Modal decomposition without *a priori* scale information

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Abstract: The modal decomposition of an arbitrary optical field may be done without regard to the spatial scale of the chosen basis functions, but this generally leads to a large number of modes in the expansion. While this may be considered as mathematically correct, it is not efficient and not physically representative of the underlying field. Here we demonstrate a modal decomposition approach that requires no *a priori* knowledge of the spatial scale of the modes, but nevertheless leads to an optimised modal expansion. We illustrate the power of the method by successfully decomposing beams from a diode-pumped solid state laser resonator into an optimised Laguerre-Gaussian mode set. Our experimental results, which are in agreement with theory, illustrate the versatility of the approach.

OCIS codes: (070.6120) Spatial light modulators; (120.3940) Metrology; (090.1995) Digital holography; (120.5060) Phase modulation.

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1. Introduction

The decomposition of a light field into a superposition of orthonormal basis functions, so-called modes, has been known for a long time [1, 2]. There are clear advantages in executing such modal decompositions of superpositions (multimode) of laser beams, and several attempts have been made with varying degrees of success [2–6]. To be specific, if the underlying modes that make up an optical field are known (together with their relative phases and amplitudes), then all the physical quantities associated with the field may be inferred, e.g., intensity, phase, wavefront, beam quality factor, Poynting vector and orbital angular momentum density. Despite the appropriateness of the techniques, the experiments were nevertheless rather complex or customised to analyse a very specific mode set. Recently this subject has been revisited by employing computer-generated holograms for the modal decomposition of emerging laser beams from fibres [7–9], for the real-time measurement of the beam quality factor of a laser beam [10], for the determination of the orbital angular momentum density of light [11, 12] and for measuring the wavefront and phase of light [13]. All these techniques rely on knowledge of the scale parameter(s) within the basis functions chosen. For example, in the case of free space modes the beam width of the fundamental Gaussian mode is the scale parameter (see later). There exists a particular basis without any scale parameters, the angular harmonics, but as this is a one dimensional (azimuthal angle) basis, it requires a scan over the second dimension (radial coordinate) to extract the core information [14]. In short, all the existing modal decomposition techniques have relied on *a priori* information on the modal basis to be used, and the scale parameters of this basis. Clearly this is a serious disadvantage if the tool is to be used as a diagnostic for arbitrary laser sources.

In this paper we outline a new approach using digital holography for an optimal modal decomposition without any prior knowledge of the scale parameters of the basis functions. We show that in a simple two-step process both the scale and the optimal mode set can be found. The result, as we will show, is a complete modal decomposition without any initial scale information.

2. Concept

Modal decomposition is a powerful tool to characterise laser beams; accordingly, an optical field can be described as a superposition of basis functions, called the modes, each weighted with a complex expansion coefficient. To determine these coefficients is the main task of each modal decomposition, mapping all necessary information about the field onto a one-dimensional set of coefficients. Mathematically the problem reduces to finding the unknown modal weights (ρ_n^2) and phases ($\Delta\phi_n$) so that an unknown field $U(\mathbf{r})$ can be expressed as a phase dependent superposition of a finite number of modes, $\psi_n(\mathbf{r})$ [8]

$$U(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n \psi_n(\mathbf{r}). \tag{1}$$

The orthonormal property

$$\langle \psi_n | \psi_m \rangle = \iint_{\mathbb{R}^2} \mathrm{d}^2\mathbf{r}\ \psi_n^*(\mathbf{r})\, \psi_m(\mathbf{r}) = \delta_{nm}, \tag{2}$$

of the basis may be exploited to uniquely determined the unknown coefficients

$$c_n = \rho_n \exp(\mathrm{i}\Delta\phi_n) = \langle \psi_n | U \rangle. \tag{3}$$

The modal weights (ρ_n^2) and phases ($\Delta\phi_n$) can be found experimentally with a simple optical set-up for an inner product measurement, as will be discussed later. However an optimal decomposition, yielding the minimum number of nonzero coefficients, requires knowledge of the scale of the basis; we will refer to this as the adapted basis set. To date there are no reports on techniques to find this adapted set.

To illustrate the problem, consider the basis set of Laguerre-Gaussian modes LG_{pl} with radial and azimuthal indices p and l, which at the waist position may be written as:

$$\mathrm{LG}_{pl}(\mathbf{r}; w_0) = \sqrt{\frac{2p!}{\pi w_0^2 (p+|l|)!}} \left(\frac{\sqrt{2}r}{w_0}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w_0^2}\right) \exp\left(-\frac{r^2}{w_0^2}\right) \exp(il\phi) \tag{4}$$

where $\mathbf{r} = (r, \phi)$, and $L_p^{|l|}$ is the Laguerre polynomial of order p and l. The basis functions have an intrinsic (generally unknown) scale, w_0, corresponding to the Gaussian (fundamental mode) radius. Now an arbitrary scalar optical field U can be decomposed into a Laguerre-Gaussian set of any size:

$$U(\mathbf{r}) = \sum_{pl} c_{pl}^a \mathrm{LG}_{pl}(\mathbf{r}; w_a) = \sum_{pl} c_{pl}^b \mathrm{LG}_{pl}(\mathbf{r}; w_b) \tag{5}$$

where $c_{pl}^{a,b}$ denote the complex expansion coeffient for different basis set sizes w_a and w_b, respectively. From Eq. (5) it becomes clear that the modal spectrum c_{pl} changes with the scale of the basis set. To attain a mode set of adapted size, we propose the following simple two-step approach: (i) determine the second moment size of the beam w and the beam propagation ratio M^2. The scale of the adapted basis set can then be inferred [15]:

$$w_0 = w/\sqrt{\mathrm{M}^2}, \tag{6}$$

enabling the second step, (ii) an optimal decomposition in the adapted mode set. The latter may be used to deduce the "actual" modes constituting the field, and as a check of the previously determined M^2 and w_0. It is possible to implement the first step by any ISO-compliant method [16], for example, a modal decomposition [8,17], or with a recently introduced digital approach [18]. The core idea therefore is that we can relate the (yet unknown) scale of the basis functions directly to the size of the embedded fundamental Gaussian beam, w_0 [15]. This is clear from Eq. (4), where it is noted that the size of the Gaussian term is carried through to provide the scale of all the functions in the expansion. With this observation noted, the question becomes how to find this embedded fundamental Gaussian size from a measurement of the arbitrary input field? We exploit the fact that since the beam quality factor of a fundamental Gaussian beam is $M^2 = 1$, and since the second moment beam size for all beams scales as $w = w_0\sqrt{M^2}$ [15], we can infer the unknown scale by measuring the field size and its M^2. Thus while Eq. (6) is simple to implement, its impact on the ability to optimise modal decomposition is significant, as we shall show in the sections to follow.

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3. Experimental methodology

The laser resonator used to create the beams under study was a stable plano-concave cavity with variable length adjustment (300-400 mm), and is shown as a schematic in Fig. 1.

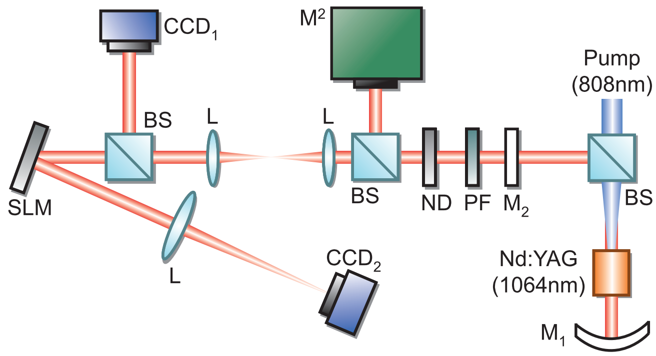

Fig. 1. Schematic experimental setup of the end-pumped Nd:YAG resonator, where the output beam is 1:1 imaged onto a camera (CCD_1) and a Spatial Light Modulator (SLM), whose diffraction pattern is observed in the far field with CCD_2. $M_{1,2}$: curved ($R = 500$ mm) and flat mirror ($R = \infty$), BS beam splitter, PF pump light filter, ND neutral density filter, L lens, M^2: M^2 meter.

The back reflector was chosen to be high reflective with a curvature of $R = 500$ mm whereas the output coupler was flat with a reflectivity of 98%. The gain medium, a Nd:YAG crystal rod (30 mm $\times$ 4 mm), was end-pumped by a 75 W Jenoptik multimode fibre coupled laser diode (JOLD 75 CPXF 2P W). In order to select specific transverse modes, an intra-cavity amplitude mask was inserted near the flat output coupler. The amplitude mask used in our experiments was custom made and consisted of lithographically produced thin aluminium absorbing rings fabricated on a 700 μm thick borosilicate glass with a 93% transmission at 1064 nm wavelength. The mask consisted of a 5 x 4 grid of ring structures, each designed to select a specific LG mode by overlapping the strong absorbing ring structures with the nulls of the desired fields. The adjustment of the resonator length (L), which alters the Gaussian mode size, can be viewed as a means to vary the scale parameter of the modes, while the type of ring structure on the mask selects the type of modes to be generated. The fundamental Gaussian waist size on the flat mirror satisfies $w_0^2 = (\lambda/\pi)[L(R-L)]^{1/2}$ and so each LG mode size also scales as $w_{p,l}^2 = w_0^2(2p+l+1)$ [19]; note that the final field may be some superposition of these modes. Thus adjusting the resonator length results in a change in the fundamental Gaussian mode size, and hence the positions of the zeros of the LG modes. By selecting an appropriate ring structure for a given resonator length, the laser could be forced, e.g., to oscillate either on the first radial Laguerre Gaussian mode ($LG_{1,0}$), a coherent superposition of $LG_{0,\pm4}$ beams (petal profile) or a mixture of the $LG_{1,0}$ and $LG_{0,\pm4}$ modes.

The resonator output at the plane of the output coupler was relay imaged onto a CCD camera (Spiricon LBA USB L130) to measure the output beam size in the near field, and could be directed to a laser beam profiler device (Photon ModeScan1780) for measurement of the beam quality factor. The same relay telescope was used to image the beam from the output coupler to the plane of the phase-only spatial light modulator (SLM) (Holoeye HEO 1080 P). The SLM, calibrated for 1064 nm wavelength, was used for complex amplitude modulation of the light to execute an inner product measurement with a Fourier transforming lens ($f = 150$ mm). The modulation achieved by the holograms was implemented with phase-only holograms coded to achieve any desired function using standard coding approaches. In our case, the LG modes were

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encoded following Eq. (4) using a modulation technique suitable for a phase-only SLM [20]. The method used to implement the inner product measurement has been reported previously [9], but is briefly reviewed here with the aid of Fig. 2.

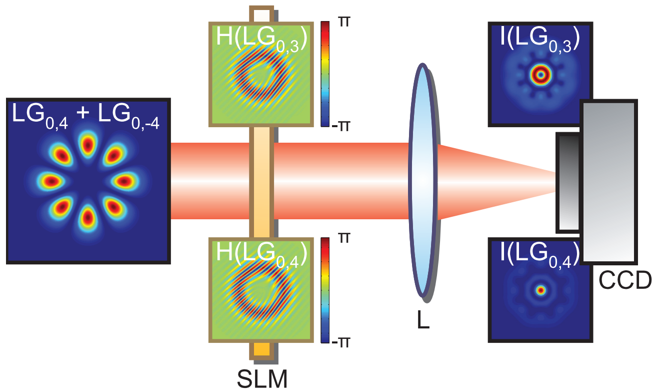

Fig. 2. Illustration of the inner product measurement scheme using a $2f$-setup. The correlation of an incoming beam with the hologram pattern (H) results in a correlation signal I at the optical axis in the back focal plane of a lens L.

Consider the case where the unknown field incident on the SLM is in fact a superposition of $LG_{0,4}$ and $LG_{0,-4}$ modes (but as yet unknown). The SLM is programmed with a series of match filters, each the complex conjugate of the modal functions in the chosen expansion, e.g. the LG basis functions from Eq. (4). If the SLM (match filter) is set in the front focal plane of a lens, then in the far-field (back focal plane) the signal on the optical axis (at the origin of the detector plane) is proportional to the modal power [1,2]. In our example, if the match filter was set to test for the modes $LG_{0,3}$ and $LG_{0,4}$, then the signal at the origin of the detector would return zero for the former and a strong signal for the latter, indicative of the weighting in the original field. In Fig. 2 we show examples of just such holograms as well as the light seen at the plane of the CCD detector. Our measurement scheme therefore comprises digital holograms as the match filters, and a monitor (CCD) of the on-axis signal in the Fourier plane of a lens. The measured intensities return the desired coefficients, ρ_n^2, for each mode. The modal phases $\Delta\phi_n$ are accessible analogously by creating a match filter, which depicts the superposition of the desired mode with a (previously chosen) reference mode as detailed in [8].

4. Results

Without any loss of generality, we tested our approach on the coherent superposition of Laguerre-Gaussian modes $LG_{0,4}$ and $LG_{0,-4}$, with nearly equal weighting, as seen in Fig. 3(a). To demonstrate the influence of the scale of the beam on the modal decomposition result, the scale of the hologram functions used for the decomposition was changed from the optimal $w_0 = 208\,\mu$m, yielding non-adapted basis sets. Results of these measurements are depicted in Fig. 3(b)through 3(d). Mismatching the relative scale, from an ideal w_0 to $0.75\,w_0$, $2\,w_0$ and $3\,w_0$, yields a concomitant increase in the number of modes in the non-adpated basis sets. We find that solely radial modes (with azimuthal orders $l = \pm 4$) respond, with the non-adpated set now containing modes of $LG_{p,\pm 4}$ with $p \geq 0$. At the same time, the power content of the $LG_{0,\pm 4}$ modes drops from initially 99% to 48%, 13% and 2%, while the power is dispersed among more and more modes – up to 30 for a basis scale of $3\,w_0$, compared to 2 for the adapted set. This is seen more clearly in Fig. 4(a) for a continuous change in the mismatch between the basis scale and the fundamental mode radius. The theoretical prediction for the change in

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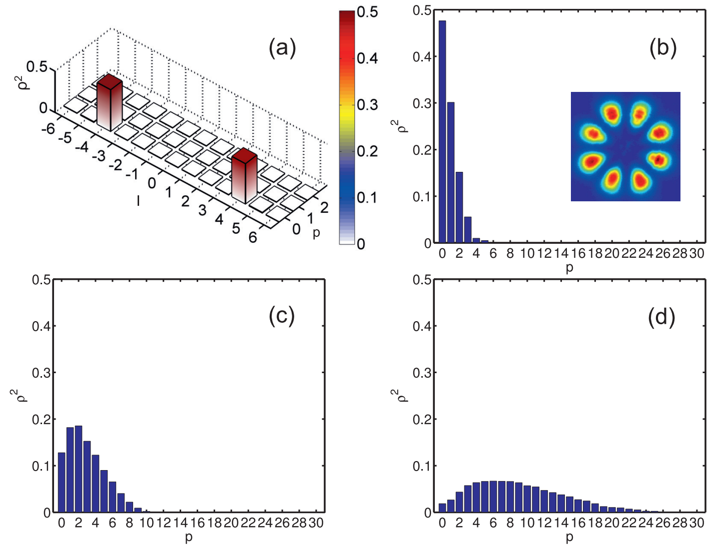

Fig. 3. Modal decomposition into adapted and non-adapted basis sets regarding scale. (a) Modal decomposition into $\mathrm{LG}_{p,\pm 4}$ modes of adapted basis scale w_0. (b) Decomposition into $\mathrm{LG}_{p,\pm 4}$ modes with scale $0.75\,w_0$, (c) $2\,w_0$, and (d) $3\,w_0$. Inset in (b) depicts the measured beam intensity.

$\mathrm{LG}_{0,\pm 4}$ power as a result of the scale mismatch (solid curve) is in good agreement with the experimental data points. As noted, the modal power is dispersed amongst a large number of radial modes (Fig. 4(b)) and in general the greater the scale mismatch, the greater is the modal power dispersion.

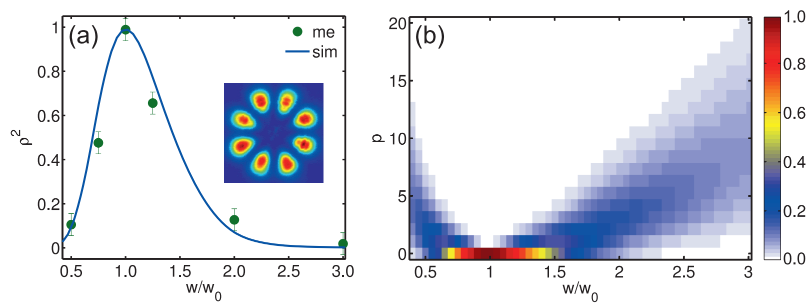

Fig. 4. Influence of basis set scale on mode spectrum. (a) Relative power ρ^2 of mode $\mathrm{LG}_{0,\pm 4}$, measured (me) and simulated (sim), as a function of normalised beam radius w/w_0. (b) Simulated power spectrum of $\mathrm{LG}_{p,\pm 4}$ modes ($p = 0\ldots 20$) as a function of normalised beam radius w/w_0. Inset in (a) depicts corresponding beam intensity.

While all the modal decompositions shown in Figs. 3 and 4 are mathematically equivalent, this behaviour emphasises the importance to decompose into an adapted set: there is an order of magnitude decrease in the number of significant signals. Moreover, one could argue that this is the only set with an intuitively meaningful realization behind the measurement, namely, that the beam really does consist of a coherent superposition of two azimuthal modes and not a superposition of a large number of radial modes. From these results it is also clear that while

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the first step of our suggested procedure may be performed at any scale, a large deviation from the adapted set scale will result in a laborious measurement and low modal power levels, i.e., low signal to noise, if the modal decomposition method is used for this step too.

Next we apply our two-step approach to find the adapted set assuming that we do not know what the scale parameter is. In the first step we decompose our beam into a non-adapted basis set, and use the result to find the beam diameter and beam propagation factor [17]. The modal decomposition results (Reconstruction) are compared to the measured values (Measurement) using the ISO standard approach, and are summarised in Table 1. It is clear that both approaches

Table 1. Diameter and M^2 of measured and reconstructed intensity.

	$2w$(μm)	M^2	$2w/\sqrt{M^2}$ (μm)
Measurement	945.7	5.2	413.4
Reconstruction	913.6	5.0	408.6

are in good agreement. This step returns the "unknown" scale parameter with an average value of $2w_0 = 411 \pm 2\,\mu$m which compares well with the theoretical value of $416\,\mu$m (based on the known resonator parameters). Next, the modal decomposition is executed with the correct scale, results of which are shown in Fig. 5. The measurement of amplitudes and phases of the correctly scaled modes (Fig. 5(a) and 5(b)) enables the reconstruction of the optical field in the adapted basis. As expected, the modal decomposition returns the two original azimuthal modes. Using the modal decomposition results, the intensity of the field is reconstructed and compared with the measured intensity: Fig. 5(c) and 5(d). Both are in good agreement, proving the decomposition to be correct.

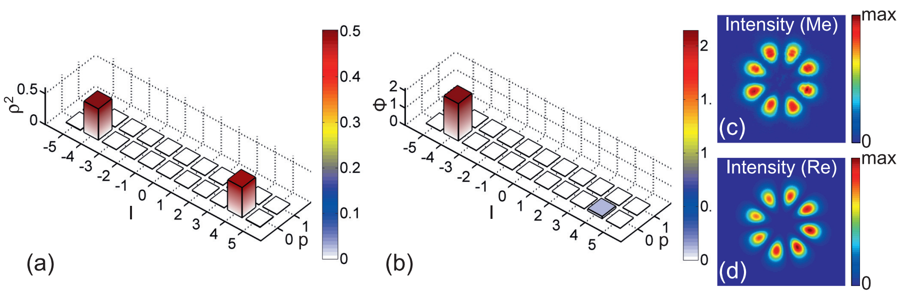

Fig. 5. Reconstruction of the beam by modal decomposition into $LG_{p,l}$ modes of previously determined scale. (a) Modal power spectrum (total power normalised to one). (b) Modal phases. (c) Measured intensity (Me). (d) Reconstructed intensity (Re).

The same two-step approach was applied to a beam consisting of the radial Laguerre Gaussian mode $LG_{1,0}$ as seen in Fig. 6(a), and of a superposition of the $LG_{1,0}$ and $LG_{0,\pm 4}$ modes, as depicted in Fig. 6(b).

It is important to note that if the first step of the procedure is executed with the recently mooted digital approach to M^2 measurements [18], then the entire technique can be implemented with a single spatial light modulator necessitating only a changing digital hologram. As holograms are easy to create and may be refreshed at high rates, the entire procedure can be made all-digital and effectively real-time.

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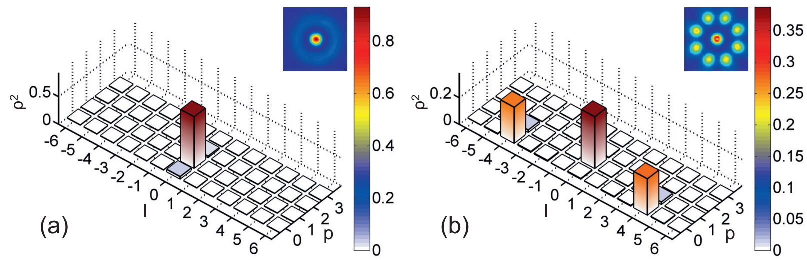

Fig. 6. Modal decomposition after determination of correct basis set scale of (a) a Laguerre-Gaussian $LG_{1,0}$ beam, and (b) of a superposition of an 8-petal beam and a $LG_{1,0}$ beam. Insets depict corresponding beam intensities.

5. Conclusion

We have outlined an improved method for the modal decomposition of an arbitrary field that requires no *a priori* scale information on the basis functions used. Our approach makes use of digital holograms written to a spatial light modulator, and exploits the relationship between the scale parameters within the basis and the beam propagation factor of the beam. We have demonstrated the approach on LG modes and have successfully reconstructed the modes and their sizes; we note that the procedure may readily be extended to other bases too. The advance of our method will be of relevance to studies of resonator perturbations, e.g. thermal effects and aberrations, and in the study of multimode fibre lasers.

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