# Experimental validation of tip over stability of a tracked mobile manipulator

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Abstract—Mobile manipulators are highly susceptible to tip over due to the motion of the manipulator or the gradient of the slope being traversed by the robot's mobile base. This paper presents the experimental validation of the tip over stability analysis of a tracked mobile manipulator. The Force Angle stability measure is used to compute the stability index of the platform. A refined model of a tracked mobile manipulator is used to determine the ground contact points which form an input into the stability formulation. A Vicon motion capture system is used to determine the actual time of tip over of the robot. It is found that the tip over model is able to determine the tip over occurrence ahead of the actual tip over time in the majority of the test cases.

# I. INTRODUCTION

Mobile manipulators are robotic platforms that consist of a robotic arm mounted on a tracked, legged or wheeled base. They are used in a number of different applications such as mining robotics, bomb disposal, and search and rescue operations. The CSIR is developing a Mine Safety Platform (MSP) for post blast safety inspection of South African gold and platinum mines. Within South African underground gold and platinum mines, mining occurs in areas called stopes. Mining occurs in three phases, drill, blast and scrape. After blasting, the mine hanging wall (roof has to be inspected for loose rock that could potentially fall and cause injuries. This inspection is typically performed by a miner who is in constant danger while performing the task. The aim of the MSP project is to remove the miner form danger by sending in a robot to perform the post-blast inspection. The platform will explore and create a map of the mine stope and it's manipulator will then perform inspections of the hanging wall.

An iRobot PackBot510 robot is currently being used during the development of the software for the MSP project. While in operation, mobile manipulators, such as the PackBot, are at risk of tip over due to the motion of the manipulator or the gradient of the slope being traversed by the robot. In order to perform post-blast inspections safely, tip over stability of the robot has to be maintained. This paper presents the experimental validation of the tip over stability analysis for a tracked mobile manipulator, the PackBot510 (see Figure 1).

A number of stability measures and algorithms have been developed by various researchers to assess the stability of a robot and predict tip over conditions. They include the Zero-Moment Point (ZMP), Force-Angle stability measure



Fig. 1. Tracked mobile manipulator - PackBot510

(FA), and Moment-Height Stability measure (MHS). The Zero-Moment Point is a point on the ground where the sum of all the forces and moments acting on the robot platform can be replaced by a single force [1]. It was originally derived for stability analysis of bipedal robots, and it has been adapted to other types of mobile robots [2]. A different approach to stability analysis was proposed by Papadopoulos [3] and Rey [4], which they termed the Force-Angle stability measure (FA). The FA algorithm measures the angle of the total applied force on the center of mass of the platform with reference to the support polygon, which is derived from the ground contact points of the robot [5]. Similar to the FA measure, Moosavian and Alipour proposed the Moment-Height Stability measure [6] with also accounts for the robot's inertia about each axis of the support polygon and scales results by the height of the robot's center of mass.

Few studies have attempted to verify the effectiveness of the tip over measures experimentally. Roan et al [7] performed a preliminary real-world comparison of the ZMP, FA, MHS measures using a modified iRobot PackBot Fido. The mobile robot platform was dynamically modeled in software, and the ZMP, FA, and MHS algorithms were coded. The robot was then fitted with an inertial measurement (IMU) based data collection system and driven over various obstacles and the data used to calculate the tip over measures over time [7]. Roan et al results show that the FA and MHS had very similar performance and performed better than the ZMP. In their test, the flipper and manipulator are kept at a single configuration and so the results do not take into consideration their effects on the stability of the platform.

Full validation of tip over algorithms requires extensive real world testing with different robot platforms at varied configurations. In this work, the real world validation of the FA measure for an iRobot PackBot510 at differing manipulator and flipper configurations is carried out. The PackBot has a manipulator with a 4 degree of freedom (DOF) arm and a 2 DOF camera and two flippers at the front of the platform as shown in Figure 1. The static Force Angle stability measure formulated by Papadopoulos and Rey [5] is used here to compute the stability index of the platform. A model of the PackBot in relation to the flipper's state of contact with the ground was developed by this auther [8]. This paper extends the PackBot model to take into account the change in wheel position due to the flipper motion. The effectiveness of the extended model and the stability index are then tested using a Vicon motion capture system to monitor the tip over motion of the PackBot.

Section one of the paper gives a brief description of the tip over measure used. In section two the extended PackBot model is given. Section three describes the experimental set up. The experimental results and discussion are given in section four. Concluding remarks are then given in the final section.

### II. THE FORCE ANGLE STABILITY MEASURE [5]

The FA measure [5], is used in this study and is summarised in this section. The FA measure is [5]:

$$\beta = \min\left(\theta_i \cdot \|\boldsymbol{d_i}\| \cdot \|\boldsymbol{f_r}\|\right)$$

Tip over instability occurs when  $\beta$  approaches zero. The angle  $\theta_i$  captures the effect of changes in the system center of mass (C.M.) height along the net force vector  $f_r$  and the distance  $||d_i||$  captures the effect of changes in the moment contribution of the net force. Weighing by the magnitude of  $f_r$  captures effects of total weight of the system since the disturbance force required to tip the vehicle becomes smaller as the magnitude of  $f_r$  decreases.

To derive the general case of the Force Angle measure, the location of a ground contact point of the platform is  $p_i$  and  $p_c$  represents the location of the system C.M. The lines which join the ground contact points are the candidate tip over mode axes,  $a_i$ . (See Figure 2.)

$$a_i = p_{i+1} - p_i$$

The tip over axis normals  $l_i$  which pass through the C.M. are:

$$l_i = \left( \boldsymbol{I} - \hat{\boldsymbol{a}}_i \hat{\boldsymbol{a}}_i^T \right) \left( \boldsymbol{p}_{i+1} - \boldsymbol{p}_c \right)$$

where I is the  $3 \times 3$  identity matrix.

The force component acting about each tip over axis is:

$$f_i = (I - \hat{a}_i \hat{a}_i^T) f_r$$

and the moment component is:



Fig. 2. Force Angle Stability Measure - 3d, (adapted from [5])

$$\boldsymbol{n_i} = (\boldsymbol{\hat{a}_i} \boldsymbol{\hat{a}_i}^T) \, \boldsymbol{n_r}$$

The effective net force vector is:

$$egin{aligned} egin{aligned} egin{aligned} egin{aligned} eta_i^* &= eta_i + rac{ell_i imes eta_i}{\left\| \hat{l}_i 
ight\|} \end{aligned}$$

The distance of the force from the tip over axes is:

$$oldsymbol{d}_{oldsymbol{i}} = -oldsymbol{l}_{oldsymbol{i}} + \left(oldsymbol{l}_{oldsymbol{i}}^T \cdot \hat{oldsymbol{f}}_{oldsymbol{i}}^st 
ight) \hat{oldsymbol{f}}_{oldsymbol{i}}^st$$

The angle of the force from the tip over axes is:

$$\theta_i = \sigma_i \cos^{-1} \left( \hat{l}_i \cdot \hat{f}_i^* \right)$$

where:

$$\sigma_i = \begin{cases} +1 & \left( \hat{f}_i^* \times \hat{l}_i \right) \cdot \hat{a} \\ -1 & otherwise \end{cases}$$

## III. MODELLING THE PACKBOT

The modelling of the PackBot manipulator and flipper pose is described in this section. This formulation can be generalised for any robot with a similar configuration to the PackBot. The position of the PackBot  $p_c$  is measured at the center of the bottom of the PackBot base. The direction vector of the PackBot is  $s_{pb}$ , the normal of the PackBot is  $n_{pb}$ , and the front of the PackBot is  $w_{pb}$ , as shown in figure 3. Given the joint angles of the manipulator links, the manipulator links are modelled using the Denavit-Hartenburg convention to obtain the forward kinematics of robot arms [9]. From the forward kinematics, the location  $q_j^*$  of the *j*th link is obtained. The center of mass of each link  $cm_{qj}$  is placed at the center of the link and the base center of mass is placed at the center of the PackBot base.

### A. Ground contact points

The location of ground contact points are governed by the flipper angle and the orientation of the PackBot. Flippers can either be up or in contact with the ground as shown in Figure 3. The minimum and maximum angles, for which the flippers will be in contact with the ground are,  $\phi_{min}$  and  $\phi_{max}$ .



Fig. 3. Flipper positions and ground contact points

The previous model of the ground contact points [8], assumed that the ground contact points  $p_1$  and  $p_4$  remain stationary when the flippers come into contact with the ground. However, the contact points move along the circumference of the back of the track to positions  $p_{1a}$  and  $p_{4a}$  as shown in Figure 3. The previous model also did not take into account the width of the flippers in determining the contact points. The model of the PackBot is thus extended in this section to take into account the above mentioned points.

1) Flippers up: The flippers are up when the flipper angle  $\phi_f$  is outside the range of  $\phi_{min}$  and  $\phi_{max}$  i.e.  $\sin(\phi_f) > \sin(\phi_{min})$ . The position  $p_0$  is:

$$oldsymbol{p_0} = oldsymbol{p_c} - oldsymbol{s_{pb}} imes rac{L_{pb}}{2}$$

The ground contact points are measured relative to  $p_0$  along the robot's direction, normal and the front vectors:

$$egin{aligned} p_1 &= p_0 - w_{pb} imes rac{W_{pb}}{2} \ p_4 &= p_0 + w_{pb} imes rac{W_{pb}}{2} \ p_2 &= p_1 + s_{pb} imes L_{pb} \ p_3 &= p_4 + s_{pb} imes L_{ab} \end{aligned}$$

Where  $L_{pb}$  is the length of the base,  $W_{pb}$  is the width of the base,  $H_{pb}$  is the height of the base. The slope vector and normal vector of the environment in the direction of the PackBot are equal to the PackBot direction and normal vectors.

2) Flippers down: Once the flippers are in contact with the ground, i.e. if  $\sin(\phi_f) \leq \sin(\phi_{min})$ , the flipper motion changes the configuration of the PackBot. The position of  $p_0$  changes along the circumference of the tracks. To calculate the configuration based on the flipper down angle, the contact points become, given the direction, normal and the front vectors of the PackBot:

$$oldsymbol{p_{0a}} = oldsymbol{p_c} - oldsymbol{s_{pb}} imes rac{L_{pb}}{2} + oldsymbol{n_{pb}} imes H_{wc} - oldsymbol{s_{pb}} imes V_1 + oldsymbol{n_{pb}} imes V_2$$

where  $H_{wc}$  is the height of the wheel centre, and  $V_1$  and  $V_2$  represent the distance that the contact point  $p_0$  moves from its original position along the PackBot direction and normal vectors. The ground contact points are then:

$$p_{1a} = p_{0a} - w_{pb} \times \frac{W_{pb}}{2}$$
$$p_{4a} = p_{0a} + w_{pb} \times \frac{W_{pb}}{2}$$
$$p_{2a} = p_{1a} + s_p \times L_3 - w_{pb} \times W_f$$

$$p_{3a} = p_{4a} + s_p \times L_3 + w_{pb} \times W_f$$

Where  $L_3$  is the perpendicular distance of the back contact points from the front contact points. From  $\phi_7$ , the angle of the PackBot above the ground, the slope vector and normal of the ground in the direction of the PackBot are given as:

$$s_{p} = s_{pb} \times \cos(\phi_{7}) + n_{p}b \times \sin(\phi_{7})$$

$$m{n_p} = m{s_p} imes m{w_p}m{b}$$

The lengths and angles as shown in figure 4 are as follows:



Fig. 4. Flipper down contact points

 $V_2$  and  $V_1$  capture the distance that the back contact points move by from the original position:

$$V_2 = \sqrt{H_{wc}^2 - V_1^2}$$
$$V_1 = \frac{H_{wc}\sin(\phi_7)}{\sin(\pi)}$$

 $L_3$  is the perpendicular distance of the back contact points from the front contact points

$$L_3 = \sqrt{L_4^2 - (H_{wc} - R_f)^2}$$

 $\phi_7$  is the angle of the PackBot above the ground:

$$\phi_7 = \pi - \phi_1 - \phi_5 - \phi_6 - \phi_B$$

The lengths  $L_3$ ,  $V_1$ ,  $V_2$ , and the angle  $\phi_7$  are found using trigonometry. The angles used are:

$$\phi_{B} = \pi - \phi_{1} - \phi_{f}$$

$$\phi_{1} = \arccos\left(\frac{L_{pb}}{L_{1}}\right)$$

$$\phi_{5} = \arccos\left(\frac{L_{3}^{2} + L_{4}^{2} - (H_{wc} - R_{f})^{2}}{2L_{3}L_{4}}\right)$$

$$\phi_{6} = \arccos\left(\frac{L_{f}^{2} + L_{4}^{2} - L_{pb}^{2}}{2L_{f}L_{4}}\right)$$

$$\phi_{2} = \arccos\left(\frac{L_{1}^{2} + L_{2}^{2} - L_{f}^{2}}{2L_{1}L_{2}}\right)$$

$$\phi_{4} = \frac{\pi}{2} - \phi_{1}$$

and the lengths are:

$$L_4 = \sqrt{L_2^2 + H_{wc}^2 - 2H_{wc}L_2\cos(\phi_4 + \phi_2)}$$
$$L_2 = \sqrt{L_1^2 + L_f^2 - 2L_1L_f\cos\phi_B}$$
$$L_1 = \sqrt{L_{pb}^2 + H_{wc}^2}$$

# IV. EXPERIMENTAL SET UP

The PackBot's tip over stability was modelled in Matlab and C++, using the FA stability measure and the PackBot model described in the previous sections. In this work, the static FA measure is used, which takes into account only the gravitational forces acting on the robot. It is assumed that the manipulator and flipper motion are at low enough speeds that the dynamic effects of their motion can be ignored. It is also assumed that all manipulator links have a center of mass at the geometric center of each link.

Pose data of the platform is collected using a Vicon motion capture system. Markers are placed on the PackBot and these then give the position and orientation of the PackBot within the motion capture system. Joint angles of the manipulator are also recorded to give the position of the links. The flipper angle is recorded to determine the ground contact points of the PackBot. Figure 5 shows the experimental set up.

The orientation of the PackBot, obtained from the motion capture system, is used to determine the tip over state of the platform. The stable state is when the motion capture orientation is the same as the PackBot's staring orientation. When tip over occurs, the orientation of the PackBot changes as the robot tips over a particular axis.



Fig. 5. Test set up - PackBot at starting position

Overall, 16 tests are conducted. First, for Test-Set A, the PackBot flippers are placed in the up position, with the PackBot on flat ground, and the manipulator is moved to specified joint angles. For Test-Set B, the PackBot flippers are placed in the down position at  $65^{\circ}$ , putting them into contact with the ground. In total, 8 tests are conducted for each Test-Set. The shoulder and the two elbow joints for all of the Test-Sets move from angles  $S_{init}$ ,  $E1_{init}$ ,  $E2_{fin}$  to  $S_{fin}$ ,  $E1_{fin}$ ,  $E2_{fin}$ . Table I shows the initial and final angles. The turret angle for the Test-Sets moves from  $T_{init}$  and changes at intervals from  $0^{\circ}$  to  $360^{\circ}$  for each separate test. This is in order to test tip over at different axis of the PackBot. Table II and Figure 6 show the initial and final angles for the turret.

TABLE I Test Joint Angles

Joint	Initial angle	Final angle
Shoulder	$-151^{o}$	$-160^{o}$
Elbow 1	$151^{o}$	$38^o$
Elbow 2	$-180^{o}$	$-23^{o}$

TABLE II Test Turret joint angles.

Test Number	Initial angle	Final angle
Test .1	00	00
Test .2	00	$45^{o}$
Test .3	00	90°
Test .4	00	135°
Test .5	$0^{o}$	$-180^{o}$
Test .6	00	$-135^{o}$
Test .7	00	$-90^{o}$
Test .8	$0^{o}$	$-45^{o}$



Fig. 6. Manipulator turret joint test angles

Each test starts with the PackBot manipulator joint angles at their initial or starting positions. The manipulator joint angle and PackBot pose are set to remain stationary for 50 time frames and the motion of the manipulator then starts. A single joint angle command with the final joint angles of the manipulator is sent at frame 51. The PackBot manipulator's motion planner then plans and executes the motion for the joints to reach the desired final position. The robot is allowed to tip over freely and is then caught by an operator to prevent collision with the ground. Both the manipulator joint angle data and the robot's pose data are collected.

While on-line tip over computation is possible, the tip over computation is conducted off-line. For each of the different tests, two computations of the tip over measure are examined. The first uses the assumption that the contact points of the PackBot are in contact with the ground throughout the test. This is similar to the case when motion planning is computed off line to predict if a desired motion path of the manipulator would result in tip over. The second case uses the pose of the PackBot obtained from the Vicon motion capture system as input to determine the ground contact points. This is similar to a real-time case when the stability is determined for the current pose and configuration of the platform.

### V. RESULTS

Figure 7 shows the results for Test B.7. The robot's manipulator and flipper joint angles, base orientation angles and both stability indexes are displayed. Motion of the manipulator joint angles starts at frame 51. The starting orientation angles of the base are set to  $0^{\circ}$ . Tip over can be seen by the rapid increase in any one of the orientation angles, in this case the roll angle and to a smaller extent the yaw angle. The leveling off of the roll and yaw angles seen is due to the PackBot being caught and steadied by an operator to avoid collision of the robot with the floor.

Tip over starts at the frame when one of the orientation angles is greater than a threshold  $t_o$ . The angle with the largest overall change is used to determine tip over time. A threshold of  $0.6^o$  was chosen for the tip over point. This corresponds to a lift off of the relevant contact points from the floor of about 3 to 5 mm depending on the tip over axis.



Fig. 7. Results Test - B.7

Of Test-Set A, with the flippers up, 3 of the tests (A.1, A.2, A.8) result in no tip over occurring. (See Table III ). These correspond to a turret position that places the manipulator towards the back of the PackBot. With the flippers in contact with the ground, Test-Set B, only two tests (B.3, B.7) result in tip over. These correspond to when the manipulator is placed at either side of the PackBot. With flippers down, the support polygon along the length of the robot is larger, making the PackBot more stable thus there are fewer tip over events for Test-Set B.

The lead time, the difference between the computed and the actual tip over time, of both the real time and predictive FA measures is computed. The cases where no actual tip over occur are ignored as they do not provide any information on the actual tip over time. Table III shows the lead time results for all the tests. In most cases, the computed tip over using the FA measure occurs before actual tip over commences, this is indicated by a positive lead time.

The predictive and real time FA indexes have similar results until a tip over index of zero is reached. The real time index then decreases more rapidly below zero than the predictive. The real time tends to reach tip over slightly faster than the predictive one as can be seen in Figure 7. The real time FA measure predicts tip over slightly faster than the predictive by about 2 time frames as is shown in Table III.

TABLE IIITIP OVER EXPERIMENT RESULTS - LEAD TIME.

Experiment	Predictive	Real-Time			
Set A - Flippers up					
Test A.1	no tip over	no tip over			
Test A.2	no tip over	no tip over			
Test A.3	65	67			
Test A.4	-9	-7			
Test A.5	6	8			
Test A.6	-6	-5			
Test A.7	28	28			
Test A.8	no tip over	no tip over			
Set B - Flippers down					
Test B.1	no tip over	no tip over			
Test B.2	no tip over	no tip over			
Test B.3	63	65			
Test B.4	no tip over	no tip over			
Test B.5	no tip over	no tip over			
Test B.6	no tip over	no tip over			
Test B.7	40	42			
Test B.8	no tip over	no tip over			

Tests A.3 and A.7 have a larger lead time than the other tests in Test-Set A. Tests B.3 and B.7 also have a large lead time. These cases are when tip over happens over the sides of the PackBot. Tip over occurring over the front of the PackBot, test A.5, has a much smaller lead time, while the ones at the corners, tests A.4 and A.6 have a small negative lead. The negative lead time means that they predict tip over after the actual tip over has started to occur.

Table IV shows the computed tip over indexes at the actual time of tip over. An index of 1 is equivalent to stability when the manipulator is stowed at rest. An index of 0 means that tip over is supposed to be occurring. The tests with high positive lead times, A.3, A.7, B.3, B.7, have highly negative stability indexes at the time of actual tip over. The tests with slightly negative lead time, A.4, A.6, show stability indexes that are close to 0. This shows that while they predict the tip over time slightly after the actual tip over has occurred, they still predict that the robot is highly unstable at the actual tip over time. For test A.5, the stability index does not change between the computed tip over time and actual tip over time.

### VI. CONCLUSIONS

The validation of the Force Angle stability measure conducted in this study shows that the FA index is capable of predicting tip over before tip over starts in the majority of cases. Even when tip over is predicted after actual tip over has commenced, the resulting stability index at tip over time

 TABLE IV

 TIP OVER INDEX AT ACTUAL TIME OF TIP OVER

Experiment	Predictive	Real-Time			
Set A - Flippers up					
Test A.3	-0.76	-0.85			
Test A.4	0.05	0.04			
Test A.5	0.00	0.00			
Test A.6	0.01	0.01			
Test A.7	-0.14	-0.14			
Set B - Flippers down					
Test B.3	-0.55	-0.64			
Test B.7	-0.25	-0.30			

is very close to zero, indicating a highly unstable position. To use the FA measure effectively, it would be advisable to pick a stability threshold which demarcates the start of the highly unstable region. The results obtained from this study were from a single implementation of each test. For statistical significance each test should ideally be conducted a large number of times. The tests conducted had the manipulator in motion while the base was stationary. Further testing should include motion of both the base and the manipulator.

### REFERENCES

- Q. Huang, S. Sugano, and I. Kato, "Stability control for a mobile manipulator using a potential method," in *Proceedings of the the IEEE/RSJ/GI International Conference on Intelligent Robots and Systems*, 1994, vol. 2, pp. 839-846.
- [2] J. Kim, W. K. Chung, Y. Youm, and B. H. Lee, "Real-time zmp compensation method using null motion for mobile manipulators," in *Proceedings of the the IEEE International Conference on Robotics and Automation*, 2002
- [3] E. Papadopoulos and D. Rey, "A new measure of tip over stability margin for mobile manipulators," in *Proceedings of the the IEEE International Conference on Robotics and Automation*, 1996, vol. 4, pp. 3111-3116.
- [4] D. A. Rey and E. G. Papadopolous, "On-line automatic tip over prevention for mobile manipulators," in *Proceedings of the the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1997, September
- [5] E. Papadopoulos and D. Rey, "The Force-Angle Measure of Tip over Stability Margin for Mobile Manipulators," Vehicle System Dynamics, 2000, 33, pp. 29-48.
- [6] S. A. Moosavian and K. Alipour, "Stability evaluation of mobile robotic systems using moment-height measure," in *Proceedings of the the IEEE International Conference on Robotics, Automation, and Mechatronics*, 2006, June, pp. 97-102.
- [7] P. R. Roan, A. Burmeister, A. Rahimi, K. Holz, and D. Hooper, "Real-World Validation of Three Tip over Algorithms for Mobile Robots," in *Proceedings of the the IEEE International Conference on Robotics and Automation*, 2010, May, pp. 4431-4436.
- [8] C. Dube, "Modeling the manipulator and flipper pose effects on tip over stability of a tracked mobile manipulator," in *Proceedings of the Robotics* and Mechatronics Conference of South Africa, Nov. 2011.
- [9] Sciavicco, L., Siciliano, B., Modeling and Control of Robot Manipulators, London: Springer-Verlag, 2005.