

Unraveling light with digital holograms

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ABSTRACT

Modal decomposition of optical fields as a concept has been in existence for many decades, yet despite its clear applications to laser beam analysis it has nevertheless remained a seldom used tool. With the commercialization of liquid crystal devices, digital holography as an enabling tool has become accessible to all, and with it modal decomposition has come of age. Here we outline the basic principles of modal decomposition of laser beams with digital holograms, and review recent results on the modal decomposition of arbitrary optical fields. We show how to use the information to infer the intensity, phase, wavefront, Poynting vector and orbital angular momentum density of the light. In particular, we show how to achieve optimal modal decomposition even in the absence of key information about the field, such as its scale and wavefront. We demonstrate the techniques on optical fields from fibers, diode-pumped solid-state lasers, and structured light by laser beam shaping.

Keywords: Digital holography, modal decomposition, match filters

1. INTRODUCTION

The decomposition of some light field into a superposition of orthonormal basis functions, so-called modes, has been known for a long time¹⁻⁵. Previously modal decomposition was executed with match filters encoded as physical diffractive optical elements and thus while extremely accurate were nonetheless expensive and required specialist knowledge of such tools. Using these optics, often called correlation filters, one could show excellent reconstruction of intensity, phase and the beam quality factor of a variety of laser beams⁶⁻⁸. Despite the appropriateness of the techniques, the experiments were nevertheless rather complex or customized to analyze a very specific mode set. Recently this subject has been revisited by employing digital holograms for the modal decomposition of laser beams from fibers, with the first modal decomposition by digital holography done on Laguerre-Gaussian modes from a diode-pumped solid-state laser⁹ and then subsequently applied to fiber modes¹⁰, for the real-time measurement of the beam quality factor of a laser beam¹¹, for the determination of the orbital angular momentum density of light¹²⁻¹⁴ and for measuring the wavefront and phase of light¹⁵.

In this paper we review the recent work on the use of digital holograms for modal decomposition. We also outline a new approach using digital holography for an optimal modal decomposition without any prior knowledge of the scale parameters within the basis functions¹⁶. We show that in a simple two-step process both the scale and the optimal mode set can be found. The result, as we will show, is that the complete decomposition can be achieved without any initial scale information.

2. CONCEPT

Consider an expansion of an arbitrary optical field, U :

$$U(\vec{u}) = \sum_n^{n_{\max}} c_n \Psi_n(\vec{u}) \quad (1)$$

where the expansion is in terms of the basis functions Ψ . The central idea in this section is how to measure the modal coefficients c_n directly and therefore obtain all the required information about a scalar field distribution. Importantly we wish to do this with computer generated holograms (CGH) in the form of digital holograms written to a spatial light modulator (SLM). If the transmission function of the CGH is

$$T(\bar{u}) = \Psi_n^*(\bar{u}) \quad , \quad (2)$$

then the field directly behind the diffractive CGH will read as

$$W_0(\bar{u}) = \Psi_n^*(\bar{u})U(\bar{u}) \quad . \quad (3)$$

The far field diffraction is realised experimentally by using a lens with a focal length f and observing the field at the focal plane:

$$\begin{aligned} W_f(\bar{u}) &= \frac{k_0}{2\pi if} \exp(2ik_0 f) \iint_{\mathfrak{R}^2} d^2\bar{u}' W_0(\bar{u}') \exp\left(-i \frac{k_0}{f} \bar{k}_\perp \bar{u}'\right) \\ &= -2\pi i \frac{k_0}{f} \exp(2ik_0 f) \tilde{W}_0\left(\frac{k_0}{f} \bar{u}\right) \\ &= A_0 \tilde{W}_0\left(\frac{k_0}{f} \bar{u}\right) \end{aligned} \quad (4)$$

where A_0 is a constant factor. By applying the following convolution theorem

$$\mathfrak{T}\{f(x)g(x)\} = [\tilde{f} * \tilde{g}](k_x) \quad (5)$$

to the previous equation we get

$$W_f(\bar{u}) = A_0 \iint_{\mathfrak{R}^2} d^2\bar{u}' \tilde{\Psi}_n^*\left(\frac{k_0}{f} \bar{u}'\right) \tilde{U}\left(\frac{k_0}{f} (\bar{u} - \bar{u}')\right) \quad . \quad (6)$$

If the measurement is done at the origin of the focal plane then from

$$c_n = \rho_n \exp(i\phi_n) = \langle \Psi_n, u \rangle = \iint_{\mathfrak{R}^2} d^2\bar{u} \tilde{\Psi}_n^*(\bar{u})U(\bar{u})$$

and

$$c_n = \iint_{\mathfrak{R}^2} d^2\bar{k}_\perp \tilde{\Psi}_n^*(\bar{k}_\perp)U(\bar{k}_\perp)$$

we find that the unknown coefficients may be related to the signal at this point from

$$W_f(0) = A_0 \langle \Psi_n, u \rangle \propto c_n \quad . \quad (7)$$

Thus a transmission function as in Eq. (2) produces a diffraction pattern containing information about the modal coefficients: we are able to determine a signal that is proportional to $|c_n|^2$ with a detector that is positioned directly in the center of the output focal plane. With this approach we are only able to measure one mode. It is possible, in principle, to measure all modes by modifying the transmission function

$$T(\bar{u}) = \sum_n^{n_{\max}} \Psi_n^*(\bar{u}) \exp(i\vec{V}_n \bar{u}) \quad (8)$$

so that it contains all possible modes, each of them multiplied by a grating of spatial frequency \vec{V}_n . In the CGH plane we then find

$$W_0(\bar{u}) = \sum_n^{n_{\max}} \Psi_n^*(\bar{u})U(\bar{u}) \exp(i\vec{V}_n \bar{u}) \quad . \quad (9)$$

To evaluate the field in the focal plane of the Fourier lens, a simplification is possible by using the shifting theorem

$$\mathfrak{F}\{f(x)\exp(i\nu_0x)\} = \tilde{f}(k_x - \nu_0) \quad (10)$$

From which we get

$$W_f(\bar{u}) = A_0 \sum_n \left[\iint_{\mathfrak{R}^2} d^2\bar{u}' \tilde{\Psi}_n^* \left(\frac{k_0}{f} \bar{u}' \right) \tilde{U} \left(\frac{k_0}{f} (\bar{u} - \bar{u}') - \vec{V}_n \right) \right]. \quad (11)$$

This means that the output pattern consists of a superposition of cross correlation functions between the incident field $U(\bar{u})$ and the mode field distribution Ψ_n . These correlation answers are shifted transversely by a spatial frequency \vec{V}_n and therefore each is projected to a particular position in the detector plane – simultaneous correlations of the various modes. A similar approach is followed to find the intermodal phases, where superpositions of modes are programmed onto the SLM¹⁵.

With the complete modal decomposition of the field achieved, it is possible to calculate the intensity of the field

$$I = |u(r, \phi)|^2 \quad (12)$$

the phase of the field

$$\Theta = \arg[u(r, \phi)], \quad (13)$$

the Poynting vector

$$\vec{S} = \frac{\epsilon_0 \omega c^2}{4} \left(i(u \nabla u^* - u^* \nabla u) + 2k|u|^2 \hat{z} \right), \quad (14)$$

and the OAM density

$$L_z = \frac{1}{c^2} (r \times S)_z. \quad (15)$$

In the sections to follow we illustrate the power of the technique with some typical examples of measurements of these quantities using modal decomposition with digital holograms.

3. INTENSITY AND PHASE

It is clear that if all the modal weightings and their phases are known, then the intensity and phase of the light may readily be found from Eqs. (12) and (13). In Figure 1 and Figure 2 we illustrate this for laser beams from various sources. In all cases the reconstructed intensities^{10,16} and phases¹⁵ were in excellent agreement with the experimentally measured data using CCD detectors and wavefront sensors.

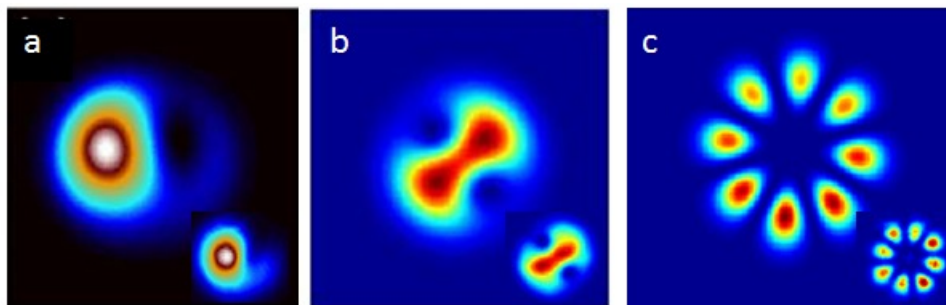


Fig. 1. Reconstructed intensity using modal decomposition for: (a) fibre laser beam and (c) a structured light beam from a SLM and (c) a solid-state laser. The insets show the measured (or created) intensity by traditional means.

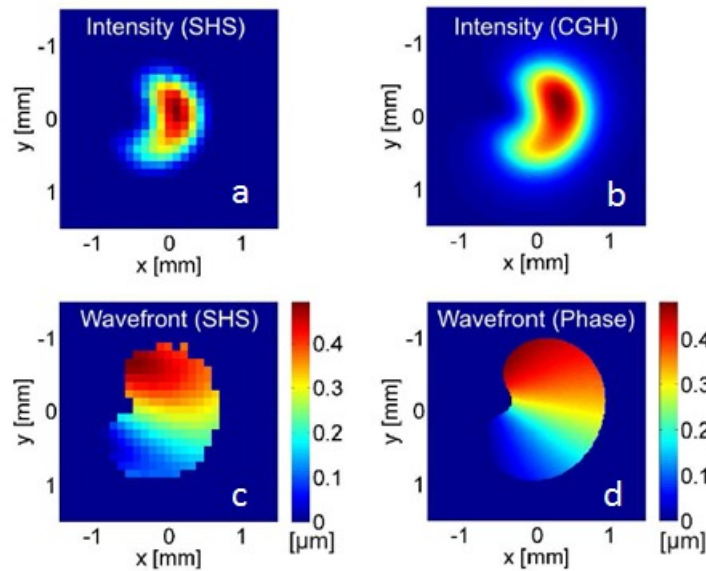


Fig. 2. (a) Measured intensity on a Shack-Hartmann wavefront sensor (SHS), (b) Reconstructed intensity using modal decomposition, (c) Measured phase on a Shack-Hartmann wavefront sensor (SHS), (d) Reconstructed phase using modal decomposition.

4. ORBITAL ANGULAR MOMENTUM DENSITY

Orbital angular momentum (OAM) is very topical of late, and the measurement of the OAM density is important for understanding the interaction of light and matter in, for example, optical trapping systems. Using Eqs. (14) and (15) we may readily calculate the OAM density from the measured modal weightings and their phases, as shown in Figure 3.

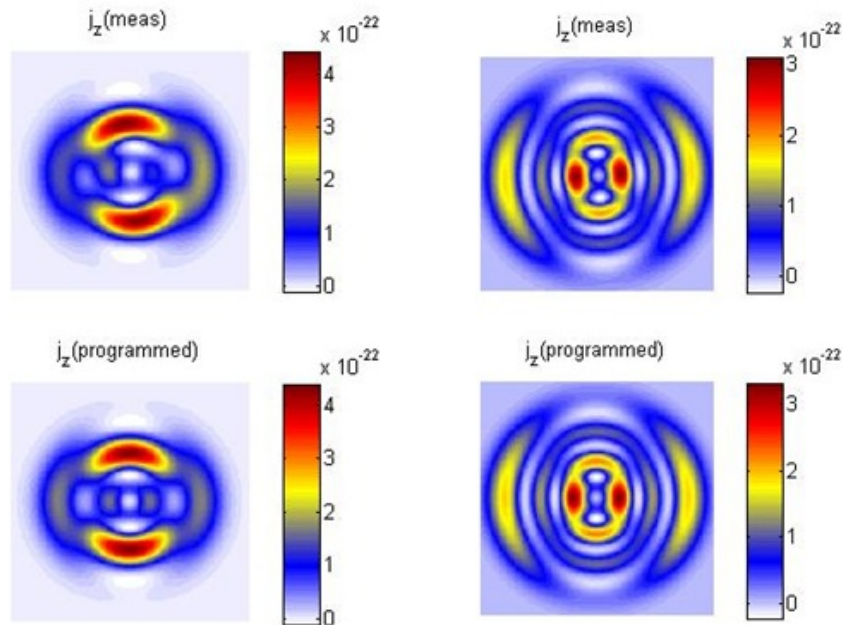


Fig. 3. Orbital angular momentum density measurement of a structured light beam by modal decomposition: (1st column) Measured and programmed super-position beam of $LG_{21}+LG_{03}$; (2nd column) Measured and programmed super-position beam of $LG_{21}+LG_{23}$.

In these cases structured light superpositions of Laguerre-Gaussian modes of radial order p and azimuthal order l , LG_{pl} , were created on an SLM and then directed to a second SLM for the modal decomposition. The measured OAM density [j_z (meas)] is in excellent agreement with the expected OAM density [j_z (programmed)] as programmed on the first SLM¹⁷.

5. SCALE INDEPENDENCE

An important limitation in modal decomposition methods is the requirement that some scale information is available on the modes to be studied. Modal decomposition is a powerful tool to characterize laser beams and allows very detailed studies of the underlying physics of a laser system. Accordingly, an optical field can be described as a superposition of basis functions, called the modes, each weighted with a complex expansion coefficient. To determine these coefficients is the main task of each modal decomposition, mapping all necessary information about the field onto a one-dimensional set of coefficients. However, a reasonable decomposition necessitates the knowledge about the scale of the beam. Mathematically, each basis set, independent from its scale, is suitable to describe an optical field of certain size. However, changing the scale of the basis set will change the expansion coefficients. Hence, a reasonable decomposition is performed into a basis set with a size adapted to the beam itself, yielding the minimum number of nonzero coefficients. Note that the size of such a best fitting mode set is understood as the corresponding fundamental mode size, which is unequal the size of the beam itself, which may consist of any arbitrary superposition of modes.

Consider for example the basis set of Laguerre-Gaussian modes with mode numbers p and l :

$$u_{p,l} = \sqrt{\frac{2p!}{\pi(p+|l|)!}} \frac{1}{w_0} \left(\frac{\sqrt{2}r}{w_0}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w_0^2}\right) \exp\left(-\frac{r^2}{w_0^2}\right) \exp(-il\phi), \quad (16)$$

where $r=(r, \phi)$ are the coordinates, w_0 the intrinsic beam radius of the set (corresponding fundamental mode radius), and L the Laguerre polynomial. An arbitrary scalar optical field U can be decomposed into the Laguerre-Gaussian set of any size:

$$U = \sum_{p,l} c_{pl}(a) u_{p,l}(ar, \phi) = \sum_{p,l} c_{pl}(b) u_{p,l}(br, \phi), \quad (17)$$

where $c_{pl}(a)$ and $c_{pl}(b)$ denotes the complex expansion coefficient for size w_a and w_b , respectively. From Eq. (17) it becomes clear that the modal spectrum c_{pl} changes with scale of the basis set. To attain a mode set of adapted size we propose the following recipe: two quantities need to be measured, the beam radius w in the plane of interest and the beam quality factor M^2 . The scale of the basis set can then be inferred from

$$w_0 = w / M^2 \quad (18)$$

enabling the decomposition in an adapted mode set. To determine beam size and beam quality factor any appropriate method may be used¹⁴, including the modal decomposition approach itself, by making a guess for the size of the mode set and running the decomposition¹⁶. Deducing the correct size from these measurements the decomposition can be re-run into the adapted mode set. The results from such a procedure are shown in Figure 4.

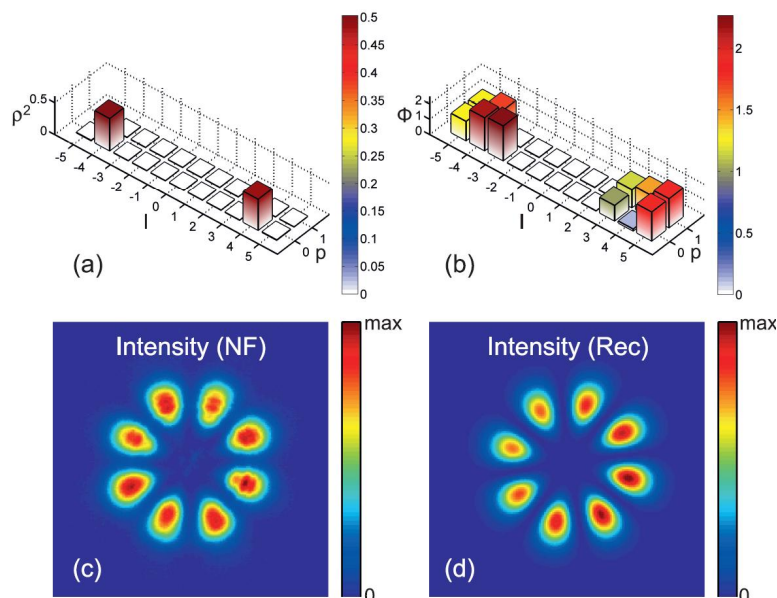


Fig. 4. (a) Measured modal amplitudes and (b) measured modal phases. With this information the measured intensity of the field, (c), can be reconstructed. (d). The reconstructed intensity profile was done without any a prior information of the beam's scale.

A similar approach can be used to deduce the unknown curvature of the basis set¹⁵. An alternative approach is to run the modal decomposition into a basis set that does not have any scale, for example the angular harmonics, in which case the scale information is ultimately stored in radially dependent coefficients of the basis¹¹.

6. CONCLUSION

We have shown that modal decomposition with digital holograms is a versatile tool for the characterization of laser beams. With a full decomposition of the field, it is possible to retrieve all the physical quantities associated with the field, including the intensity, phase, wavefront, Poynting vector and orbital angular momentum density of the light. In particular, we have shown how to achieve optimal modal decomposition even in the absence of key information about the field. Our results have included examples from fibers, diode-pumped solid-state lasers, and structured light by laser beam shaping

REFERENCES

- [1] J. W. Goodman, Introduction to Fourier Optics (McGraw-Hill Publishing Company, 1968).
- [2] Tervonen, E., Turunen, J. and Friberg, A., "Transverse laser mode structure determination from spatial coherence measurements: experimental results," *Appl. Phys. B* 49, 409-414 (1989).
- [3] Cutolo, A., Isernia, T., Izzo, I., Pierri, R. and Zerni, L., "Transverse mode analysis of a laser beam by near- and far-field intensity measurements," *Appl. Opt. B* 34, 7974-7978 (1995).
- [4] Santarsiero, M., Gori, F., Borghi, R. and Guattari, G., "Evaluation of the modal structure of light beams composed of incoherent mixtures of Hermite-Gaussian modes," *Appl. Opt. B* 38, 5272-5281 (1999).
- [5] Xue, X., Wei, H. and Kirk, A. G., "Intensity-based modal decomposition of optical beams in terms of Hermite-Gaussian functions," *J. Opt. Soc. Am. A* 17, 1086-1091 (2000).
- [6] Flamm, D., Schmidt, O. A., Schulze, C., Borchardt, J., Kaiser, T., Schroter, S. and Duparre, M., "Measuring the spatial polarization distribution of multimode beams emerging from passive step-index large-mode-area fibers," *Opt. Lett.* 35(20), 3429-3431 (2010).
- [7] Kaiser, T., Flamm, D., Schroter, S. and Duparre, M., "Complete modal decomposition for optical fibers using CGH-based correlation filters," *Opt. Express* 17(11), 9347-9356 (2009).
- [8] O. A. Schmidt, C. Schulze, D. Flamm, R. Bruning, T. Kaiser, S. Schoter, and M. Duparre, "Real-time determination of laser beam quality by modal decomposition," *Opt. Express* 19, 6741-6748 (2011).

- [9] Naidoo, D., Ait-Ameur, K., Brunel, M., & Forbes, A., "Intra-cavity generation of superpositions of Laguerre-Gaussian beams," *Appl. Phys. B* 106, 683-690 (2012).
- [10] D. Flamm, D. Naidoo, C. Schulza, A. Forbes and M. Duparre, "Mode analysis with a spatial light modulator as a correlation filter," *Opt. Lett.* 37, 2478-2480 (2012).
- [11] Litvin, I., Dudley, A., Roux, F. S. and Forbes, A., "Azimuthal decomposition with digital holograms," *Opt. Express* 20(10), 10996-11004 (2012).
- [12] Litvin, I. A., Dudley, A. and Forbes, A., "Poynting vector and orbital angular momentum density of superpositions of Bessel beams," *Opt. Express* 19(18), 16760-16771 (2011).
- [13] Dudley, A., Litvin, I. A. and Forbes, A., "Quantitative measurement of the orbital angular momentum density of light," *Applied Optics* 51(7), 823-833 (2012).
- [14] C. Schulze, D. Flamm, M. Duparre and A. Forbes, "Beam quality measurements using a spatial light modulator" *Opt. Lett.* 37, 4687-4689 (2012).
- [15] C. Schulze, D. Naidoo, D. Flamm, O. Schmidt, A. Forbes and M. Duparre, "Wavefront reconstruction by modal decomposition" *Opt. Express* 20, 19714-19725 (2012).
- [16] C. Schulze, S. Ngcobo, M. Duparre and A. Forbes, "Modal decomposition with a prior scale information" *Opt. Express* 20, 27866-27873 (2012).