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A novel implementation of an ISO standard method for primary vibration calibration by laser interferometry

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Abstract
An implementation of the sine-approximation method of ISO 16063-11 (primary vibration calibration by laser interferometry) is described. The quadrature interference signals are generated using an interferometer as described in method 3 of ISO 16063-11. The signal processing required for the successful implementation of this method of ISO 16063-11 has been implemented by various metrology institutes using various techniques. For the implementation of these techniques, different hardware and software configurations are used, ranging from expensive VAX computer systems to digital storage oscilloscopes.

A novel, cost effective, PC based system has been developed and is described herein. The hardware consists of a Pentium 200 with 256 MB of RAM. A commercial, four-channel analogue to digital (A to D) conversion card is used for performing the data capturing, while the signal processing software was developed to run under Microsoft Windows®.

The system was evaluated using data signals with and without disturbances added. The data signals were generated using software techniques. Further system validation was achieved using experimental data.

1. Introduction

Various techniques and systems were examined for extending the CSIR-NML's absolute accelerometer calibration facility. The existing system, a homodyne He–Ne laser interferometer system, employing the well-established ratio-counting method, provides the CSIR-NML with primary acceleration calibration capabilities over the frequency range 10 Hz to 1 kHz. The sine-approximation method as described in ISO 16063-11 was selected after careful consideration of various existing methods for extending the calibration range up to 10 kHz.

The sine-approximation method has already been researched and implemented by various other national metrology institutes (NMIs) [2, 3, 5]. There have also been other quadrature interferometer systems developed and implemented by NMIs [7]. The theoretical implementation of this technique by the various NMIs is most certainly the same, though countless interferometer, hardware and signal processing configurations are possible. Therefore, it is understandable that one would find different systems implemented by various NMIs.

A brief overview of the sine-approximation method is given in section 2. The laser interferometer implemented is described in section 3. The interferometer fulfils all the requirements as stipulated by ISO 16063-11 for quadrature laser interferometer systems. Of interest for discussion in this paper is the hardware configuration implemented for sampling and capturing the quadrature signals produced on the output of the photodetector circuits as well as the software developed for performing the required signal processing. The system hardware is discussed in section 4, while the signal processing is discussed in section 5. Section 6 deals with the system data processing. The investigations performed as well as the results obtained using simulated data are discussed in section 7. Some experimental results are given and discussed in section 8.

2. Sine-approximation method

The sine-approximation method is an absolute calibration method based on homodyne or heterodyne interferometry with dual optical signals in quadrature. For the application

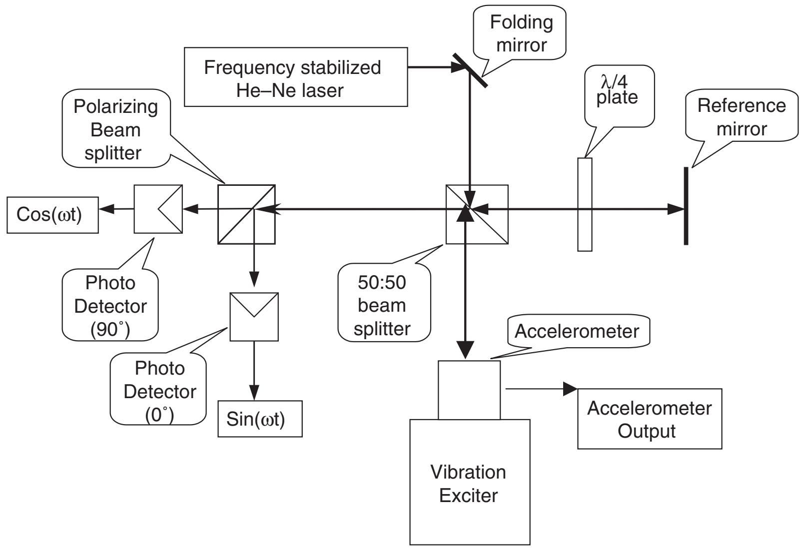

Figure 1. Block diagram of the laser interferometer with quadrature optical signals.

described, a homodyne interferometer system is used. The sine-approximation method can be implemented over a wide frequency range, typically from 1 Hz to 10 kHz, with the added advantage that the method can be used to simultaneously determine the phase shift of the accelerometer as well.

The two quadrature signals can be written in the form

$$u_1(t) = \hat{u}_1 \cos[\varphi_0 + \hat{\varphi}_M \cos(\omega t + \varphi_s)] \tag{1}$$

$$u_2(t) = \hat{u}_2 \sin[\varphi_0 + \hat{\varphi}_M \cos(\omega t + \varphi_s)]. \tag{2}$$

Both quadrature signals $u_1(t)$ and $u_2(t)$ are equidistantly sampled, giving a series of measurement values $u_1(t_i)$ and $u_2(t_i)$ over a sampling period T.

The discrete-time phase signal of the displacement signal is obtained by applying an arctangent subroutine to the two quadrature signals:

$$\varphi_M = \tan^{-1} \frac{u_2(t_i)}{u_1(t_i)} + n\pi. \tag{3}$$

An integer number n is chosen to correct for discontinuities introduced by the arctangent function, also known as phase unwrapping. The approximation of the discrete-time phase signal (φ_M) by a sine function using the sine-approximation method produces the amplitude and phase of the displacement. This is achieved by solving $N + 1$ equations for the three unknown parameters A, B and C, using the least squares sum method:

$$\varphi_M(t_i) = A \cos \omega t_i - B \sin \omega t_i + C \tag{4}$$

where $A = \hat{\varphi}_M \cos \varphi_S$; $B = \hat{\varphi}_M \sin \varphi_S$; C is a constant; $i = 0, 1, 2, \ldots, N$; $\omega = 2\pi f$; ϕ_S is the initial phase angle of the displacement and N is the number of samples.

From the values of A and B the phase amplitude and phase angle are determined using

$$\hat{\varphi}_M = \sqrt{A^2 + B^2} \tag{5}$$

$$\hat{\varphi}_0 = \arctan\left(\frac{B}{A}\right). \tag{6}$$

3. Interferometer

A block diagram of the interferometer is shown in figure 1. The interferometer is constructed around a 632 nm, frequency stabilized He–Ne laser. The laser beam is folded through 90˚ as the laser is mounted horizontally and a vertical measurement beam is required for the application. The beam passes through a 10 mm × 10 mm × 10 mm non-polarizing beam splitter, providing a 50 : 50 split of the laser beam. These two beams form the reference and measuring beams of the interferometer, respectively. All the optics is mounted on an optical equipment mounting frame, making the mounting and alignment of the optics much simpler. This configuration also improves the stability and robustness of the interferometer.

The laser is mounted with its polarization at 0˚ with the measurement beam. The quarter-wave plate is mounted in the reference arm between the 50 : 50 beam splitter and the reference mirror at 45˚ to the laser beam polarization. The reference beam passing through the quarter-wave plate is circularly polarized. On the beam's return, after being reflected from the reference mirror, the circularly polarized beam is linearly polarized on passing through the quarter-wave plate. The beam polarization is now in quadrature with the measurement beam. The adjustment of the two beams of the interferometer to obtain a 90˚ phase difference is achieved by using an adjustable rotary support. Fine adjustment of the rotary support is achieved using a micrometer. This arrangement allows for easy and fine adjustment of the phase difference between the optical signals. The combined beam is passed through a polarizing beam splitter that separates the two polarizations into the two quadrature beams.

4. Hardware

The sine-approximation method does not deliver real-time results. Its implementation requires that the relevant data be sampled and stored. The data are post-processed to obtain the measurement result. As a result, an aspect that requires careful consideration for the successful implementation of this method is the selection of the hardware to be used. Data capturing and storage hardware are of particular importance. Various hardware configurations have been implemented by NMIs with success, and each system has both advantages as well as disadvantages. The PTB implemented the method on a VAX computer system [3]. This system boasts excellent specifications as far as sampling frequency and storage capacity (50×10^6 samples per channel) are concerned. However, from a smaller NMI's point of view, it comes with too high a price tag. Other institutes use wave memory recorders [5] or digital oscilloscopes [7] with capacities of the order of 4000 samples per channel. In the latter case, the small number of samples limits the system's performance.

The criteria that were used for selecting a cost effective data sampling and storage system were

(a) sampling rate: 1 MHz and higher;
(b) number of simultaneous sampling channels: 3;
(c) minimum data points per channel: 2×10^6.

Commercially available systems seem to have a trade-off between sampling rate and the number of samples that can be captured (data throughput). After comparing various options that included instruments as well as PC based units, a suitable modular PC based system, which fulfilled all the requirements, was obtained.

The PC based A to D card system consists of a product range by Sundance, namely an SMT 320 carrier card that can accommodate four daughter boards. The three channels of data are sampled using two SMT 340, dual channel, A to D converters. Each SMT 340 is capable of sampling rates of up to 40 MHz and has a 12-bit resolution. Each A to D module is linked to an SMT 332 DSP controller via a high speed data bus. The SMT 332 DSP unit has 16 MB of SDRAM on board with a maximum data throughput of 20 MHz. In short, the complete system can store up to four million, 12-bit samples, of four simultaneously sampled channels, at a data throughput of 20 MHz.

When comparing the system specification with minimum requirements for compliance with the ISO 16063-11 method 3 for performing accelerometer calibrations over the frequency range 1 Hz to 10 kHz the following conclusions can be drawn:

- In order to fulfil the Nyquist theorem requirement for the minimum sampling rate over the specified frequency range and required minimum acceleration amplitude, a sampling rate of at least 5 MHz is required. This is determined considering the maximum fringe frequency that is obtained over the frequency and acceleration ranges to be measured: that is, $\approx$2 MHz considering a vibration frequency of 20 Hz and a peak-to-peak displacement of 10 mm. The system is capable of sampling rates up to 20 MHz.
- For the determination of phase calibration information at least three channels are required. Four channels with simultaneous sampling are available.
- It is recommended that at least one vibration cycle be sampled in order to meet the accuracies achievable using the sine-approximation method. This relates to a theoretical minimum number of 1×10^6 samples per channel (for a vibration frequency of 1 Hz at a sampling rate of 1 MHz for one vibration cycle). The system has the capability to store 4×10^6, 12-bit samples.

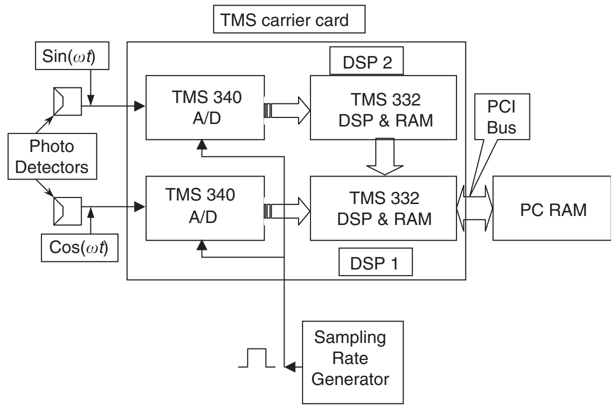

Figure 2. Block diagram showing the photodetectors with data acquisition hardware.

- ISO 16063-11 specifies a minimum analogue to digital converter resolution requirement of 8 bits for the sampling of the two interferometric signals and a 12-bit resolution for sampling the accelerometer output voltage. The system described has a 12-bit A to D resolution.

The data acquisition card is fitted into a 200 MHz Pentium PC with 256×10^6 bytes of RAM. Originally, it was thought that the long processing time might be a drawback during the signal processing stage. In actual fact, disk storage could be a bigger problem, as the data files generated from the large number of data points are stored in ASCII format prior to processing. The most time consuming process during a single measurement is the downloading of the data from the acquisition card and writing it to the hard disk drive of the PC.

Figure 2 shows a block diagram of the flow of data from the output of the two photodiodes to the PC RAM. The quadrature signals are digitized by the two A to D converters (TMS 340) which are simultaneously clocked using an external square wave generator. The generator is used to set the sampling rate. The data stream is read from the FIFO buffer of each A to D converter into the SDRAM onboard the two individual TMS 332 DSP controllers. The first TMS 332 on the carrier card acts as the communication host between the PC and both the TMS 332 controllers. Data from channel one, stored on DSP 1, are read directly from the first DSP into the PC RAM. The data from the second channel, stored on DSP 2, are read via DSP 1 into the PC RAM.

Currently, only the modulus of the accelerometer sensitivity is determined. This requires just two channels of the data acquisition system to be used with the ability to sample up to 8×10^6 data words. The possibility of using the onboard DSP processors for all the signal processing exists. This option is currently not used as all or most of the onboard memory is used for storing the data samples. Future developments might look at this as a possibility, but with the processing powers of existing PCs this is not a requirement.

5. Software

The sine-approximation method requires extensive signal processing of the sampled data in order to obtain results. The extensive processing required for this method is partly due to the complexity of the method, but it is also due to the repetition of calculations as well as the large number of data samples that need to be processed.

The software was developed using Delphi. As a powerful Microsoft Windows® platform software development program, Delphi lends itself excellently to the task. During the development stage, the sampled data retrieved from the data acquisition card were first written to an ASCII format file before being processed. When used in this configuration, a large (up to gigabyte) disk storage capacity is required. Working with data files was convenient during the development of the system as they could be used repeatedly to develop and debug the signal processing software. The final software retrieves the sampled data directly from the acquisition card into the PC's RAM. It then performs all the signal processing, determines the UUT's sensitivity and writes only the result to a very small ASCII file. The software allows one to write the sampled data to an ASCII file prior to post-processing. This feature can be set by selecting the relevant option.

The signal processing for homodyne successive-approximation systems can be described as per the flow diagram shown in figure 3. From the flow diagram, it is evident that the following procedures and functions need to be performed by the software:

(a) *Low-pass and high-pass filtering.* The use of second-order Butterworth filters is recommended because of the flatness of their pass band. The design and implementation of such filters is simple and well documented. The filter coefficients were designed using Matlab®. Each filter is applied twice and the data are filtered forward and in reverse to obtain a fourth-order filter and to correct for phase shifts.

(b) *Quadrature correction.* When required, suppression of disturbances in the quadrature signals is corrected for by using the techniques described in [8].

(c) *Modulation phase values.* Delphi has a mathematical unit with a built in function, Arctan2(Y, X). The function determines the arctangent within the correct quadrant. Performing the phase unwrapping as described in [4, 9] was considered. Using the phase unwrapping technique in MatLab® routines proved to be a more convenient method and was therefore implemented. Figure 4 shows the modulation phase values obtained from sampled interferometer outputs U_1 and U_2.

(d) *Magnitude and phase calculation.* The acceleration magnitude (5) and phase (6) is calculated by solving for three unknowns (A, B and C) (4) in a system of $N + 1$ equations. This is achieved using the least squares approximation.

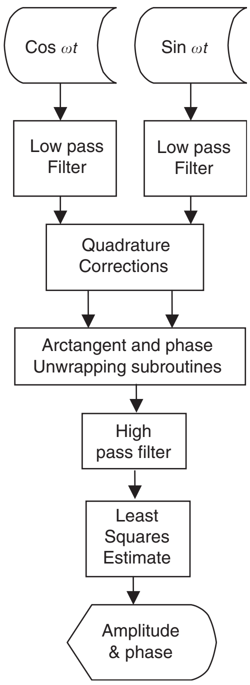

Figure 3. Signal processing flow diagram.

6. Data processing

Studying the graphical representation of the data after each step during the signal processing phase is a very simple yet efficient tool for evaluating results during the development phase. Figure 5 shows the low pass filtered sampled data points of one of the two photodiode outputs. A little more than half a vibration cycle is shown. This is a typical example of the phase modulated signal produced by the interferometer.

The quadrature corrections are achieved using the method described in [8]. The actual matrix calculations are performed using a Pascal procedure from [10]. Figure 4 shows the result after the arctangent and phase unwrapping routines. It is clearly seen from this graph that a 'smooth' sine wave with no arctangent phase jumps is obtained. This result is obtained using the MatLab® phase unwrapping algorithm.

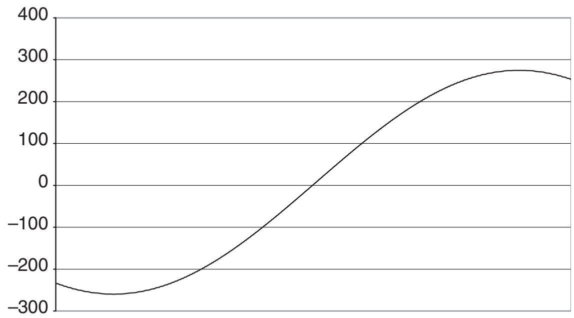

Figure 4. Modulation phase values after applying the Arctan2(Y, X) function and MatLab phase unwrapping routine.

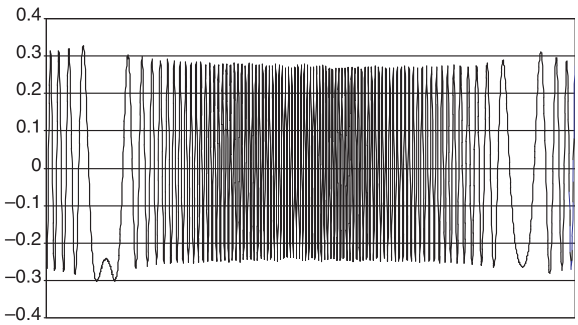

Figure 5. Low pass, filtered, phase modulated output signal from the photodiodes.

The final data processing step—determining the acceleration using (4)—has to be done carefully. The process involves, first, solving the three unknown parameters A, B and C using the least squares method for $N + 1$ equations (4). The modulation phase amplitude ($\hat{\varphi}_{\mathrm{M}}$) is then calculated using A and B from (5). This may be accomplished using one of the following sequences:

(a) Determine the average for all the A and B values obtained and then calculate the modulation phase amplitude:

$$\bar{\hat{\varphi}}_{\mathrm{M}} = \sqrt{[\bar{A}]^2 + [\bar{B}]^2}. \tag{7}$$

(b) Determine the modulation phase amplitude for each A and B calculated and then calculate the average modulation phase amplitude.

$$\bar{\hat{\varphi}}_{\mathrm{M}} = \overline{\sqrt{A^2 + B^2}}. \tag{8}$$

Although method (8) can be a little easier to accomplish in software, there exists the possibility of a small positive error being produced due to any errors present in the calculation of A and B. This is due to the fact that the squares of A and B will always be positive when used to calculate the modulation phase amplitude as per (5). The method described in (7) above is therefore preferred, as it will minimize such an error.

Another important consideration for this method is the number of data samples used. It is important to sample the data for as many acceleration cycles as possible. The successive-approximation method, as with methods 1 and 2 described in ISO 16063-11, requires that the vibratory motion be sampled over a long period for improved measurement results.

7. Data simulation

The data processing capability of the system was evaluated over the frequency range 10 Hz to 50 kHz. For this, sampled data were simulated by generating data files equivalent to the data files that would have been created during the calibration of an accelerometer. These simulated data files were created using software techniques. Table 1 summarizes, among other things, the frequencies and vibration levels that were used to evaluate the experimental results.

In an effort to ensure a realistic evaluation, using simulated data files, some error components, which can be considered as 'noise' and disturbances, were added to the experimental data. Considering (1) and (2), the disturbances were added on three levels:

- The voltage amplitudes of the detector output, $\hat{u}_1$ and $\hat{u}_2$, were randomly varied by up to 6%. This was achieved using a random number generator with a Gaussian distribution about a mean of 0.45 with a standard deviation of 0.03. In practice, the voltage amplitude is nominally 0.5 V.
- The phase modulation of both quadrature signals was randomly varied by 0.05 radians. This was achieved using a random number generator with a Gaussian distribution about a mean of 0.05 with a standard deviation of 0.000 5.
- A low frequency sinusoidal disturbance, hum, was added to both quadrature signals, $u_1(t)$ and $u_2(t)$.

Figures 6 and 7 are X–Y plots and illustrate the effect of the added disturbances on the experimental data. Figure 6 has no disturbance added to the signal, while figure 7 has all the previously described disturbances added.

The successive-approximation software was used to process the experimentally generated data. The acceleration levels obtained were compared with the acceleration levels that were simulated. Table 2 lists the acceleration levels calculated using the experimental data with and without noise and disturbances.

The acceleration levels generated compared to within 0.06%, up to 8 kHz, and 0.5%, up to 50 kHz, with the acceleration levels determined using the experimental data. A worst case standard deviation of 0.33% for determining the acceleration levels was at 50 kHz. Figure 8 shows a plot of the deviation of the calculated acceleration level from the experimentally generated acceleration level over the frequency range 10 Hz to 50 kHz. The apparently large deviation in the acceleration level determined at 500 Hz is due to the high pass filter. When the filter is not applied, no difference in the acceleration level is apparent.

Figure 9 is a plot of the standard deviation of the mean obtained by determining the acceleration levels over the frequency range 10 Hz to 50 kHz, using the successive-approximation software.

Table 1. Experimental data frequency and vibration points.

Frequency/Hz	Acceleration/ m s^{-2}	Displacement/m	Fringe frequency/Hz	Cycles	Sample rate/MHz	Number of samples
10	20	5.07×10^{-3}	503 002	4	5	2 000 000
20	50	3.17×10^{-3}	628 752	5	5	1 250 000
40	100	1.58×10^{-3}	628 752	10	5	1 250 000
80	100	3.96×10^{-4}	314 376	10	2	250 000
160	100	9.89×10^{-5}	157 188	10	2	125 000
200	100	6.33×10^{-5}	125 750	20	2	200 000
500	100	1.01×10^{-5}	50 300	25	2	100 000
1 000	100	2.53×10^{-6}	25 150	50	2	100 000
2 000	100	6.33×10^{-7}	12 575	100	2	100 000
4 000	100	1.58×10^{-7}	6 287	100	2	50 000
8 000	100	3.96×10^{-8}	3 143	200	2	50 000
10 000	100	2.53×10^{-8}	2 515	500	2	100 000
20 000	100	6.33×10^{-9}	1 257	500	5	125 000
50 000	100	1.01×10^{-9}	503	1000	5	100 000

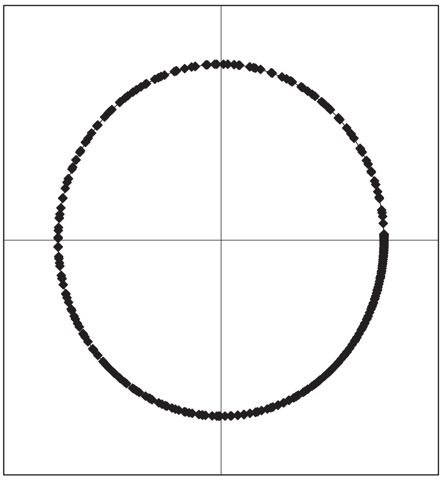

Figure 6. X–Y plot of simulated data without any disturbances added.

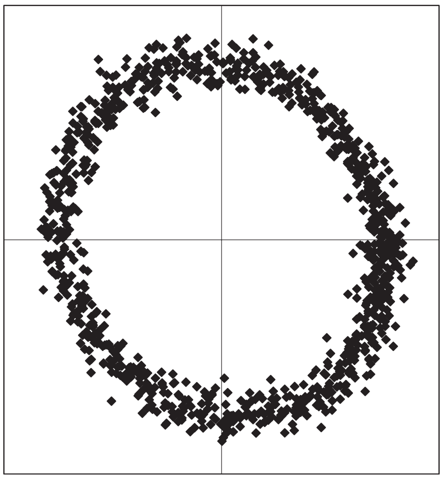

Figure 7. X–Y plot of simulated data with all the disturbances added.

8. Experimental results

The vibration calibration system was used to determine the sensitivity values of a single-ended laboratory standard

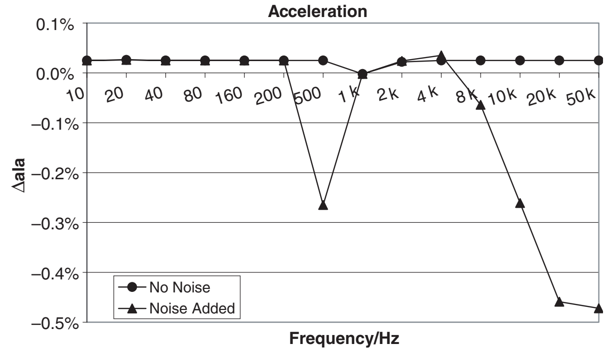

Figure 8. Acceleration levels obtained, relative to the nominal experimental acceleration level.

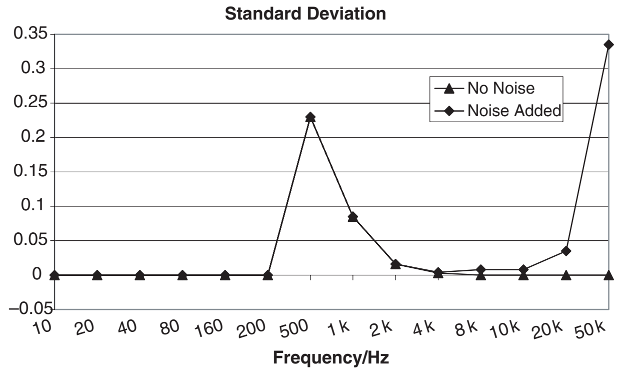

Figure 9. Associated uncertainties of measurement for experimental data with and without noise added.

accelerometer. The values obtained were compared with the values certified by the manufacturer. The certified values were obtained using a laser calibration system.

Figure 10 shows the sensitivity measurements obtained for a laboratory standard single-ended accelerometer. The results shown are the averages of three measurements per frequency point with error bars indicating the expanded measurement uncertainty associated with the measurements. The measurement results shown from 500 Hz to 10 kHz were obtained using the system described herein.

Table 2. Uncertainty values obtained with experimental data.

Frequency/Hz	Nominal acceleration/m s^{-2}	No noise added		Noise added	
		Acceleration/m s^{-2}	Uncertainty/m s^{-2}	Acceleration/m s^{-2}	Uncertainty/m s^{-2}
10	20	20.005	0	20.005	0
20	50	50.013	0	50.013	0
40	100	100.025	0	100.025	0
80	100	100.025	0	100.025	0
160	100	100.025	0	100.025	0
200	100	100.025	0	100.025	0
500	100	100.025	0.23	99.735	0.23
1 000	100	99.998	0.085	99.998	0.085
2 000	100	100.022	0.016	100.024	0.016
4 000	100	100.025	0.003	100.035	0.004
8 000	100	100.025	0	99.936	0.008
10 000	100	100.025	0	99.739	0.008
20 000	100	100.025	0	99.541	0.035
50 000	100	100.025	0	99.528	0.335

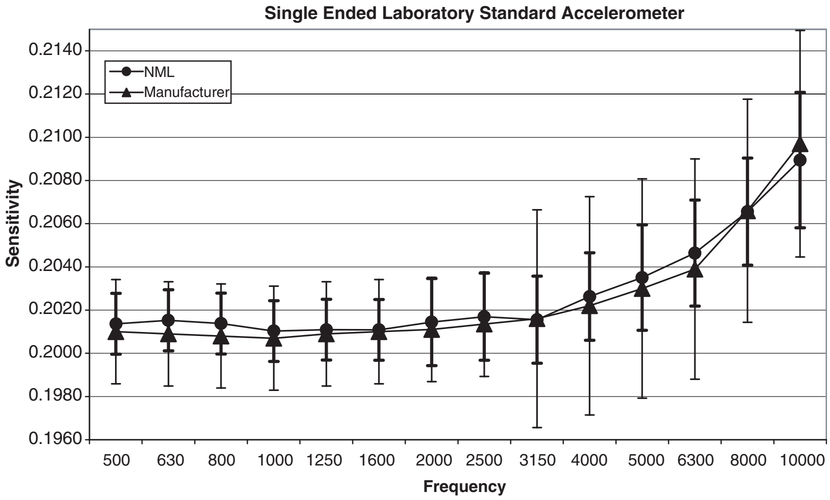

Figure 10. Sensitivity values obtained for a single-ended accelerometer.

Table 3. Sensitivity values obtained with associated uncertainties.

		Successive-approximation		Manufacturer		
Frequency/Hz	Nominal acceleration/m s^{-2}	Sensitivity/ pC m^{-1} s^2	100 × Uncertainty	Sensitivity/ pC m^{-1} s^2	100 × Uncertainty	100 × Difference (NML − manufacturer)
500	100	0.2014	0.7	0.2010	1.2	0.18
630	100	0.2015	0.7	0.2009	1.2	0.31
800	100	0.2014	0.7	0.2008	1.2	0.29
1 000	100	0.2010	0.7	0.2007	1.2	0.16
1 250	100	0.2011	0.7	0.2009	1.2	0.10
1 600	100	0.2011	0.7	0.2010	1.2	0.04
2 000	100	0.2014	1.0	0.2011	1.2	0.17
2 500	100	0.2017	1.0	0.2014	1.2	0.17
3 150	100	0.2016	1.0	0.2016	2.5	−0.02
4 000	100	0.2026	1.0	0.2022	2.5	0.21
5 000	100	0.2035	1.2	0.2030	2.5	0.25
6 300	100	0.2046	1.2	0.2039	2.5	0.36
8 000	100	0.2066	1.2	0.2066	2.5	−0.02
10 000	100	0.2089	1.5	0.2097	2.5	−0.36

In table 3, the results obtained using the method described and the values certified by the manufacturer are compared. The expanded uncertainties of measurement at the different frequencies are expressed with a 95% ($k = 2$) confidence level. It is evident from the sensitivities stated in the table that the results compare favourably within the stated levels of uncertainties. The worst-case difference of −0.4% was noted at 10 kHz.

9. Conclusions and remarks

An implementation of method 3 of ISO 16063-11 has been described. The interferometer is robust and can be aligned accurately in order to realize the required quadrature optical signals.

The data acquisition requirements for the system have been met using a modular PC plug-in card. This set-up allows for a system that can be adapted to suit the laboratory's requirements. With the acquisition card's data storage capacity, using a sampling frequency of 1 MHz, sampling periods of up to 4 s are obtainable. By selecting an optimum sampling rate, the system can currently be used to perform calibrations down to 40 Hz.

The computational power of the modern PC should not be underestimated. Implementing the described method requires, literally, millions of calculations, which are completed in a matter of seconds. This level of performance was achieved using a PC that is two generations old. Developing systems where the PC forms the soul of the measurement process greatly enhances the following:

(a) development process,
(b) system versatility,
(c) compatibility,
(d) adaptability.

A Microsoft Windows® based software program was developed and implemented. The software fulfils the minimum signal processing requirements of ISO 16063-11. A modular design approach was followed with the required algorithms developed in-house. The software program compiles into a single executable file that can be run from any Windows® based PC. Provided that the sampled data of the X, Y and accelerometer output are stored in a comma-delimited ASCII file, data from any data acquisition system can be processed by the program. The results can be analysed step by step, or the complete process can be performed automatically.

Good agreement exists between the results obtained using the successive-approximation method and the results obtained using the ratio-counting method at the frequency points 500 Hz, 630 Hz and 800 Hz. The results obtained using the new system compared well ($<1\%$) with those specified by the manufacturer over the frequency range 500 Hz to 10 kHz.

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