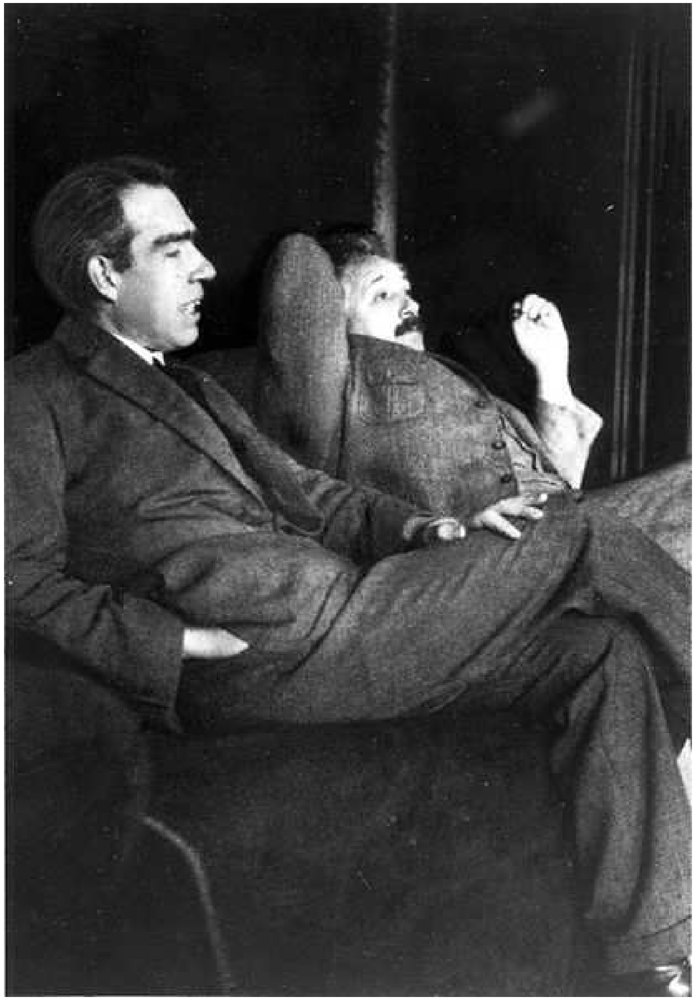
Einstein-Bohr debate

Quantum mechanics:

- ▷ Wave-particle duality
- ▷ Hilbert space
- ▷ Quantum superposition
- ▷ Heisenberg uncertainty

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



Albert Einstein
and Neils Bohr
debating quantum
mechanics

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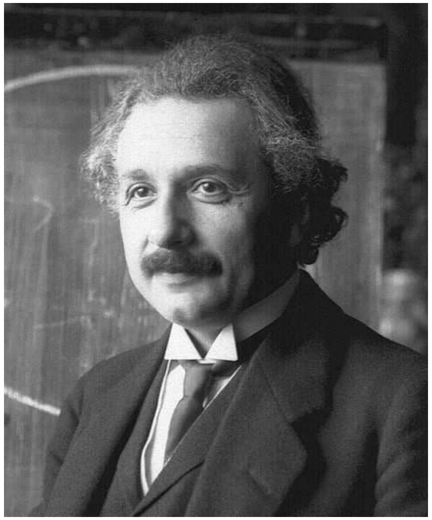
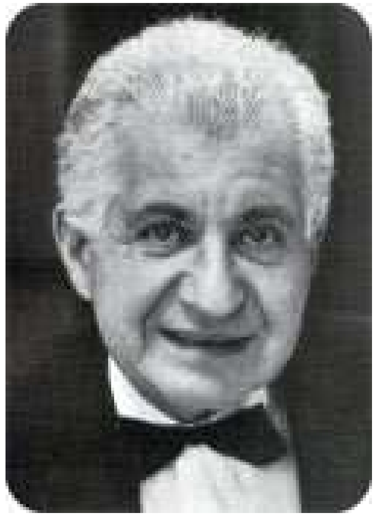
Einstein-Podolsky-Rosen

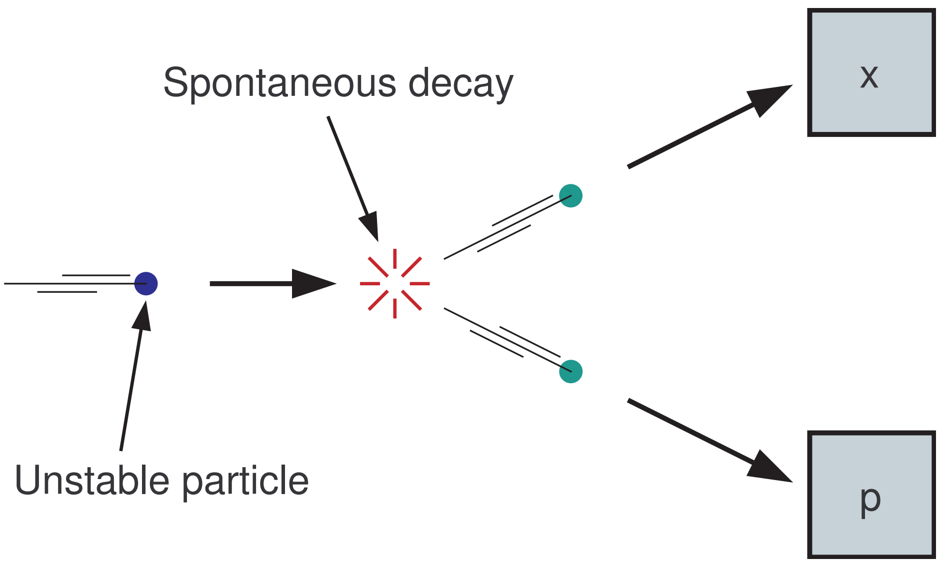


Albert Einstein



Boris Podolsky

Nathan Rosen



Quantum mechanics:
measurements on one particle <u>dictate</u> the state of the other particle.

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Bell's inequality

John S. Bell

Assumptions:

- ▷ Only local interactions
  (no "spooky action at a distance")
- ▷ Unique reality
  (not multiple realities)

Then:

$$B(a,b,a',b') = \left|C(a,b) - C(a,b')\right| + \left|C(a',b) + C(a',b')\right| \leq 2$$

where C is the correlation between two measurements:

$$C(a,b) = E\left\{A(a)B(b)\right\}$$

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4 possibilities

	Unique reality	Multiple realities
Local	Hidden variables (Classical theories)	Copenhagen interpretation
Non-local	Alternative interpretation	Undesirable

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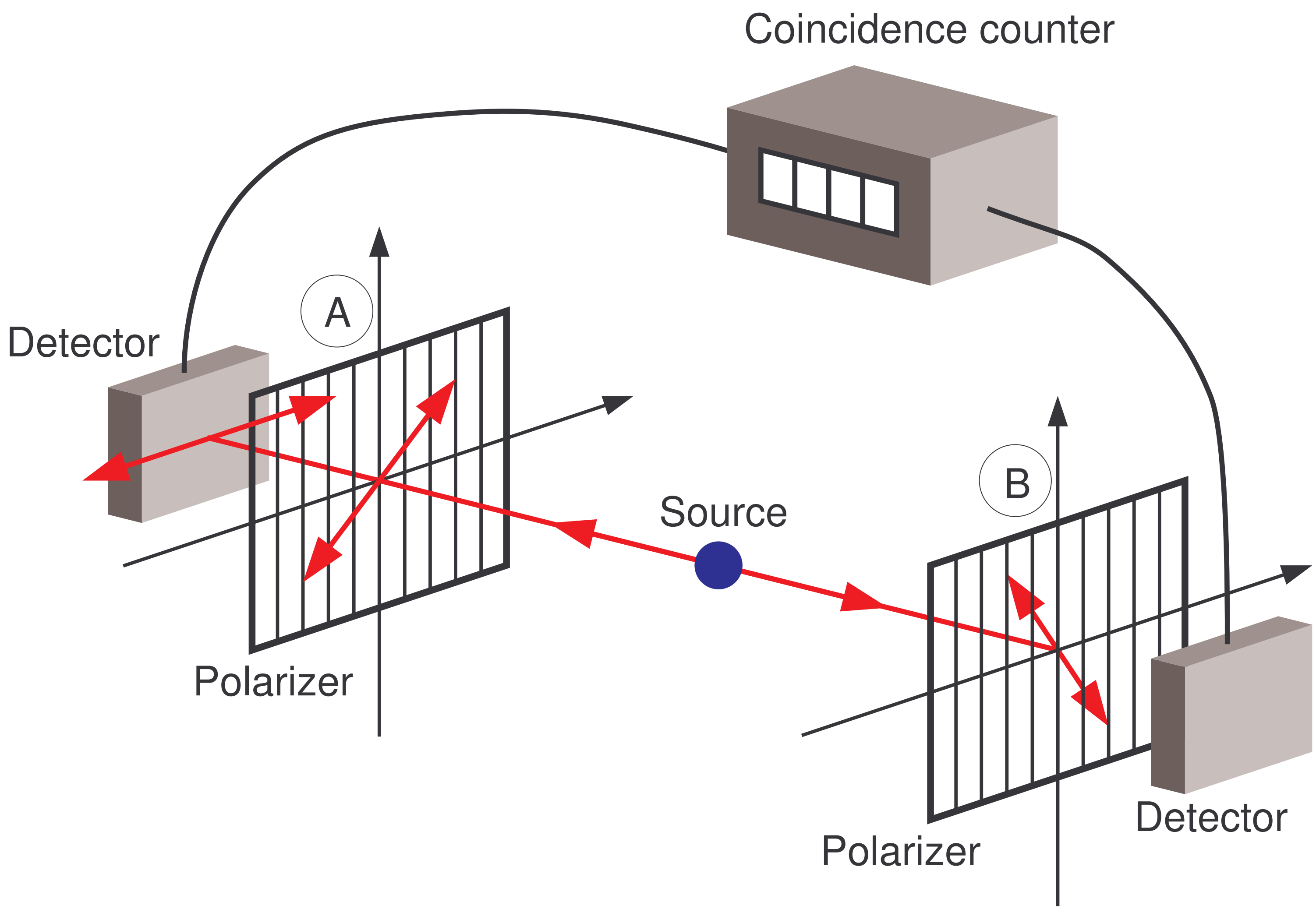
Aspect experiment



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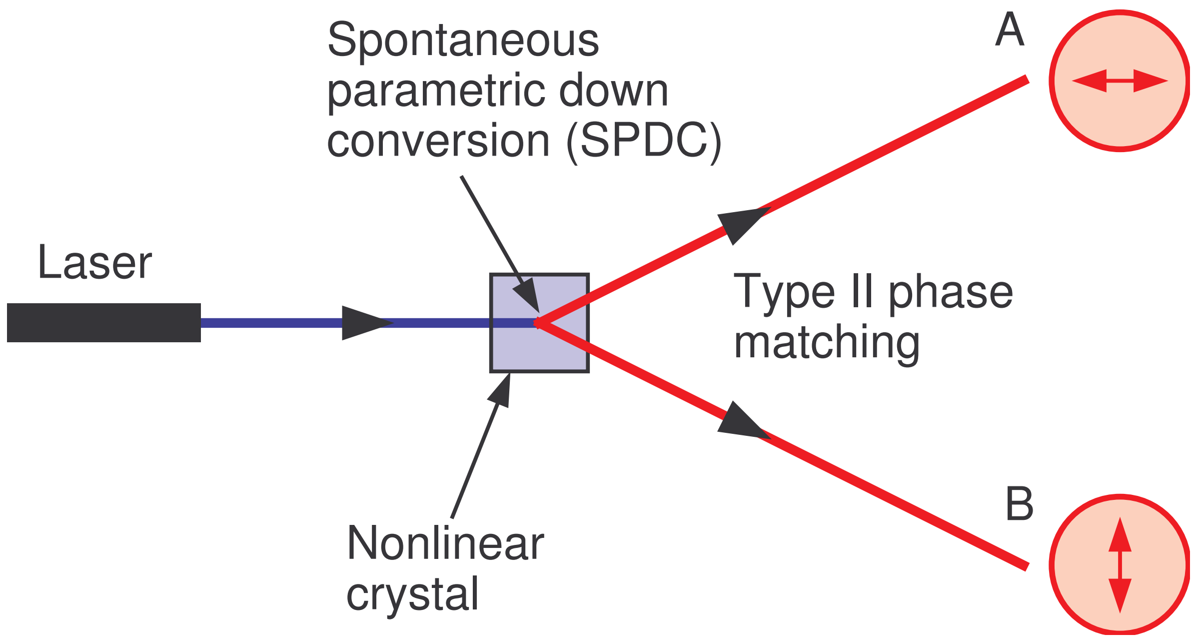
Parametric down conversion



Type II phase matching $\Rightarrow$ photons have perpendicular polarization: $\theta_B = \theta_A - \pi/2$

However, each beam is unpolarized
— contains all states of polarization.

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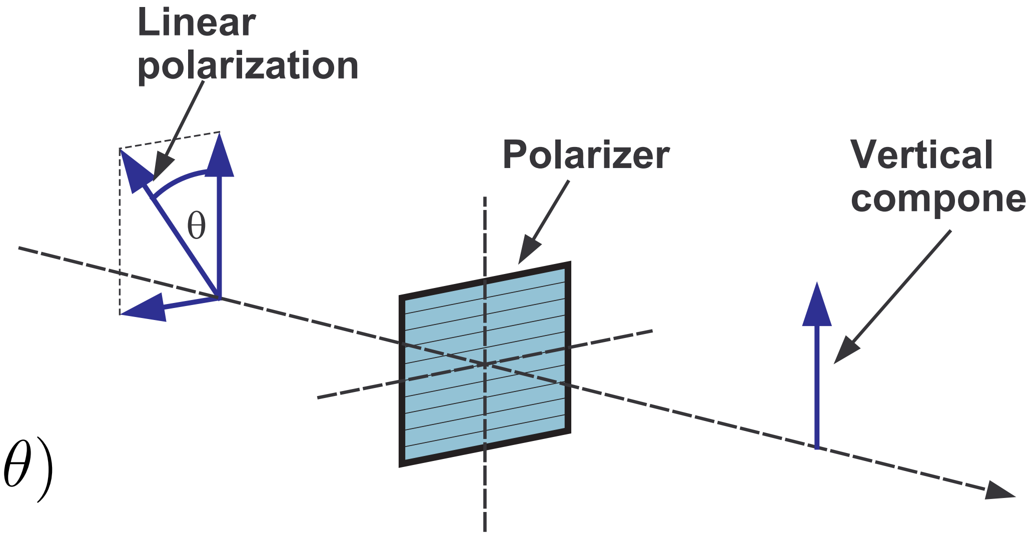
Dichroic polarizers

Before considering the calculations, we first need to review dichroic polarizers.



Intensity: $I_{ver} = I_{in} \cos^2(\theta)$

where θ is the angle between vertical polarization and the input polarization.

Probability to detect photon: $P(\theta) = \cos^2(\theta)$

Classical calculations

Orientation angles of the two polarizers: α_A and α_B

Probability to detect photons at respective detectors:

$$P(A|\theta_A, \alpha_A) = \cos^2(\theta_A - \alpha_A)$$

$$P(B|\theta_B, \alpha_B) = \cos^2(\theta_B - \alpha_B) = \sin^2(\theta_A - \alpha_B)$$

Detectors are far apart $\rightarrow$ assume the probabilities of detecting photons are statistically independent

$$\Rightarrow P(AB|\theta_A, \alpha_A, \alpha_B) = P(A|\theta_A, \alpha_A)P(B|\theta_A, \alpha_B)$$

$$= \frac{1}{4}\left[\sin(\alpha_A - \alpha_B) + \sin(2\theta_A - \alpha_A - \alpha_B)\right]^2$$

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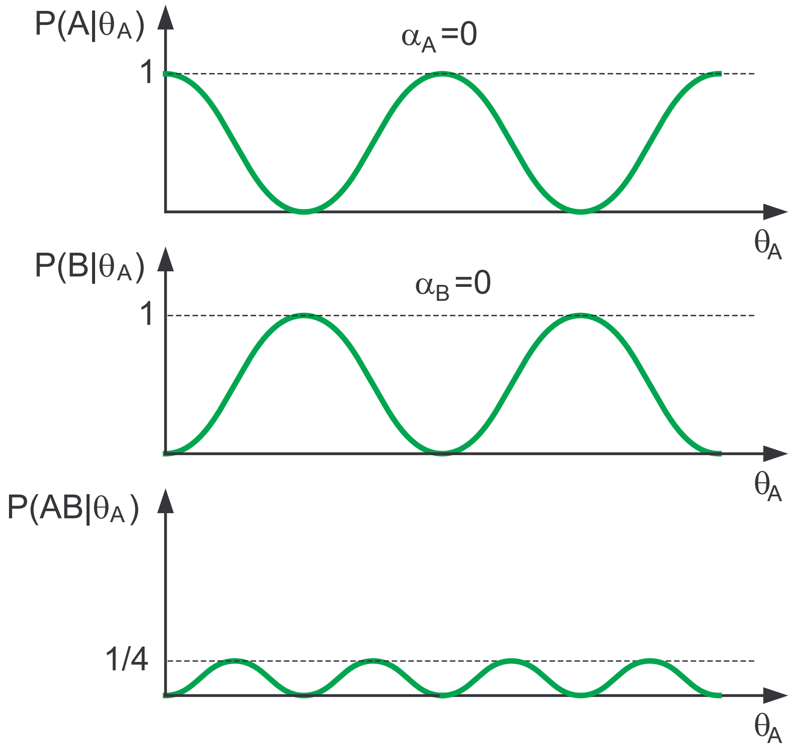
Classical case



For the same orientation: angle: ($\alpha_A = \alpha_B = \alpha$):

$$P(AB|\alpha,\alpha) = \frac{1}{8}$$

Integrate over all polarization states:

$$P(AB|\alpha_A,\alpha_B) = \frac{1}{2\pi}\int_0^{2\pi} P(AB|\theta_A,\alpha_A,\alpha_B)\,\mathrm{d}\theta_A$$

$$= \frac{1}{8} + \frac{1}{4}\sin^2(\alpha_A - \alpha_B)$$

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Quantum calculations

2-state maximally entangled bi-partite (2 photon) system:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|hor, ver\rangle - |ver, hor\rangle\right)$$

$$a_n = a_{n,hor}\cos\alpha_n + a_{n,ver}\sin\alpha_n \qquad n = A, B$$

$${a_n}^\dagger = {a_{n,hor}}^\dagger\cos\alpha_n + {a_{n,ver}}^\dagger\sin\alpha_n \qquad n = A, B$$

$$P(A, B|\alpha_A, \alpha_B) = \langle\psi|\, {a_A}^\dagger {a_B}^\dagger a_A a_B\,|\psi\rangle = \frac{1}{2}\sin^2(\alpha_A - \alpha_B)$$

So for $\alpha_A - \alpha_B = 0$ (same orientation) $P(A, B|\alpha, \alpha) = 0$

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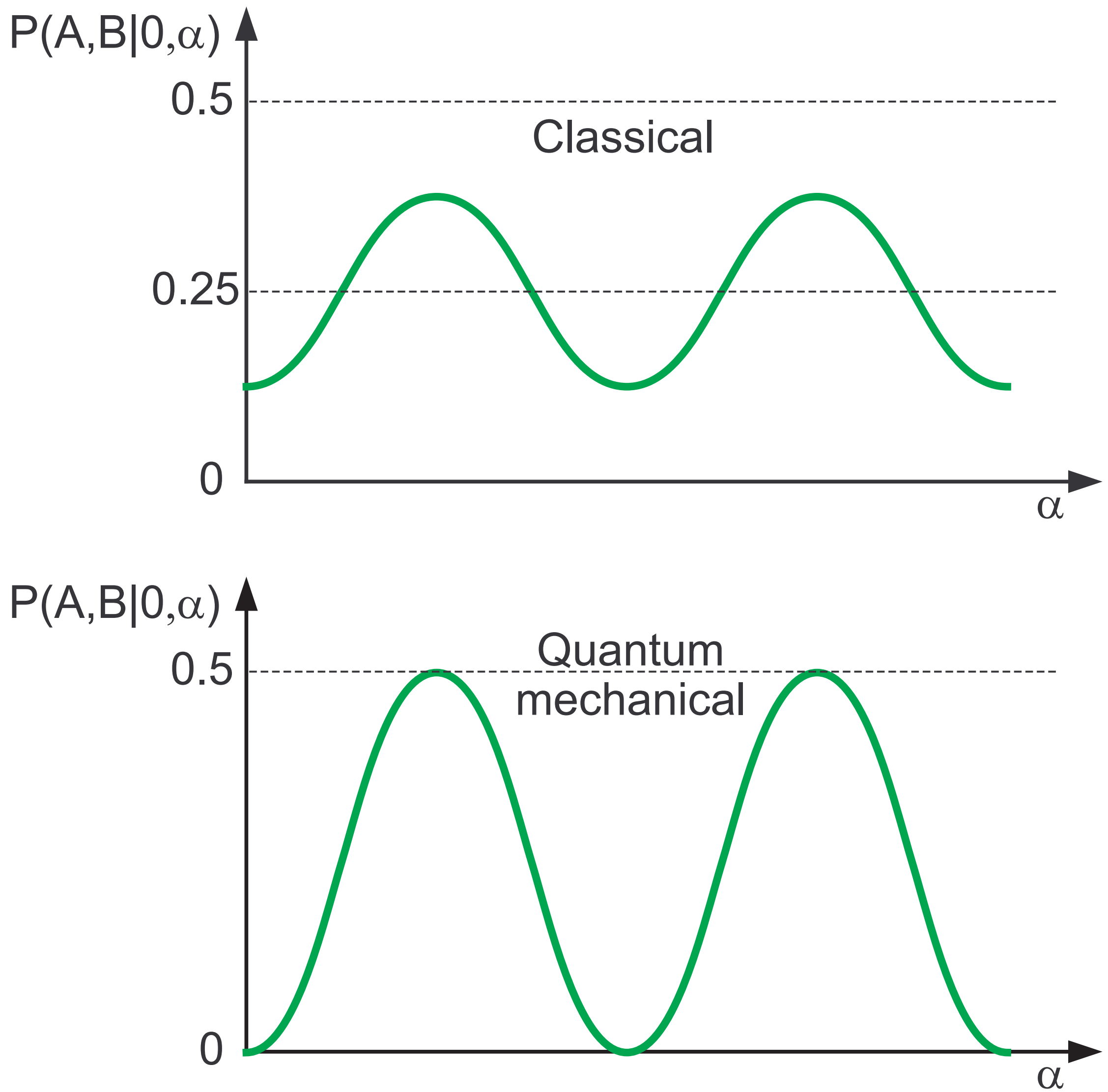
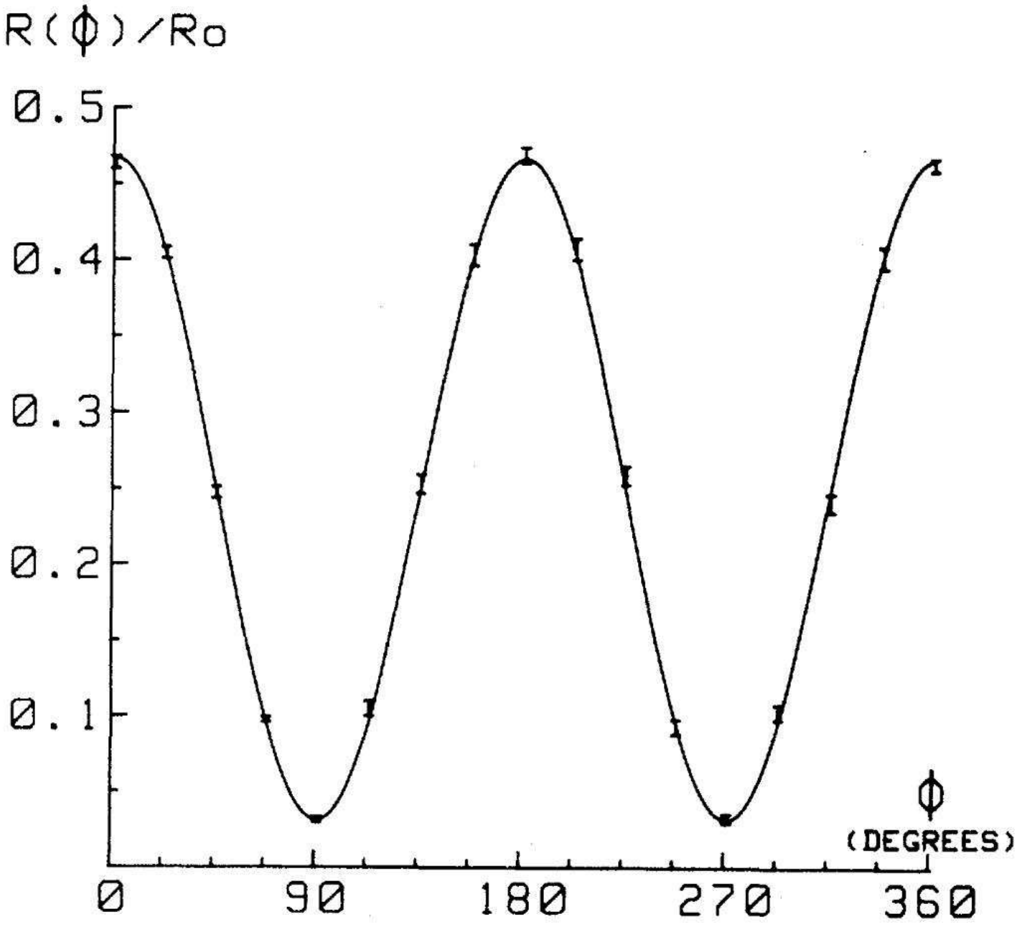
Comparison



Experimental results

For polarization ... [a]



[a] A. Aspect, et al., Phys. Rev. Lett., Vol 47, p. 460 (1981).

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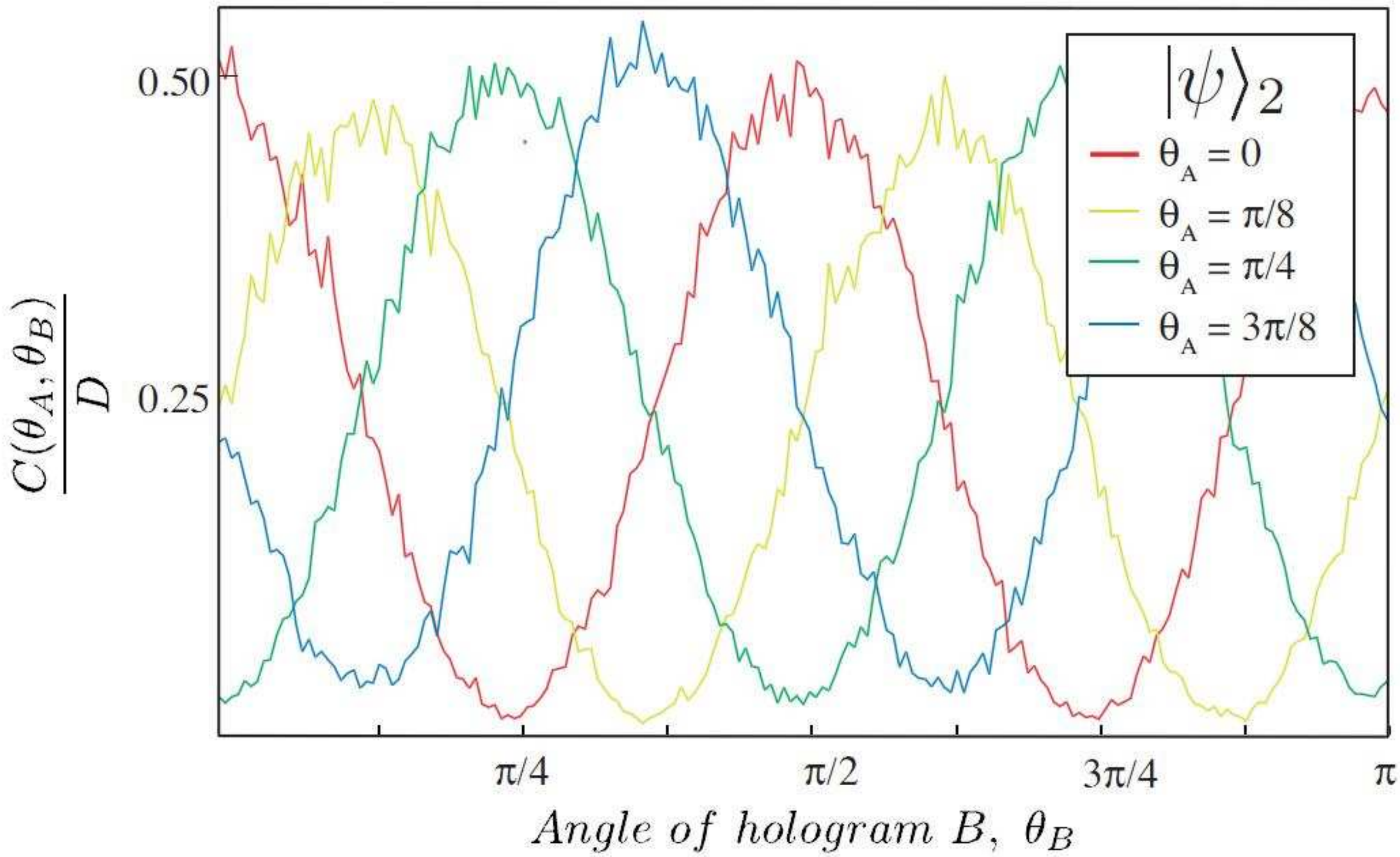
More experimental results

(... and for orbital angular mumentum) [a]



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[a] J. Leach, et al., Optics Express, Vol 17, p. 8287 (2009).

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Test of Bell's inequality

Theoretically:

Classical case: $C_C(\alpha_A, \alpha_B) = -\frac{1}{2}\cos(2\alpha_A - 2\alpha_B)$

Quantum case: $C_Q(\alpha_A, \alpha_B) = -\cos(2\alpha_A - 2\alpha_B)$

Maximum violation for:

$$\alpha_A = 0, \alpha_B = \frac{3\pi}{8}, \alpha_A{}' = -\frac{\pi}{4}, \alpha_B{}' = \frac{\pi}{8}$$

Gives

$$B_C(\alpha_A, \alpha_B, \alpha_A{}', \alpha_B{}') = 1.4 < 2$$

and

$$B_Q(\alpha_A, \alpha_B, \alpha_A{}', \alpha_B{}') = 2.8 > 2$$

Experimentally Bell's (or related) inequality was violated by several standard deviations.

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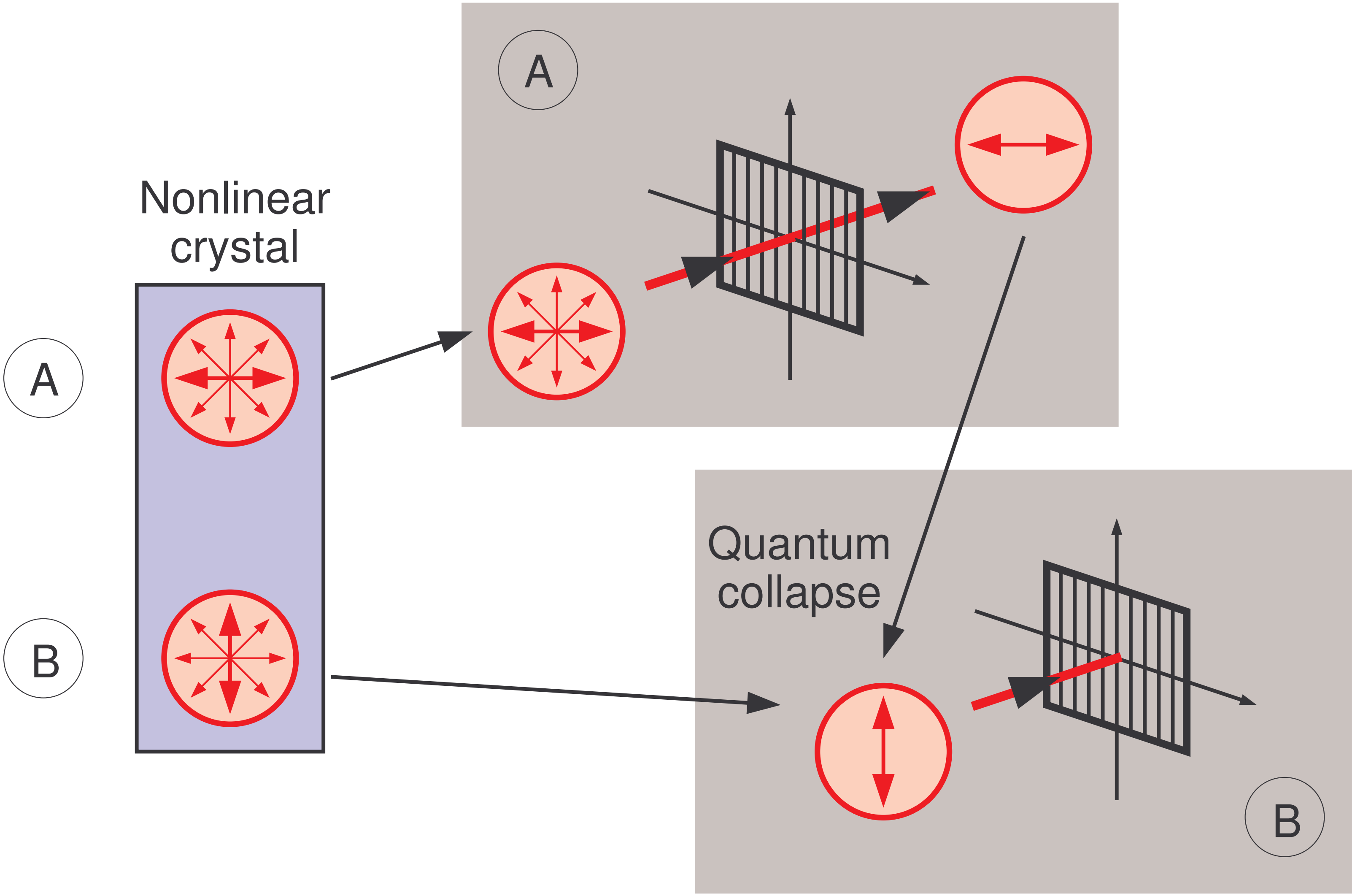
How does it work?



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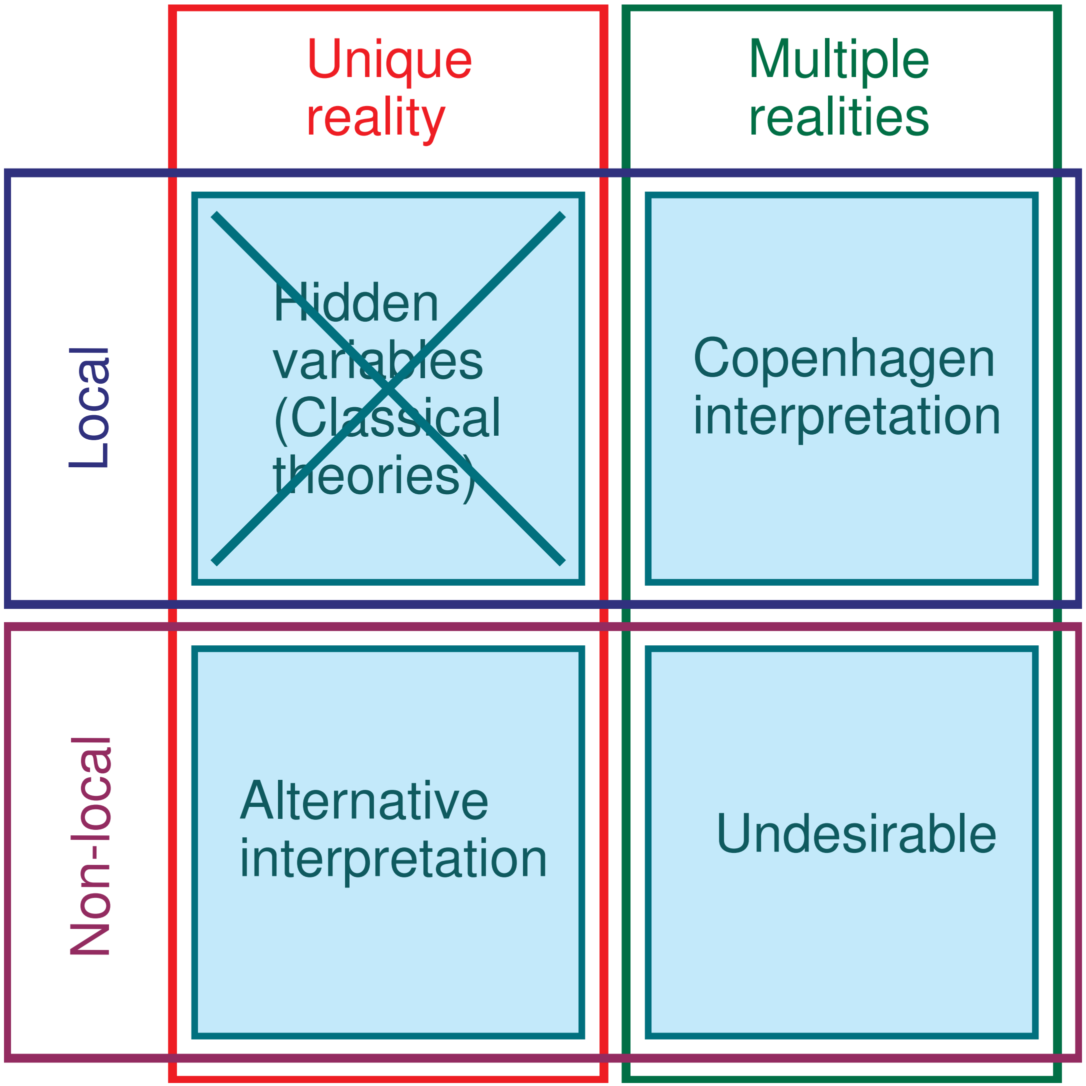
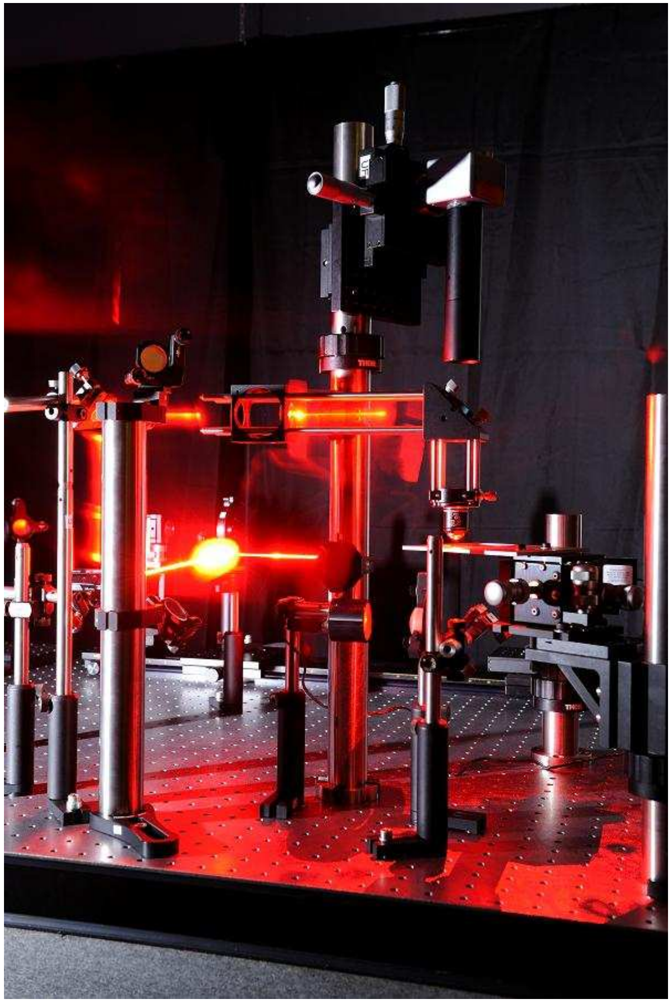
What does it mean?



Where can we use it?

- Quantum communication
  — quantum cryptography
- Quantum computing
  — efficient factorization
- Ghost imaging
- Exotic photon sources



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