

# A new radiative transfer scattering phase function discretisation approach with inherent energy conservation

Thomas H. Roos a,<sup>†</sup>, Thomas M. Harms b,1

a CSIR, PO Box 395, Pretoria 0001, South Africa

bDepartment of Mechanical and Mechatronic Engineering, University of Stellenbosch, Private Bag X1, Matieland 7602, South Africa

## Abstract

In the popular Discrete Ordinates Method (DOM) formulation of the Equation of Radiative Transfer (ERT), the  $4\pi$  solid angle range of directions is divided into a finite number of discrete directions or ordinates. This requires that the continuous distribution of the scattering phase function of the medium under consideration must be discretised to suit the different number, weightings and directions of the  $S(\text{sub}n)$  ordinate set being used. This must be done such that the sum of scattered energy is conserved relative to the continuous distribution, and that the asymmetry factor  $g$  is similarly conserved. This paper introduces a discretisation technique with inherent energy conservation, suitable for any quadrature scheme. The technique was tested on two large sphere scattering phase function distributions of interest for packed bed radiative heat transfer: the analytic distribution for a diffusely reflecting sphere (a backscattering test case) and the distribution for a transparent sphere ( $n = 1.5$ ) obtained by ray tracing (a test case with strong forward scatter and some back-scatter). In both cases the resultant discretised phase function distributions for the  $S(\text{sub}4)$ ,  $S(\text{sub}6)$  and  $S(\text{sub}8)$  ordinate sets produced errors for the sum of scattered energy conservation of less than 0.035% and errors for  $g$  less than 1.3%. This demonstrates the inherent energy conservation of the method, as well as visible reductions in  $g$  errors. The phase function values for each case are tabulated in the paper. The major benefit of the method is the fact that computationally costly matrix calculations are avoided at run-time: the discretisation for a given scattering medium using a quadrature scheme of given order is performed only once beforehand, and the resultant distributions can be stored in an input file or look-up table for future computations with different boundary conditions, different meshes and even different geometries.