

Techniques to measure complex-plane fields

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ABSTRACT

In this work we construct coherent superpositions of Gaussian and vortex modes which can be described to occupy the complex-plane. We demonstrate how these fields can be experimentally constructed in a digital, controllable manner with a spatial light modulator. Once these fields have been generated we illustrate, with three separate techniques, how the constituent components of these fields can be extracted, namely by measuring the intensity of the field at two adjacent points; performing a modal decomposition and a new digital Stokes measurement.

Keywords: vortex modes, complex-plane, spatial light modulator, modal decomposition, Stokes measurements

1. INTRODUCTION

An optical vortex is a beam of light whose phase-fronts vary along a helical path, resulting in angular momentum being carried along the beam's axis [1]. The angular momentum can be defined by an orbital and a spin component, where the orbital contribution is determined by the azimuthal phase dependence of the beam and is equivalent to $l\hbar$ per photon. These light beams which carry orbital angular momentum (OAM) were first experimentally realized as Laguerre-Gaussian laser modes [2]. The visual characteristic of these beams is that they have a ring-shaped intensity profile. Nowadays these beams can be generated with the use of many optical devices, such as spiral phase plates (SPP) [3], computer generated holograms [4, 5] or spatial light modulators (SLMs) [6, 7].

Since the discovery of optical vortices [8, 9] they have become very important in optical tweezing [10] for the steering of micro-machines [11-13], as well as quantum entanglement [14]. In many investigations into the quantum nature of these beams, it is necessary to be able to detect the amount of OAM carried by the beam. There already exists a number of methods used in determining the topological charge of an optical vortex such as computer-generated holograms [14], interferometers [15, 16] and SLMs [17, 18].

In this work we create a coherent superposition between a Gaussian beam ($l = 0$) and an optical vortex ($l = 1$) with the use of an SLM where one is able to control the amplitude ratios between the two superimposed modes. We demonstrate with three separate techniques how the constituent components of these fields can be extracted. The first method measures the peak intensities at two adjacent points in the resultant field, while the second requires an additional SLM to perform a modal decomposition on the resultant field and the third involves only one SLM and a polarization grating (PG) to perform digital Stokes polarimetry [19]. Reconstructing superimposed Gaussian and vortex modes, where one has control over the weighting of the modes, can be very useful in quantum entanglement experiments.

2. CONCEPT AND THEORY

Superpositions of Gaussian and vortex modes

Experimentally, the first SLM will be used to create a field which is a superposition of a Gaussian and a vortex beam. The field produced, for our particular case for a superimposed Gaussian and vortex mode of charge +1, can be described as follows

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$$\begin{aligned}
 A(r, x, y, z) &= \alpha(\text{Gaussian}) + \beta(\text{Vortex}) \\
 &= \alpha \left(\exp \left(\frac{-r^2}{(\omega(z))^2} \right) \right) + \beta \left((x + iy) \left(\exp \left(\frac{-r^2}{(\omega(z))^2} \right) \right) \right), \quad (1)
 \end{aligned}$$

where α and β are the weightings of the Gaussian and vortex modes respectively.

The phase function (or hologram) used to create a vortex beam is defined as $\exp(il\phi)$, where ϕ is the azimuthal angle and l is the topological charge of the beam. In order to experimentally generate a field which consists of a Gaussian and a vortex component, the incident beam is displaced off of the hologram's phase singularity. When the incident Gaussian beam is centered perfectly on the singularity, the field produced is purely vortex. By displacing the incident beam from the centre of the singularity to a distance of $\sim 3\omega_0$ from the singularity, the fields produced are superpositions of Gaussian and vortex modes. The closer the incident beam is to the singularity, the stronger the weighting of the vortex mode and the further away the incident beam is from the singularity, the higher the Gaussian mode. Figure 1 contains both experimental and theoretical cross-sectional intensity profiles of the fields produced when the incident beam is displaced from the singularity.

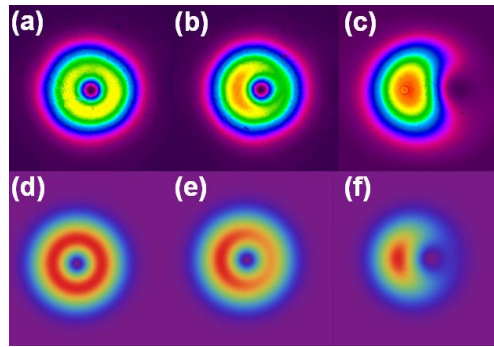


Fig. 1. Experimental (theoretical) images of the cross-sectional intensity profile of the field produced when the incident beam is (a) ((d)) centered on the singularity of the hologram and (b) ((e)) and (c) ((f)) displaced from the singularity.

Measurement method 1: Intensity ratio

By determining the intensity of the field described in Eq. 1 (illustrated in Fig. 2) and solving for $d(I) = 0$, the positions, x_1 and x_2 , of the two peaks of the intensity profile, I_1 and I_2 , can be determined. The position for each peak of the intensity profile can be expressed as follows

$$x = \frac{-R \pm \sqrt{R^2 + 2}}{2}, \quad (2)$$

where $R = \alpha/\beta$. In Eq. 1 the beam size, $\omega(z)$, is equated at $z = 0$ and the beam waist, ω_0 , is neglected ($\omega_0 = 1$).

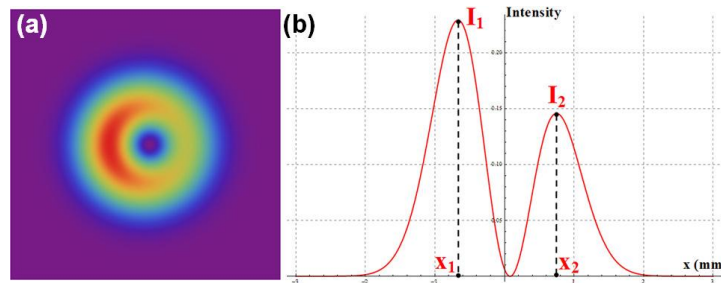


Fig. 2. (a) The cross-sectional intensity profile of the field produced when the incident beam is displaced from the singularity. (b) The corresponding intensity profile.

Substituting the position for each of the two intensity peaks, given in Eq. 2, into Eq. 1, the intensities, I_1 and I_2 , can be determined. By experimentally measuring I_1 and I_2 the ratio (R) of the two modes can be established with the following relationship

$$\frac{I_2}{I_1} = \frac{\left(\exp\left(-2R\sqrt{R^2+2}\right)\right)\left(R-\sqrt{R^2+2}\right)^2}{\left(R+\sqrt{R^2+2}\right)^2}. \quad (3)$$

Measurement method 2: Modal decomposition

Another method to measure the constituent components of these superimposed fields is to project them onto their orthogonal basis states. All modes form orthogonal basis functions where the weightings (c_l) of their azimuthal modes can be extracted by performing an azimuthal decomposition (or inner-product)

$$c_l = \langle U, t_l \rangle = \iint U t_l^* dx dy. \quad (4)$$

Here U denotes the field of interest as described in Eq. 1 and $t_l = \exp(il\phi)$ is the match-filter. When the azimuthal mode index, l , of the match-filter is (is not) the complex conjugate of one of the azimuthal modes present in the field of interest, the weighting of the azimuthal mode, c_l is non-zero (zero).

Measurement method 3: Stokes polarimetry

Stokes polarimetry requires four separate intensity profile measurements to determine the following two Stokes parameters (S_2 and S_3)

$$S_2 = I_{45^\circ} - I_{135^\circ} \text{ and } S_3 = I_{Right} - I_{Left} \quad (5)$$

The intensity profiles, I_{45° and I_{135° , can be measured behind a polarizer at angular orientations of 45° and 135° , while I_{Right} and I_{Left} are obtained by introducing a preceding quarter-waveplate. These four manual measurements can be reduced to two digital measurements, by replacing the polarizer with a PG and the quarter-waveplate with an additional $\pi/2$ phase term encoded on a SLM [19]. The digital Stokes measurements are then used to extract the wavefront as follows:

$$\phi = 0.5 \arctan(S_3/S_2) \quad (6)$$

3. EXPERIMENTAL METHODOLOGY

Superpositions of Gaussian and vortex modes

The experimental setup is depicted in Fig. 3. A HeNe laser ($\lambda = 633\text{nm}$) was directed on to the liquid crystal display of SLM1 which was addressed with a spiral hologram of charge +1. When the singularity was perfectly centred within the incident beam, a vortex beam of charge +1 was produced. As the singularity was displaced from the centre of the incident beam, the field produced became a mixture of a Gaussian beam and a vortex beam of charge +1. By increasing the displacement, the resulting field became more Gaussian and the weighting of the vortex mode decreased.

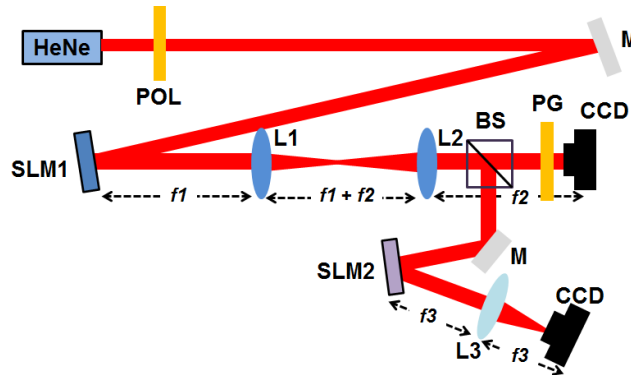


Fig. 3. A schematic of the experimental setup used to generate (with the use of SLM 1) and verify (with the use of SLM 2) the construction of mixed Gaussian and vortex modes.

Measurement method 1: Intensity ratio

The field generated at the plane of SLM1 in Fig. 3 was relay-imaged with the use of lenses L1 and L2 to a CCD camera where the cross-sectional intensity profile of the mode was recorded.

Measurement method 2: Modal decomposition

The field at SLM1 was relay-imaged on to SLM2 with the use of lenses L1 and L2 and the assistance of the beam-splitter (BS). Here the inner-product (given in Eq. 4) was executed experimentally by encoding the match-filter onto the second SLM (SLM2) and viewing the Fourier transform, with the use of lens L3, on the CCD camera.

Measurement method 3: Stokes polarimetry

Again the modes generated at the plane of the SLM were relay-imaged onto a CCD detector, preceded by a PG, where the Stokes measurements, S_2 and S_3 , were recorded. Since the SLM can be dynamically addressed, we first displayed the hologram required to create our field of interest followed by the same hologram encoded with an additional $\pi/2$ phase term. For each of the two holograms, the corresponding intensity profiles (I_{45° , I_{135° and I_{Right} , I_{Left}) were recorded to determine the two Stokes parameters, S_2 and S_3 , respectively necessary for reconstructing the wavefront as defined in Eq. 6.

4. RESULTS AND DISCUSSION

Superpositions of Gaussian and vortex modes

The setup depicted in Fig. 3 was implemented to create superpositions of Gaussian and vortex modes which are presented in Fig. 4 for movement with respect to the phase singularity along the (a) horizontal axis; (b) vertical axis; (c) diagonal axis and (d) around a circle.

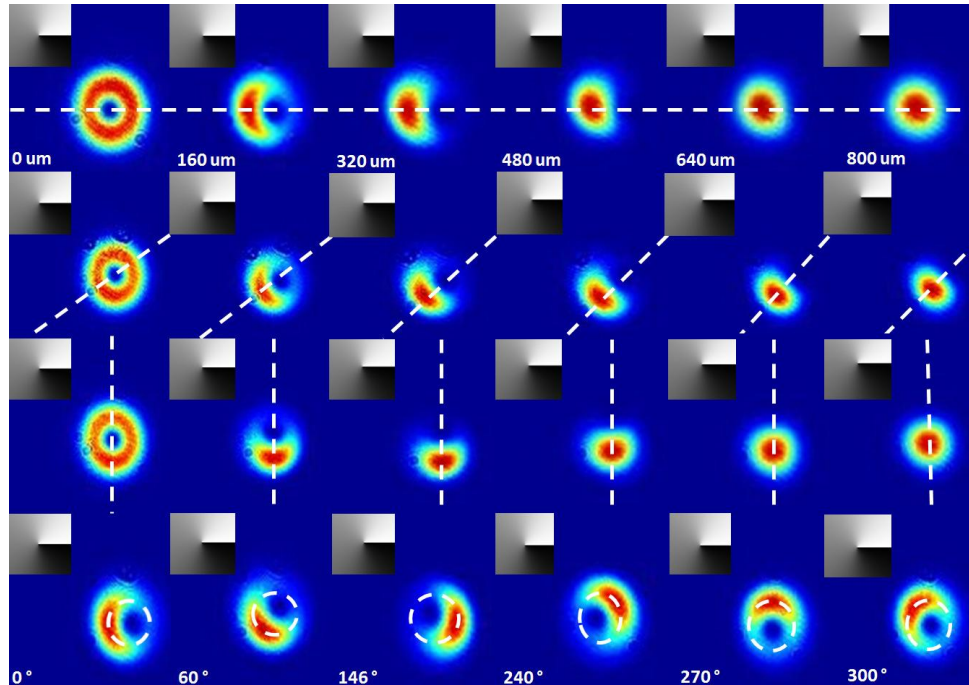


Fig. 4. The intensity profiles of the superimposed Gaussian and vortex modes where the singularity in the hologram (inserts) was displaced along the (a) x-axis, (b) y-axis, (c) x-y-axis and (d) around a circle. Displacement distances and angles are given as inserts.

Measurement method 1: Intensity ratio

The ratio between the two peak intensities as a function of the displacement from the phase singularity was measured and is presented in Figs 5 (a) for a displacement along the x-axis and (b) along the y-axis. It is evident that there is very good agreement between the measured ratios and the theoretical prediction defined by Eq. 3. As the singularity is displaced further away from the centre of the incident Gaussian beam, the ratio between the two intensities varies from 0 to 1, illustrating the evolution from a pure vortex mode to a pure Gaussian mode with weighted superpositions in between.

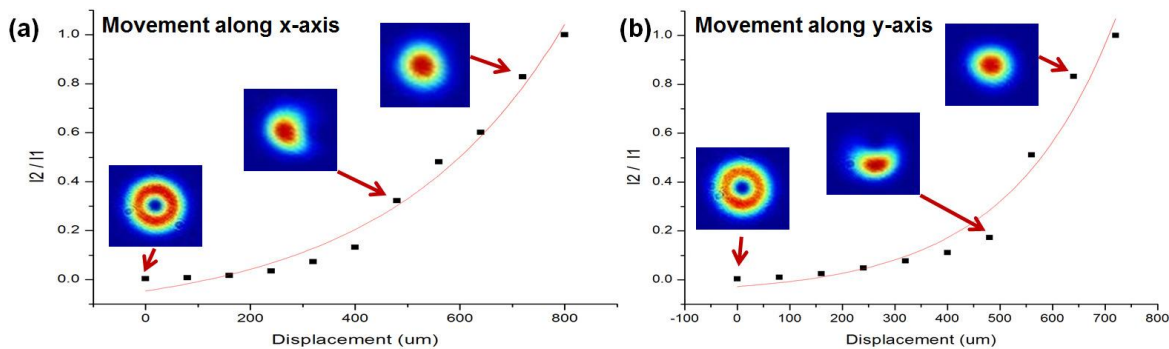


Fig. 5. The ratio of the peak intensities plotted against the displacement of the singularity along (a) the x-axis and (b) the y-axis. The red curve denotes the theoretically predicted ratio.

Measurement method 2: Modal decomposition

The weightings of the Gaussian ($l = 0$) and vortex mode ($l = 1$) for a range of displacements were measured via the inner-product approach defined in Eq. 4 and are presented in Figs 6 (a) for a displacement along the x-axis and (b) along the y-axis. Similarly as in the case above, it is evident that as the singularity in the hologram is displaced further away from the

centre of the incident Gaussian beam, the mode transforms from a pure vortex mode to a pure Gaussian mode with weighted superpositions in between.

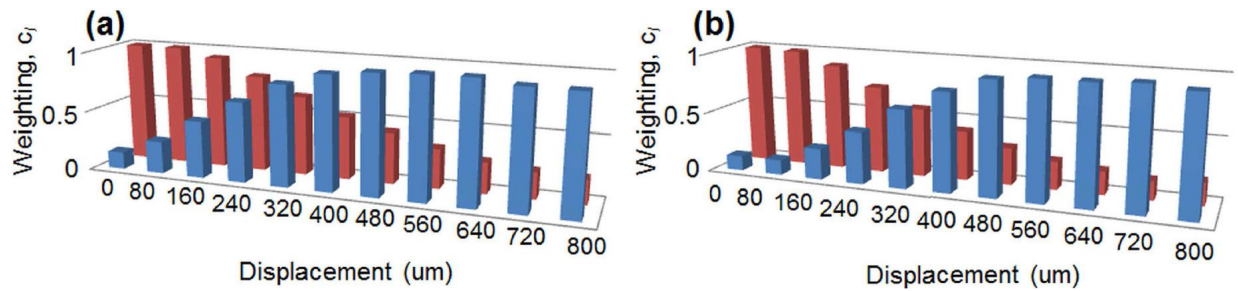


Fig. 6. The weighting of the two azimuthal modes plotted against the displacement of the singularity along (a) the x-axis and (b) the y-axis.

Measurement method 3: Stokes polarimetry

The wavefronts of off-axis optical vortices [intensity profiles given in Fig. 7 (a)] were measured via the digital Stokes polarimetry and are depicted in Fig. 7 (b). It is evident that as the intensity null moves from left to right in Fig. 7 (a) so the reconstructed phase extracted via digital Stokes polarimetry is in agreement: also moving from left to right [Fig. 7 (b)].

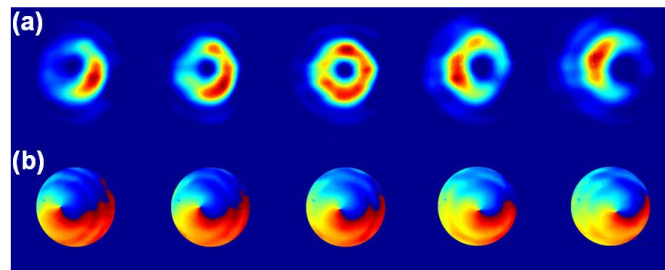


Fig. 7. (a) Experimentally measured intensity profiles of the superimposed Gaussian and vortex mode with (b) their corresponding reconstructed phase profiles.

5. CONCLUSION

In this work we have generated a field which is a superposition of a Gaussian beam and a vortex beam of charge +1. We have illustrated that by displacing the initial Gaussian beam off of the singularity in the hologram we are able to control the weighting of the two modes (Gaussian and vortex) in the superposition. We show how these fields can subsequently be broken down into their fundamental (Gaussian or vortex) components by measuring the ratio of the peak intensities at two adjacent points in the field, performing a simple modal decomposition or performing digital Stokes polarimetry. Since there is a lot of interest into fields which carry OAM, especially in quantum entanglement experiments, being able to have control over the weighting of OAM states is of some importance. Due to the programmability of SLMs, this experiment can be easily adapted for higher-order vortex modes as well as superpositions involving a number of different modes.

REFERENCES

- [1] Allen, L., Beijersbergen, M. W., Spreeuw, R. J. C., and Woerdman, J. P., "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," *Phys. Rev. A* **45**, 88185 (1992).
- [2] Beijersbergen, M. W., Allen, L., Van der Veen, H. E. L. O., and Woerdman, J. P., "Astigmatic laser mode converters and the transfer of orbital angular momentum," *Opt. Comm.* **96**, 123 (1993).
- [3] Beijersbergen, M. W., Coerwinkel, R. P. C., Kristensen, M., and Woerdman, J. P., "Helical-wave-front laser-beams produced with a spiral phaseplate," *Opt. Comm.* **112**, 321 (1994).

- [4] Yu Bazhenov, V., Vasnetsov, M. V., and Soskin, M. S., "Laser beams with screw dislocations in their wavefronts," *JETP Lett. (USA)* **52**, 429 (1990).
- [5] Heckenberg, N.R., McDuff, R., Smith, C. P., and White, A. G., "Generation of optical phase singularities by computer-generated holograms," *Opt. Lett.* **17**, 221 (1992).
- [6] Reicherter, M., Haist, T., Wagemann, E. U., and Tiziani, H. J., "Optical particle trapping with computer-generated holograms written on a liquid-crystal display," *Opt Lett.* **24**, 608 (1999).
- [7] Curtis, J. E., Koss, B. A., and Grier, D. G., "Dynamic holographic optical tweezers", *Opt. Comm.* **207**, 169 (2002).
- [8] Nye, B. A., and Berry, M. V., "Dislocations in wave trains," *Proc. R. Soc. London Ser. A* **336**, 165 (1974).
- [9] Dirac, P.A.M., "Quantised singularities in the electromagnetic field," *Proc. R. Soc. London Ser. A* **133**, 60 (1931).
- [10] He, H., Friese, M. E. J., Heckenberg, N. R., and Rubinsztein-Dunlop, H., "Direct observation of transfer of angular momentum to absorptive particles from a laser beam with a phase singularity," *Phys. Rev. Lett.* **75**, 826 (1995).
- [11] Higurashi, E., Ukita, H., Tanaka, H., and Ohguchi, O., "Optically induced rotation of anisotropic micro-objects fabricated by surface micromachining," *Appl. Phys. Lett.* **64**, 2209 (1994)
- [12] Galajda, P., and Ormosa, P., "Complex micromachines produced and driven by light," *Appl. Phys. Lett.* **78**, 249 (2001).
- [13] Knöner, G., Parkin, S., Nieminen, T. A., Love, V. L. Y., Heckenberg, N. R., and Rubinsztein-Dunlop, H., "Integrated optomechanical microelements," *Opt. Express* **15**, 5521 (2007).
- [14] Mair, A., Vasiri, A., Weihs, G., and Zeilinger, A., "Entanglement of the orbital angular momentum states of photons," *Nature* **412**, 313 (2001).
- [15] Leach, J., Padgett M. J., Barnett, S. M., Frank-Arnold, S., and Courtial, J., "Measuring the orbital angular momentum of a single photon," *Phys. Rev. Lett.* **88**, 257901 (2002).
- [16] Leach, J., Courtial, J., Skeldon, K., Barnett, S. M., Franke-Arnold, S., and Padgett, M. J., "Interferometric methods to measure orbital and spin, or the total angular momentum of a single photon," *Phys. Rev. Lett.* **92**, 013601 (2004).
- [17] Gibson, G., Courtial, J., Padgett, M. J., Vasnetsov, M., Pas'ko, V., Barnett, S. M., and Franke-Arnold, S., "Free-space information transfer using light beams carrying orbital angular momentum," *Opt. Express* **12**, 5448 (2004).
- [18] Moreno, I., Davis, J., Pascoguin, B., Mitry, M., and Cottrell, D., "Vortex sensing diffraction gratings," *Opt. Lett.* **34**, 2927 (2009).
- [19] Dudley, A., Milione, G., Alfano, R. R., Forbes, A., "All-digital wavefront sensing for structured light beams," *Opt. Express* **22**(11), 14031-14040 (2014).