33 Target Tracking Using a 2D Radar

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CONTENTS

33.1 INTRODUCTION

This chapter briefly outlines a few mathematical techniques to track targets in 3D using a 2D radar. 2D radars are relatively cheap and efficient sensors that often form the first line of defence in airspace control. In military applications they are often used as early-warning devices because they can detect approaching enemy aircraft or missiles at great distances. In case of an attack, early detection of the enemy is vital for a successful defence against attack. Depending on the threat evaluation of tracked aircraft the tracking process is passed along to 3D search radars or fire control tracking radars once it comes into range of those sensors. A key component in the above hierarchy is the threat evaluation component. It relies on many factors such as angle of incidence towards defended assets, time to approach to defended asset, speed of target and so forth. The normal 2D radar provides range and azimuth but the altitude of the target is omitted. This can be an important consideration as aircraft altitude limits the attack profiles a target can fly [1].

33.2 HEIGHT ESTIMATION

The current literature regarding height estimation restricts itself to computations involving two or more 2D radars where the height can be completely determined by simple geometric computations. In this section we present some mathematical methods to infer aircraft altitude from two updates given by a single 2D radar [2]. A single 2D radar source cannot directly determine the altitude of aircraft, thus naturally, the method presented here is either coupled with a number of assumptions and limitations or is a mere approximation. The terms height and altitude are used interchangeably. Height often refers to the height of an aircraft above ground level, and altitude the height of the aircraft above mean sea level. The proposed techniques do not consider terrain,

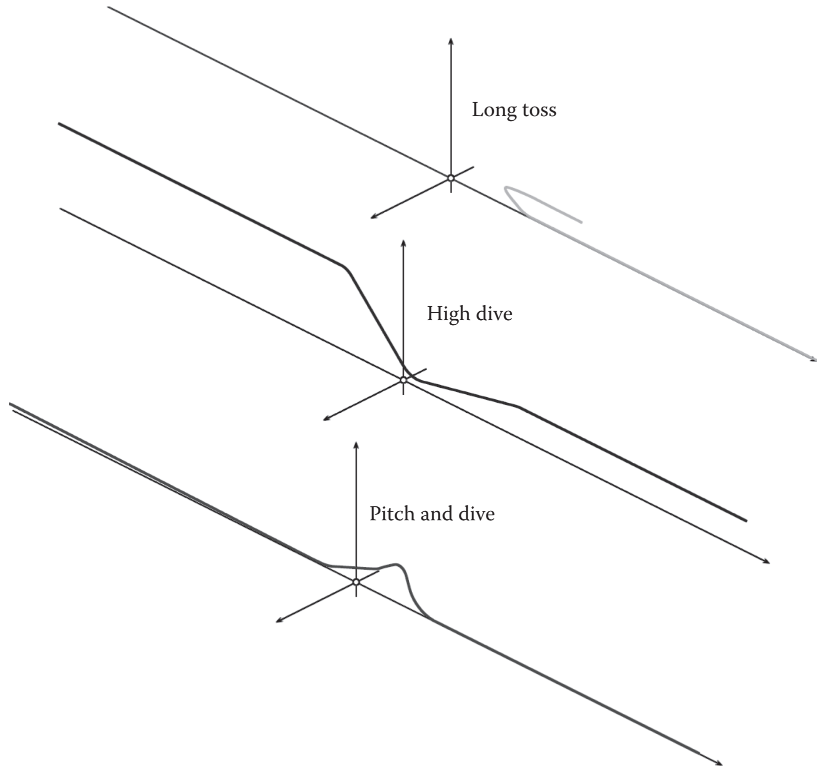

FIGURE 33.1 Flight profile examples.

terrain height, or height above mean sea level, but rather the difference in height of the sensor and observed aircraft.

It should also be noted that if the aircraft is flying perfectly tangential to the radar beam, then the radial speed component is zero and it is impossible to estimate its altitude. Conversely, however, it is more accurate to determine the altitude an aircraft flying at great speeds at a 45° angle to the radar beam, than a slow flying aircraft that is flying towards the radar. The aircraft speed is instrumental in determining the aircraft altitude. The accuracy to which these speeds are known is directly proportional to the accuracy to which the altitude can be determined. Knowledge of aircraft speed can be obtained in a variety of ways. For example, due to the volatile nature of their payload, the speed at which bombers fly is usually controlled by doctrine, similarly cruise missiles fly at known speeds. We will make use of the three known flight profiles depicted in Figure 33.1 as examples. The defended asset as well as the sensor is located at the origin.

In the discussion that follows we will make use of two sequential sensor readings at time t_1 and t_2. The given data sets will consist of a slant range and azimuth reading, denoted (r_1, θ_1) and (r_2, θ_2). If the aircraft speed, denoted v_2, is known then we can easily determine the distance travelled between time t_1 and time t_2 as $u_2 = v_2\,(t_2 - t_1)$. In the figures (Figure 33.2) that follow we will, without loss of generality, assume that $r_1 \geq r_2$.

33.2.1 Using Doppler Measurements

Modern 2D radar sensors allow Doppler measurements of one (radial) component of the velocity of the aircraft it is observing. In other words, Doppler measurements do not give us the velocity vector, $\vec{v}_2$, but only the magnitude of its radial component, denoted $\hat{v}_2$, at time t_2. From the measured radial

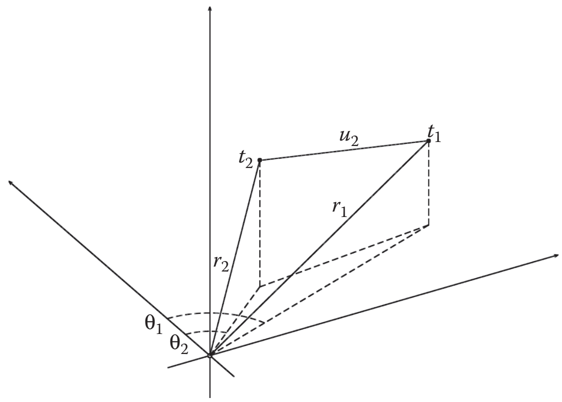

FIGURE 33.2 Radar tracking in 2D.

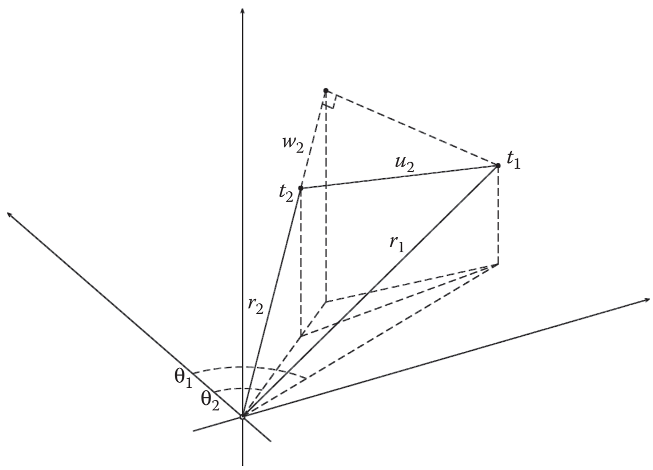

FIGURE 33.3 Radar tracking in 2D with Doppler data.

speed $\hat{v}_2$ we can easily determine the radial distance travelled $w_2 = \hat{v}_2(t_2 - t_1)$ and from that the total distance travelled is easily obtainable:

$$u_2 = \sqrt{r_1^2 - (r_2 + w_2)^2 + w_2^2} \tag{33.1}$$

It is clear from Figure 33.3 that the height can still not be directly calculated, even with Doppler measurements at hand, since the relations between these known values are valid at any height. In short, without any height-dependent data, there is no general way to directly compute the height.

33.2.2 Special Case

If we make an assumption that the aircraft is flying radially towards/away from the radar at level height and known speed (Figure 33.4), then we can compute the height with simple trigonometry as

$$h_1 = h_2 = r_1\sqrt{1 - \left(\frac{r_2^2 - r_1^2 - u_2^2}{-2r_1u_2}\right)^2} \tag{33.2}$$

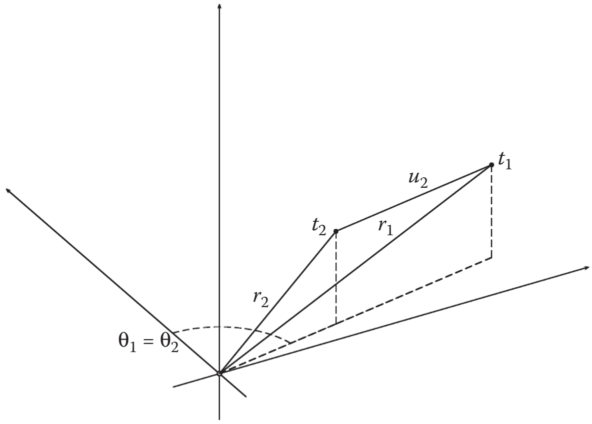

FIGURE 33.4 Simple case.

We will refer to this method as the AASC (Altitude Approximation via Simple Case) method to find an approximation to h_1 without knowing whether or not the assumptions of radial, straight flight hold true. The accuracy of this approximation is entirely dependent on the flight profile of the aircraft as illustrated in Figure 33.5. The model is usable without Doppler data as long as we have an accurate estimate of the speed of the aircraft, or more precisely u_2 so it seems natural to proceed with a sensitivity analysis to see how sensitive this model is to changes in the given u_2.

Assume that the aircraft is flying level and radially towards the sensor. We have $0 < h = h_1 = h_2 < r_1$ and $\theta = \theta_1 - \theta_2 = 0$. Thus

$$u = u_2 = \sqrt{r_1^2 + r_2^2 - 2h^2 - 2\sqrt{r_1^2 - h^2}\sqrt{r_2^2 - h^2}} \tag{33.3}$$

Differentiation yields

$$\frac{\partial h}{\partial u} = \sqrt{r_1^2 - h^2}\sqrt{r_2^2 - h^2}\left(h\sqrt{r_1^2 + r_2^2 - 2h^2 - 2\sqrt{r_1^2 - h^2}\sqrt{r_2^2 - h^2}}\right)^{-1} \tag{33.4}$$

from which it is clear that

$$\frac{\partial h}{\partial u} \to \infty \ \text{ as } h \to 0 \quad \text{and} \quad \frac{\partial h}{\partial u} \to 0 \ \text{ as } h \to r_1$$

This implies that as the aircraft's height approaches zero, the estimated value of h will become infinitely sensitive to changes in u and as the aircraft's height approaches its range, the estimated height h will become infinitely insensitive to changes in u.

33.3 VERTICAL ACTIVITY ESTIMATION

Understanding the vertical activity of an aircraft enables airspace control to predict aircraft intent which means more accurate situation awareness. The problem of estimating the behaviour of an aircraft is well known with applications in airspace control and threat evaluation.

The existing literature typically makes use of two or more separate radars to compute aircraft altitude. The advantages are immediately obvious as an aircraft can be tracked in 3D and the flight path can be compared to known flight profiles to offer a strong basis for threat evaluation.

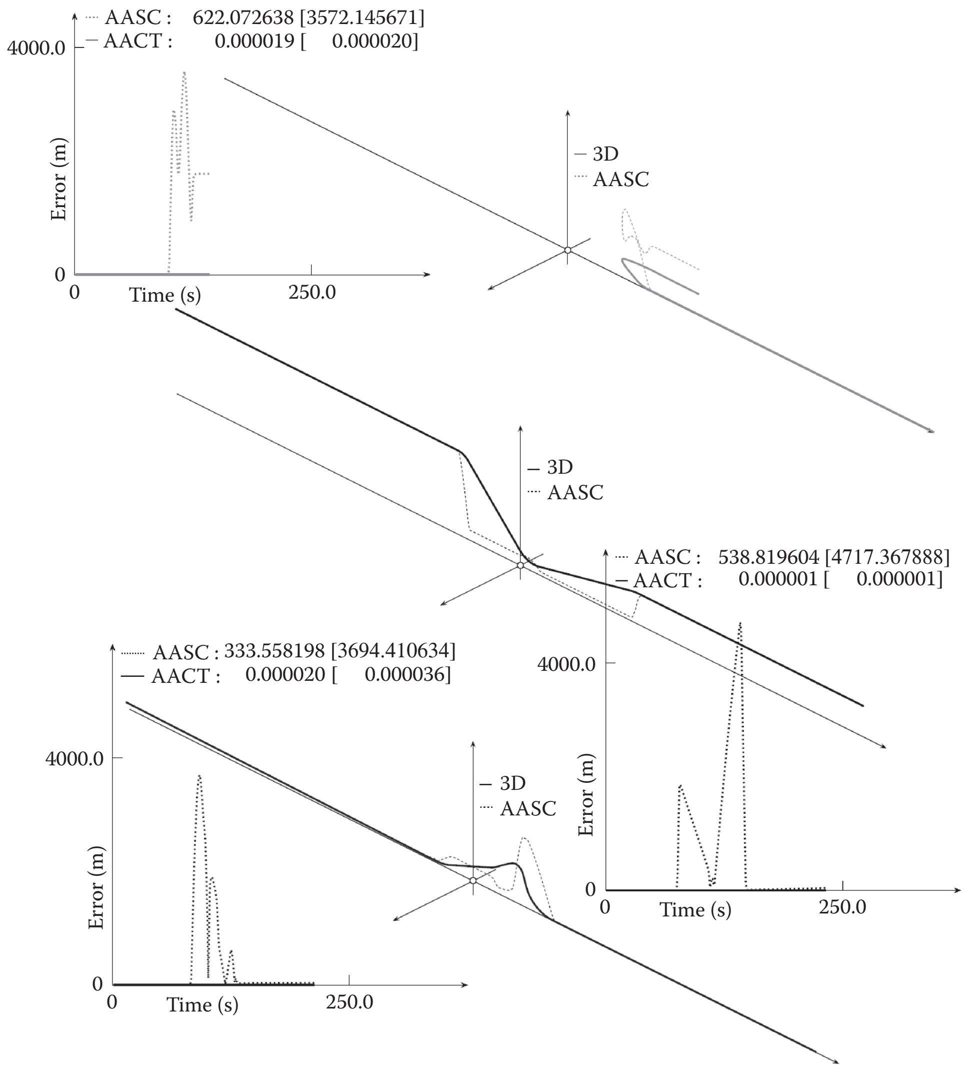

FIGURE 33.5 Height approximation using the AACT method.

33.3.1 Using Doppler Measurements

Vertical manoeuvres are easily and accurately recognizable when we have Doppler data available. Furthermore, it will give us a method to track the target in 3D.

If we have a relatively accurate estimation for h_1, then we can compute h_2 by solving the equation

$$-w_2 = r_2 - r_1\cos(\theta_1 - \theta_2)\cos(\epsilon_1)\cos(\epsilon_2) - r_1\sin(\epsilon_1)\sin(\epsilon_2) \tag{33.5}$$

for ϵ_2 once we know the value of ϵ_1 as illustrated in Figure 33.6. For simplicity, we define

$$c_1\sin(\epsilon_2) + c_2\cos(\epsilon_2) + c_3 = 0 \quad \text{where} \quad \begin{aligned} c_1 &= r_1\sin(\epsilon_1) = h_1 \\ c_2 &= r_2\cos(\theta_1 - \theta_2)\cos(\epsilon_1) \\ c_3 &= -w_2 - r_2 \end{aligned} \tag{33.6}$$

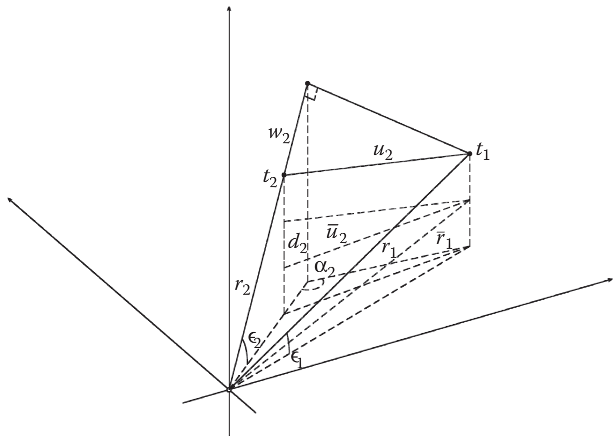

FIGURE 33.6 Calculation of change in altitude.

Then for $c_1^2 + c_2^2 - c_3^2 \geq 0$ we have

$$\epsilon_2 = \begin{cases} \epsilon_{2:1} = 2\,\mathrm{atan}\left(\dfrac{c_1 - \sqrt{c_1^2 + c_2^2 - c_3^2}}{c_2 - c_3}\right) & \text{if } \alpha_2 > 90° \text{ and} \\ \epsilon_{2:2} = 2\,\mathrm{atan}\left(\dfrac{c_1 - \sqrt{c_1^2 + c_2^2 - c_3^2}}{c_2 - c_3}\right) & \text{otherwise.} \end{cases} \tag{33.7}$$

We do not have enough information to calculate the projected angle α_2 so we define two functions covering all possible projection angles.

$$h_{2:1}(h) = r_2 \sin(\epsilon_{2:1}) \quad \text{and} \quad h_{2:2}(h) = r_2 \sin(\epsilon_{2:2})$$

where $\epsilon_{2:1}$ and $\epsilon_{2:2}$ is computed as above with $h_1 = h$. We start by correcting the given initial height h if the error in range $\gamma(h) = |r_1 - \bar{r}_1(h)|$ is too big (e.g., >0.001). We do this by adjusting the fixed height h to

$$h = \begin{cases} h + \gamma(h) \\ h - \gamma(h) \end{cases} : \gamma(h \pm \gamma(h)) \text{ is a minimum.}$$

and continue to do so until $\gamma(h) \leq 0.001$.

Accuracy can further be increased by making this threshold smaller but at a computational cost. The absolute difference in height $d_2 = |h_2 - h_1|$ can be computed by projecting coordinates onto a plane of fixed height h and using the Euclidean distance $\bar{u}_2(h)$ between the projected coordinates to compute

$$d_2(h) = \sqrt{u_2 - \bar{u}_2(h)}. \tag{33.8}$$

Using this, we define the following two conditions which will assist us in deciding whether the aircraft gained or lost altitude.

$$\begin{aligned} \mu_1(h) &= \left| |h_{2:1}(h) - h| - d_2(h) \right| \\ \mu_2(h) &= \left| |h_{2:2}(h) - h| - d_2(h) \right| \end{aligned} \tag{33.9}$$

The vertical manoeuvre can be approximate if two acceptable conditions are found at a given fixed height h (~h_1). The method fails when there is no solution for ϵ_2 in the calculation of $h_{2:1}(h)$ and $h_{2:2}(h)$ or when the conditions are very close. This can be handled by using the previous change in altitude $h_1 - h_0$ and in such, assuming that the vertical manoeuvre graph is smooth. We have $h_2 \sim h + \delta_2(h)$ where

$$\delta_2(h) = \begin{cases} d_2(h)\ \mathrm{sign}(h_1 - h_0) & \text{if } \nexists \text{ a solution} \quad \text{or} \quad |\mu_2(h) - \mu_1(h)| < 0.001. \\ d_2(h)\ \mathrm{sign}(h_{2:1}(h) - h) & \text{if } \exists \text{ a solution} \quad \text{and} \quad \mu_1(h) < 0.001 \text{ and } \mu_2(h) > \mu_1(h) \\ d_2(h)\ \mathrm{sign}(h_{2:2}(h) - h) & \mu_2(h) < 0.001 \text{ and } \mu_1(h) > \mu_1(h) \\ d_2(h)\ \mathrm{sign}(h_{2:1}(h) - h) & \text{otherwise if } \mu_1(h) < \mu_2(h) \\ d_2(h)\ \mathrm{sign}(h_{2:2}(h) - h) & \text{if } \mu_1(h) > \mu_2(h). \end{cases} \tag{33.10}$$

The vertical manoeuvre graphs for our three examples using this method to calculate the change in altitude are illustrated in Figure 33.7. The numbers displayed in the legends of the error graphs are of the form Calculated: Average Error (m) [Maximum Error (m)]. The errors in the beginning

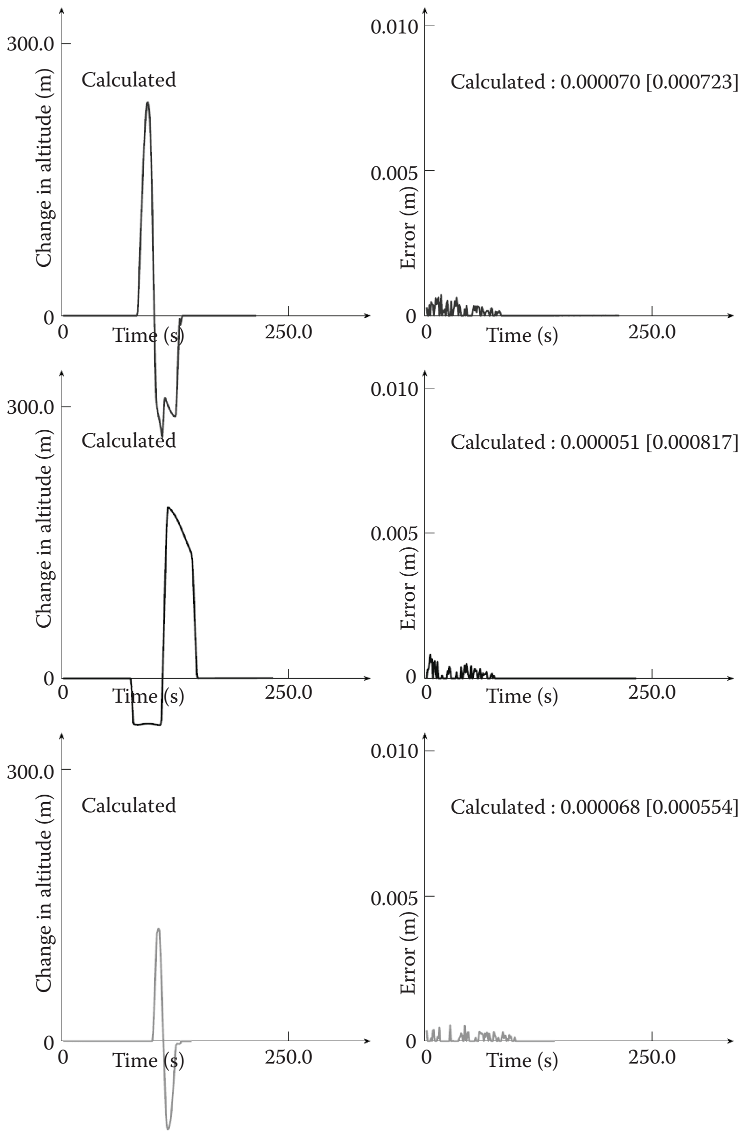

FIGURE 33.7 Vertical manoeuvre graphs.

TABLE 33.1
Numerical Values of the Various Error-Metrics (Change in Altitude Calculation)

	Sensor Offset Radius			Change in Altitude Errors		
Scenario	**Range [m]**	**Dir [°]**	**Altitude [m]**	**Avg**	**Max**	**Errors**
Pitch and dive	0–1000	0–360	0–1000	0.000003	0.001094	0.000000
Standard	0	0	0	0.000000	0.000034	0
Max average	330	260	110	0.000015	0.001047	0
Max single	600	110	280	0.000010	0.001094	0
High dive	0–1000	0–360	0–1000	0.000000	0.001126	0.000000
Standard	0	0	0	0.000000	0.000001	0
Max average	110	110	670	0.000005	0.001126	0
Max single	110	110	670	0.000005	0.001126	0
Long toss	0–1000	0–360	0–1000	0.000003	50.764000	0.000008
Standard	0	0	0	0.000000	0.000001	0
Max average	720	0	180	0.357493	50.764000	1
Max single	260	0	90	0.357493	50.764000	1

are due to rounding errors in the calculation of d_2 giving a false indication that there was a small change in altitude when the aircraft was actually in straight flight. The whole method was set up with a threshold of 1 mm which suggests that we should ignore changes in altitude smaller than this threshold. By doing this, we end up with average and maximum errors of less than 0.0001 mm. Moving the sensor away from the defended asset will affect the calculation of the vertical manoeuvre in the sense that the occasional error (not present in the standard setup) might arise. These errors refer to instances where the algorithm gave an increase in altitude when there was actually a decrease and vice versa. This is entirely dependent on the flight profile through. Table 33.1 displays the average error, maximum error and error probability (or number of errors for specific scenarios) when moving the sensor within a 1 km radius away from the defended asset. Table 33.1 is further expanded to include the standard and extreme scenarios where maximums are found. Figure 33.8 illustrates this for a special case. All the errors were due to the system having no solution and caused by using the previous update based on the assumption of continuous flight. Table 33.2 displays the average error, maximum error and error probability (indicating how likely the vertical manoeuvre calculation

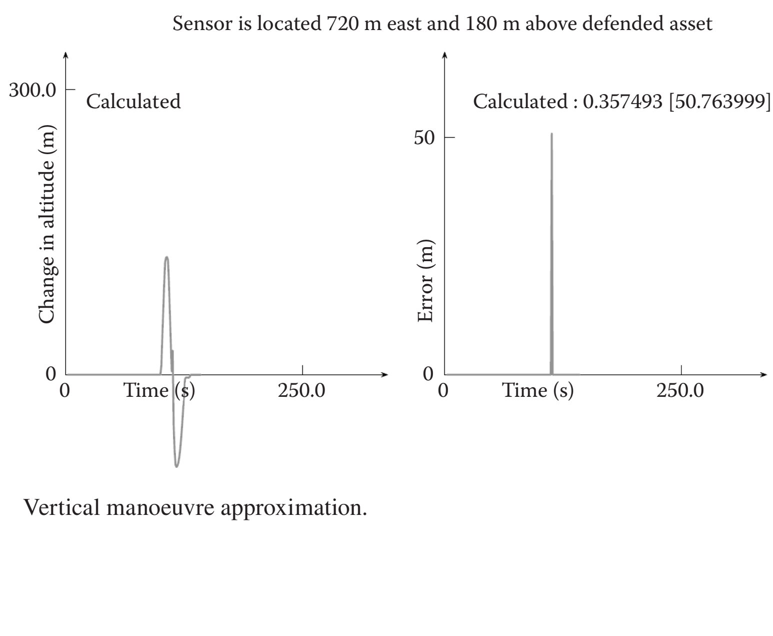

FIGURE 33.8 Vertical manoeuvre approximation.

algorithm is to fail) for the three given profiles as the sensor moves further away from the defended asset. All vertical activity errors are still due to the system having no solution.

33.4 CONTINUOUS TRACKING

As aforementioned, this gives us a method to track a target in 3D from the data sets (r_1, θ_1, $\hat{v}_1$) and (r_2, θ_2, $\hat{v}_2$) accumulated from two consecutive sensor updates at time t_1 and t_2, respectively. We will refer to this method as the AACT (Altitude Approximation via Continuous Tracking) method. The method to approximate the change in altitude $h_2 - h_1$ at time t_2 described in the previous section takes as input an initial value h ($\sim h_1$) which is used as initial approximation. The algorithm will adjust this height before trying to compute the change in altitude to yield better conditions $\mu_1(h)$ and $\mu_2(h)$. This is done by using the given range r_1 at time t_1 to find a more accurate approximation $h \sim h_1$ by mapping onto the range sphere.

The AACT method uses this idea to continuously track a target in 3D by (at the nth sensor update) provide: (i) an estimate δ_n for the change in altitude $h_n - h_{n-1}$ by using the approximated height h_{n-1} (computed at the previous sensor update) as input and (ii) an estimate for the height h_n. We will take h_0 as the AASC height whilst one may just as well take it as some other constant value (Figure 33.5). Moving the sensor away from the defended asset does affect the average AACT height errors for a specific flight profile but this is entirely dependent on the actual profile. Table 33.3 displays the average error, maximum error when moving the sensor within a 1 km radius away from the defended asset. The standard and extreme scenarios where maximums are found are also given. Table 33.4 displays the average error and maximum error for the AACT method for the three given profiles as the sensor moves further away from the defended asset. In the case of the above three scenarios we are better off when the sensor is 5*km* away from the defended asset but this might not always be the case.

33.4.1 Turning Manoeuvres

More complex turning manoeuvres in the flight path will naturally increase the error probability when detecting vertical manoeuvres and a bad first AASC approximation might cause a scenario to

TABLE 33.2
Numerical Values of the Various Error-Metrics (for Given Profiles)

		Change in Altitude Errors		
Scenario	**Radius**	**Avg**	**Max**	**Err Prob**
Pitch and dive	0–1000	0.00000289	0.00109412	0.00000000
	1000–2000	0.00000288	0.00112229	0.00000000
	2000–3000	0.00000291	0.00126226	0.00000000
	3000–4000	0.00000307	6.53600000	0.00000278
	4000–5000	0.00000365	9.44600000	0.00000278
High dive	0–1000	0.00000005	0.00112551	0.00000000
	1000–2000	0.00000005	0.00113658	0.00000000
	2000–3000	0.00000056	20.73200000	0.00000556
	3000–4000	0.00000017	0.00154998	0.00000000
	4000–5000	0.00000039	0.00150087	0.00000000
Long toss	0–1000	0.00000302	50.76400000	0.00000833
	1000–2000	0.00000019	0.00151653	0.00000000
	2000–3000	0.00000049	0.00151914	0.00000000
	3000–4000	0.00000403	50.76400000	0.00001111
	4000–5000	0.00000439	50.76400000	0.00000833

TABLE 33.3
Numerical Values of the Various Error-Metrics (Altitude Approximation/for Three Scenarios)

	Sensor Offset Radius			AACT Errors	
Scenario	**Range [m]**	**Dir [°]**	**Altitude [m]**	**Avg**	**Max**
Pitch and dive	0–1000	0–360	0–1000	0.025447	1.014586
Standard	0	0	0	0.000020	0.000036
Max average	990	350	0	0.252607	1.014428
Max single	980	10	0	0.252499	1.014586
High dive	0–1000	0–360	0–1000	0.002799	0.014047
Standard	0	0	0	0.000001	0.000001
Max average	980	60	0	0.004149	0.013457
Max single	960	10	0	0.004039	0.014047
Long toss	0–1000	0–360	0–1000	0.038625	0.546741
Standard	0	0	0	0.000019	0.000020
Max average	990	350	0	0.242730	0.546741
Max single	990	10	0	0.242729	0.546741

TABLE 33.4
Numerical Values of the Various Error-Metrics or the AACT Method/Altitude Approximation

		AACT Errors	
Scenario	**Radius**	**Avg**	**Max**
Pitch and dive	0–1000	0.02544727	1.01458580
	1000–2000	0.00496901	0.03094935
	2000–3000	0.00304537	0.01616566
	3000–4000	0.00226141	0.01117682
	4000–5000	0.00183988	0.00870827
High dive	0–1000	0.00279944	0.01404736
	1000–2000	0.00210260	0.00998633
	2000–3000	0.00170983	0.00788764
	3000–4000	0.00146634	0.00655928
	4000–5000	0.00129752	0.00570002
Long toss	0–1000	0.03862496	0.54674133
	1000–2000	0.00890965	0.03202178
	2000–3000	0.00527871	0.01690597
	3000–4000	0.00377008	0.01177279
	4000–5000	0.00294581	0.00909983

have an average AACT height error that is above the average for the standard flight profile. The various results are shown in Figures 33.9 and 33.10, and Table 33.5.

33.4.2 Practical Aspects

It should be noted that the accuracy of the method to calculate a vertical manoeuvre is dependent on the accuracy to which we can measure (r_1, θ_1) and (r_2, θ_2) and w_2. It is well known that 2D radars

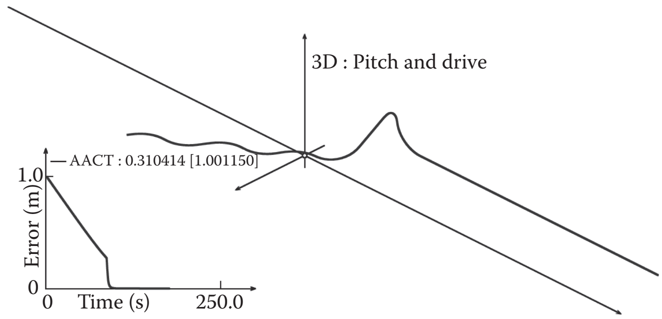

FIGURE 33.9 Height approximation using the AACT method: some results.

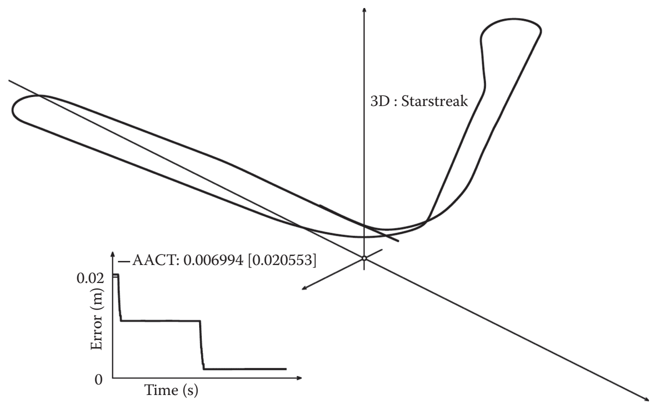

FIGURE 33.10 Height approximation using the AACT method: some more results.

TABLE 33.5
Additional Flight Profiles

	Change in Altitude Errors			AACT Errors	
Scenario	**Avg**	**Max**	**Err Prob**	**Avg**	**Max**
Pitch and dive	0.00248889	37.47600000	0.00248889	0.03543821	1.03319801
Stars treak	0.00000130	0.00082250	0.00000000	0.01112874	0.03089402

have excellent slant range measurements but poor azimuth readings. The data displayed in Table 33.6 shows the effect of randomly introduced errors in azimuth readings. The errors we have encountered here w.r.t. calculation of vertical manoeuvres are not only due to the system having no solution but due to the algorithm failing as a result of the errors in azimuth readings. As one would naturally expect, average errors and error probability goes up as the azimuth error bias does.

33.5 ALTERNATIVE METHOD

The solution suggested in Ref. [3] can be used when no Doppler data is available. It makes use of two separate motion models that independently estimate the behaviour of the tracked entity. The relative errors between predictions of these models and observations made by the 2D radar are then

TABLE 33.6
Adding Random Azimuth Errors

Scenario	Azimuth Error [mrad]	Change in Altitude Errors Avg	Max	Err Prob	AACT Errors Avg	Max
Pitch and dive	0.0000	0.000003	0.001094	0.000000	0.025447	1.014586
Radius : 0–1000	0.1000	0.016568	471.120000	0.002519	0.047542	1.741158
	0.2000	0.019945	471.120000	0.008628	0.046481	1.737893
	0.3000	0.022500	471.120000	0.016542	0.045727	1.767407
	0.4000	0.024488	471.120000	0.025508	0.045189	1.764944
	0.5000	0.026517	471.120000	0.034939	0.044771	1.761901
High dive	0.0000	0.000000	0.001126	0.000000	0.002799	0.014047
Radius : 0–1000	0.1000	0.000106	346.184000	0.000194	0.002799	0.014047
	0.2000	0.000376	346.184000	0.000411	0.002819	0.278816
	0.3000	0.001648	346.184000	0.000683	0.003003	0.499761
	0.4000	0.003521	346.184000	0.000753	0.003489	0.499910
	0.5000	0.005684	346.184000	0.000961	0.004194	0.499916
Long toss	0.0000	0.000003	50.764000	0.000008	0.038625	0.546741
Radius : 0–1000	0.1000	0.083851	194.536000	0.308378	0.040033	0.546741
	0.2000	0.132797	194.536000	0.647572	0.039996	0.546741
	0.3000	0.175531	194.536000	0.976175	0.039965	0.546741
	0.4000	0.214212	194.536000	1.255950	0.039937	0.546741
	0.5000	0.249601	194.536000	1.475436	0.039913	0.546741

used to make probabilistic statements on the current vertical behaviour of the aircraft. These models are based on inferring changes in the perceived velocity of a tracked target from a 2D radar track. The perceived changes may be due to changes in the target acceleration, changes in target altitude, or a combination of both. For this method to be useful we need to assume that the aircraft is at a fixed initial height h. We will proceed in vector notation to describe the 3D position vectors p_{n} (at time t_{n}) obtained by projecting the actual aircraft positions onto a 2D horizontal plane at height h.

Aircraft position p_n
Aircraft velocity $v_n = p_n - p_{n-1}$
Aircraft acceleration $a_n = v_n - v_{n-1}$

The first model assumes that the aircraft is flying at a constant altitude. This implies that perceived deviations (on the projection plane) from the expected flight path must be due to changes in velocity of the target. We can associate a 2D position from a single 2D radar sensor update via the above mentioned projection. A second update can then be used to estimate the perceived velocity of the target and finally a third update can be used to estimate the change in velocity of the target. We will not concern ourselves with the time term as it does not influence the result, provided updates occur at a constant rate. These values can then be used to predict position of the target at the next update.

$$p_{n+1} = p_n + v_n + a_n \tag{33.11}$$

The second model assumes that the estimated aircraft speed stays constant during updates thus all perceived deviations from the expected flight path are due to changes in altitude. This model additionally also assumes that the aircraft was in flying level during the prior update. Two predictions are made: one prediction for the case that the aircraft gained altitude and another for the case that the aircraft lost altitude. We iteratively find a new position p_n in each of the above cases. We start with a constant step size δ_0 which will be halved at every iteration and an initial value $q_0 = p_n$ for the aircraft position. We will continue to adjust the position q_i by increasing/decreasing its altitude by the current step size and mapping it onto the range sphere of the sensor until $|q_i - p_{n-1}| = |p_{n-1} - p_{n-2}|$ or the step size δ_i is small enough. At termination we update our position $p_n = q_i$ and make a prediction about where the target will be at the next update.

$$p_{n+1} = p_n + v_n \tag{33.12}$$

We can include other predictions here as the aircraft might steepen its ascent or start to level out. The predictions of these models are compared with actual observations during the subsequent sensor update. This is done by projecting the prediction to two dimensions representing slant range and azimuth. The Euclidean distance between the projected 2D position and the observed position is taken to be our measure of error. It is assumed that the smaller the error, the more likely the manoeuvre. We will abide by the following set of rules to decide as to which model to follow:

1. If the sum of all three prediction errors (representing upwards, downwards and acceleration-based errors) is less than a threshold value, then the aircraft is considered to be in straight flight. This threshold should be in the range of 5–10% of typically observed errors.
2. If the sum of all three prediction errors exceeds an upper threshold, then the aircraft is assumed to be flying a complex manoeuvre which cannot be captured by the underlying models. This threshold should be around twice the typical observed errors during normal manoeuvres.
3. If the acceleration errors is at least 10% less than the average of the upwards and downwards errors, then the aircraft is considered to be accelerating.
4. After ruling out the above three situations, we will assume that the aircraft is flying a vertical manoeuvre. The probability of the manoeuvre being upwards μ_U or downwards μ_D is given by the ratio of the corresponding errors e_U and e_D, respectively:

$${}_U = \frac{e_D}{e_D + e_U} \tag{33.13}$$

$${}_D = \frac{e_U}{e_D + e_U} \tag{33.14}$$

It is crucial to note that the distinction between upwards and downwards error is typically quite small thus not a lot of weight should be given to probabilistic statements with respect to vertical manoeuvres being up or down. Unfortunately, this is exactly what is needed to allow accurate continuous tracking of a target.

33.6 CONCLUSIONS

We have given a method to estimate vertical manoeuvres of an aircraft using a single 2D Radar when Doppler data is available and used to track a target in 3D. The average error when tracking a target in 3D using this method depends on the actual flight profile, position of the sensor w.r.t the defended asset and the accuracy of the data we get from the sensor but on average never exceeded 0.05 ms^{-1} for the standard flight profile examples we considered even with an 0.03° error bias in the azimuth readings. The maximum error at any given time was never more than 1.8 m. An alternative method that can be used when we do not have Doppler data was briefly outlined but this method statistically proven to be right only 50% of the time making it equivalent to throwing a dice when we have to decide whether the aircraft gained or lost altitude.

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