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Authors' reply

Authors' reply to Discussion by E. Siebrits and S. L. Crouch regarding the paper "A two-dimensional linear variation displacement discontinuity method for three-layered elastic media", International Journal of Rock Mechanics and Mining Sciences, Vol. 36, No. 6, pp. 719–729, 1999

Keh-Jian Shou [a,*], J.A.L. Napier [b]

[a]*Department of Civil Engineering, National Chung-Hsing University, 250, Kuo-Kuang Road, Taichung, Taiwan, 402*
[b]*Division Mining Technology, CSIR, P.O. Box 91230, Auckland Park 2006, Johannesburg, South Africa*

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The authors would like to thank Dr Siebrits and Professor Crouch for their discussion on our paper, and this Authors' Reply will follow the Discussion arguments sequentially. The authors firstly preface their reply points by emphasizing that the purpose of their paper was to apply a superposition scheme to a boundary element method (BEM) which can model, *practically* and *efficiently*, a general three-layered elastic medium. This approach provides an alternative to the traditional BEMs in which elements must be placed along the interfaces between layers.

Siebrits and Crouch point out that the superposition scheme used in this paper is an *approximate* scheme. However, the authors would rather term it the 'first order' scheme, as the 'higher order' scheme, in which the influences of the image elements further than $2H$ away from the actual source element are included (H is the thickness of the central layer), could be applied to improve the accuracy. The full solution is clearly discussed in [1], a paper describing the detailed mathematical derivation of this superposition scheme, which is in terms of infinite series as suggested by Siebrits and Crouch.

The authors agree that there is truncation error in this 'first order' scheme; however, for the first verification problem in the original paper [2], this approach properly accounts for the influence from both upper and lower layers to obtain the layer solution from the infinite plane solution. The authors would like to emphasize that the strip solution would not be obtained if the influences of the two bonding layers were not properly represented. The dimensions in examples 2 and 3 of the original paper are geometrically comparable to those in Figs. 6 and 7 (original paper). The thickness of the central layer is $2\sqrt{2}b$ for the case in Fig. 6 and $\sqrt{2}b$ for the case in Fig. 7 ($2b$ is the length of the crack in both cases).

Nevertheless, in the examples that we have analyzed, where cracks are located in the vicinity of the central layer, the results are extremely accurate. This appears to be confirmed as well by Siebrits and Crouch in their error analysis presented in Figs. 1 and 2 of their contribution. Siebrits and Crouch however seek to demonstrate that this accuracy deteriorates significantly in the case where cracks extend some distance, and at right angles, from the central layer (their Figs. 4 and 5). The argument used by Siebrits and Crouch to support this assertion is unfortunately seriously flawed.

Firstly, the results presented in their Figs. 4 and 5 do not show the absolute values of the crack opening displacement profile. We (and, more generally, the readers of this journal) therefore have no objective means of judging the accuracy of their "highly accurate semi-analytic simulator" which is

* Corresponding author.
E-mail address: kjshou@dragon.nchu.edu.tw (K-J. Shou).

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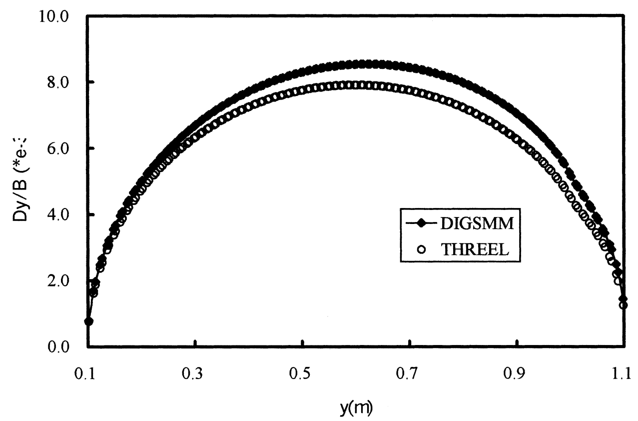

Fig. 1. The crack opening displacement for the example depicted in Fig. 4 of the Discussion by Siebrits and Crouch (*Dy* is the crack opening displacement; *B* is the length of the crack).

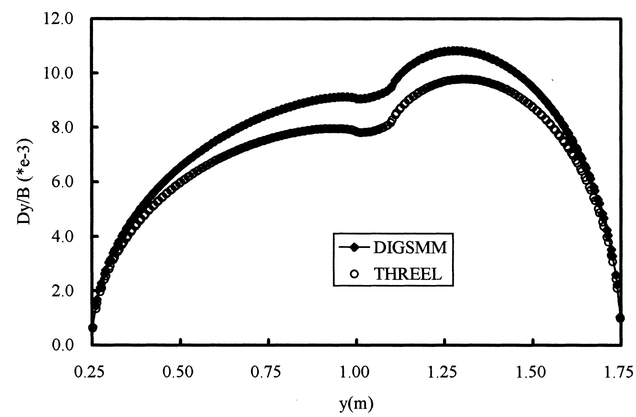

Fig. 2. The crack opening displacement for the example depicted in Fig. 5 of Siebrits and Crouch (*Dy* is the crack opening displacement; *B* is the length of the crack).

still to be published in their references [3] and [4]. Much more importantly, Siebrits and Crouch make the erroneous and unfounded assumption that the accuracy of *our* proposed method can be inferred by analyzing the truncation properties of *their* infinite series solution. This is not the case. The error plots presented in Figs. 2–5 of Siebrits and Crouch relate, in fact, only to the truncation properties of their own method and make no statement whatsoever about the accuracy of our method!

In order to arrive at some objective conclusions, we have re-analyzed the two examples depicted in Figs. 4 and 5 of Siebrits and Crouch. We have compared the results to a fine grid numerical analysis of these problems using the boundary element code DIGSMM referred to in our paper [2]. The estimated crack opening profiles using our superposition scheme are compared to the DIGSMM results in Figs. 1 and 2. These plots show clearly that our approximate scheme leads to an underestimate of the crack opening displacement of about 13% in the case of our Fig. 1 and to not more than 14% in the case of our Fig. 2. Much smaller errors are seen to occur near the crack tips. We have also examined the stress profile ahead of the crack tips and have found that our superposition scheme over-estimates this by less than 5% near to the crack tips. The error values presented by Siebrits and Crouch in their Figs. 4 and 5 can be seen to be grossly misleading (ranging up to nearly 50% in their Fig. 5). The results presented by Siebrits and Crouch therefore give a completely incorrect impression of the general validity of our approach.

In summary, we feel encouraged that our superposition method provides excellent accuracy for crack/layer interaction problems when the cracks are within a distance of two or three times the central layer thickness, from the centre of this layer. At greater distances (up to five times the central layer thickness) the accuracy appears to be satisfactory as a good "engineering" estimate both in terms of crack opening displacements and in terms of the stress field ahead of the crack tip. We acknowledge that this accuracy could be improved by modification of our method or by utilizing analytic methods such as those referenced by Siebrits and Crouch in their Discussion of our paper. We add a word of caution though relating to 'gilding the lily' without due consideration of effects such as layer delamination, crack growth deflection near layers and the complexities of the crack tip singularity near a bi-material interface.

References

[1] Shou K-J., 2000. A superposition scheme to obtain fundamental boundary element solutions in multi-layered elastic media. Accepted by Int J Num Anal Meth Geomech.

[2] Shou K-J, Napier JAL. A two-dimensional linear variation displacement discontinuity method for three-layered elastic media. Int J Rock Mech Min Sci 1999;36(6):719–29.