Accelerating flight: Edge with arbitrary acceleration

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Acknowledgements:

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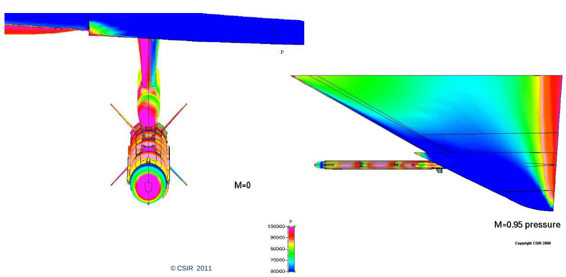
our future through science

Outline

- Context
- Background
  - CFD
  - Computational
  - Theoretical
- Navier-Stokes in relative and absolute frames
- Euranus illustrations
  - Oscillating airfoil
  - Drag through transonic range
  - Straked body
- Edge illustrations
  - Straked body
  - Diamond airfoil
- Conclusions
- Further work

Context

- CSIR Aeronautical Systems Competency Area



Context II

- While most aerodynamic analysis and missile work can be performed without worrying about acceleration,
- 4th and 5th generation missiles already execute turns at 100 *g*
- Agility and manoeuverability will increase
- The dynamics of shocks under significant acceleration are of interest in fluid dynamics
- Also: submarines, airships

Objectives: overall

- A formal framework for arbitrary manoeuvre
- CFD modelling of arbitrary manoeuvre
- Characterise dynamic loads in arbitrary manoeuvre

Objectives: 2011/12

- Include acceleration terms r and $\dot{\omega}$ in relative (body) frame formulation
- In order to assist thinking in the abolsute frame
- Note: can only characterise linear behaviour in this way; nonlinear behaviour needs real models
- Map the way toward arbitrary manoeuvre

Background

- Theory of rotating fluids: Batchelor, Greenspan, Landau and Lifshitz
- Directed largely at understanding atmospheric flows and waves
- Axial turbines
- Rothalpy and constant ω behaviour
- Overset chimera models, unstructured regridding: local
- Flight dynamics
- For arbitrary manoeuvre: progress to CFD models

CFD background

- Moving grids:
  - Chimera overset grids
  - Arbitrary Lagrangian Eulerian, ALE
  - Constant rotation ω: turbines and compressors
  - Small perturbations: aeroelasticity
  - Small perturbations: dynamic derivatives
  - Relative frame terms:
    - Roohani and Skews *2007…2011*
- In the inertial frame:
  - Inoue *et al. 1997*

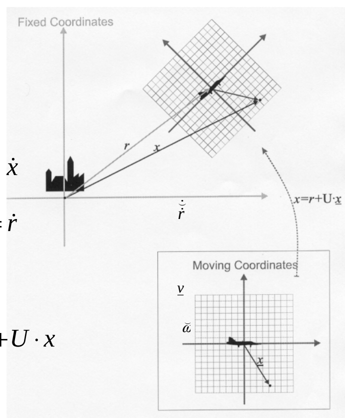
Theory background to present work

- Transformation between frames moving with constant relative velocity is trivial: Galilean
  - Transformation between frames with relative acceleration is subject of present programme
- Löfgren
  - General formulation of transforms in 4-space between inertial and relative frames
  - Invariants in transformation
- Forsberg
  - Löfgren's formulation to simpler formulation
  - Numerical implications of inertial and relative frames
  - Stability and convergence in inertial and relative frames
- Forsberg *et al. AIAA 2003, Aerosp. Sci. Tech. 2009*
  - EURANUS Implementation, validation and test cases

The parameters

Σ inertial frame: absolute

- Position vector of fluid element x
- Fluid velocity $v = \dot{x}$
- Position of *O´* r
  Velocity of *O´* $u = \dot{r}$
- Rotation vector of *Σ´* relative to *Σ* ω
- *Rotational transform* $\underline{x} = r + U \cdot x$

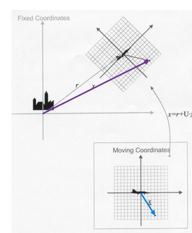


Vectors and transforms

	Σ inertial frame: absolute	***Σ´body frame: relative***
Position vector of fluid element	x	$\underline{x}$
Fluid velocity	$v = \dot{x}$	$\underline{v} = \dot{\underline{x}}$
Position of *O´* / Velocity of *O´*	r, $u = \dot{r}$	$\underline{r}$, $\underline{u} = \dot{\underline{r}}$

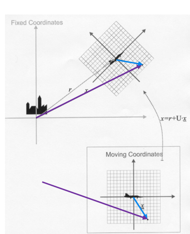
- *transform*

$$\underline{x} = r + U \cdot x$$
$$\frac{\partial U}{\partial t} = U\left(\frac{\partial}{\partial t} + \omega \times\right)$$
$$U^t = U^{-1}$$



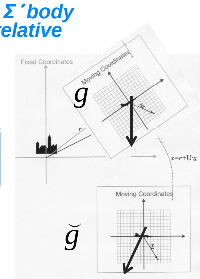
Vectors interpreted in the other frame

	Seen in Σ inertial frame: absolute		***Seen in Σ´body frame: relative***
vector	a	*as seen in Σ' is interpreted as* →	$\breve{a}$
	x	*as seen in Σ' is interpreted as* →	$\breve{x}$
	$\hat{\underline{a}}$	← *as seen in Σ' is interpreted as*	$\underline{a}$
	$\hat{\underline{x}}$	← *as seen in Σ' is interpreted as*	$\underline{x}$



Gravity vector interpreted in the other frame

	Seen in Σ inertial frame: absolute		***Seen in Σ´body frame: relative***
vector	g	*as seen in Σ' is interpreted as* →	$\breve{g}$



Scalars and intrinsic variables

Absolute, inertial

- ρ
- p
- T
- μ
- ν
- κ
- σ
- τ

Stress tensors are dependant on velocity

Relative: Conserved

- ρ
- p
- T
- μ
- ν
- κ
- $\breve{\sigma}$
- $\breve{\tau}$

Why is this notation so complex?

Why is it in the least important?

- It distinguishes in detail the frame transforms
- And provides a general framework

The general equation in conservation form for a conserved intrinsic quality *a*

Relative

$$\frac{\partial}{\partial t}(\rho a) + \nabla\cdot(\rho a\underline{v} + \breve{F}_a) = Q_a$$

Mass conservation: the equation of continuity

Relative

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\underline{v}) = 0$$

Momentum conservation

Relative

$$\frac{\partial}{\partial t}(\rho\underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v} + pI - \breve{\tau}) = -\rho\ddot{\breve{r}} - \rho\dot{\breve{\omega}}\times\underline{x} - 2\rho\breve{\omega}\times\underline{v} - \rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + \rho\breve{g}$$

- Non-dimensional forms
- Strouhal – temporal scales
- Euler – convection
- Reynolds – translational viscous
- Ekman – rotational viscous
- Translational acceleration – related to *g*
- Rotational accleration
- Rossby – Coriolis
- Centrifugal
- Gravitational

Energy conservation

Relative

$$E = e + \frac{\|\underline{v}+\underline{u}\|^2}{2}$$

$$a = E, \breve{F}_a = p\breve{v} - \breve{\tau}\cdot\breve{v} - \kappa\nabla T, Q_a = q_H + \breve{g}\cdot\breve{v}$$

$$\frac{\partial}{\partial t}(\rho E) + \nabla\cdot(\rho E\underline{v} + p\underline{v} - \breve{\tau}\cdot\underline{v} + p\underline{u} - \breve{\tau}\cdot\underline{u} - \kappa\nabla T) = q_H + \rho\underline{v}\cdot\breve{g} + \rho\dot{\breve{r}}\cdot\breve{g} + \rho\breve{\omega}\times\breve{g}$$

Generalised rothalpy *E**

Relative

$$E^* = e + \frac{\|\underline{v}\|^2}{2} - \frac{\|\underline{u}\|^2}{2}$$

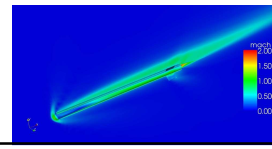
$$\frac{\partial}{\partial t}(\rho E^*) + \nabla\cdot(\rho E^*\underline{v} + p\underline{v} - \kappa\nabla T)$$
$$= q_H + \rho\breve{r}\cdot\ddot{\breve{r}} + \rho\breve{r}\cdot\dot{\breve{\omega}}\times\underline{x}$$
$$+ \rho\ddot{\breve{r}}\cdot\dot{\breve{\omega}}\times\underline{x} + \rho\dot{\breve{\omega}}\times\underline{x}\cdot\breve{\omega}\times\underline{x}$$
$$+ \rho\underline{v}\cdot\ddot{\breve{r}} + \rho\underline{v}\cdot\dot{\breve{\omega}}\times\underline{x}$$
$$+ \rho\underline{v}\cdot\breve{g} + \rho\breve{r}\cdot\breve{g} + \rho\breve{\omega}\cdot\breve{g}$$

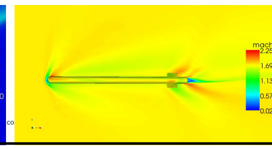
Implementation

Absolute

- Made relatively easy by moving grid and space conservation already in EURANUS
- Boundary conditions modified
- Implementation carried over to Edge

relative

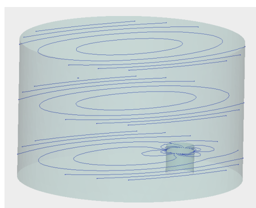


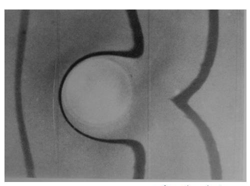


- Example: when do rotational effects dominate?
- Rossby number << 1
- Taylor-Proudman column in incompressible fluid

- Theory, Proudman 1916
- Experiment, Taylor 1917

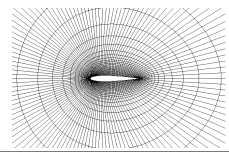
- Does not appear in Edge simulation at comparable parameters

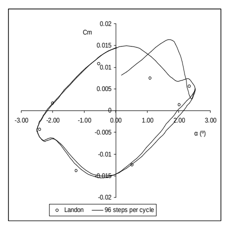


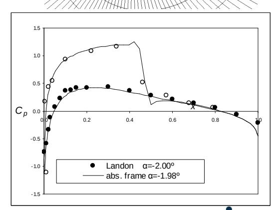


EURANUS Illustrations

- Oscillating airfoil, data Landon *1982*

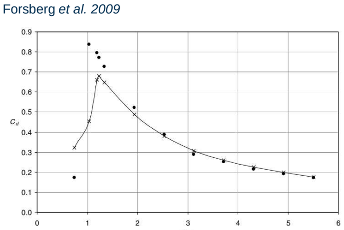


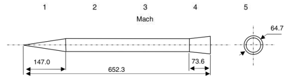




EURANUS Illustrations

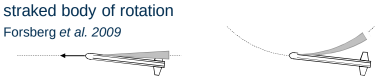
- Drag on flare through transonic region
- Forsberg *et al. 2009*

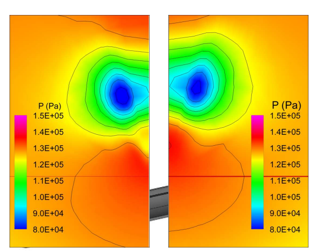




EURANUS Illustrations

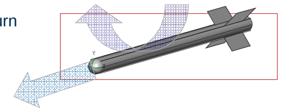
- Vortex movement on straked body of rotation
- Forsberg *et al. 2009*



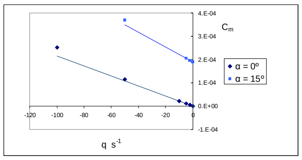


Edge illustrations

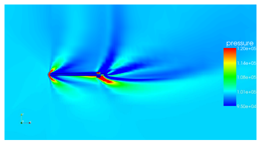
- Straked body with fins, sharp turn
- Careful validation needed
- Illustration of extreme case



- α = 0º, q = 0s^{-1}
- α = 0º, q = -100s^{-1}

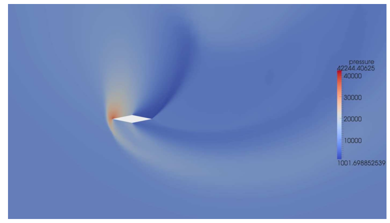


- α = 0º, q = -100s^{-1}



Edge illustrations

- Diamond wing, parabolic path
- Inoue, Sakari, Nishida 1997
- *En route*



Conclusions…

- We have a theoretical framework for arbitrary manoeuvre…
- Which allows us to keep an eye on transformations.
- In the code, absolute frame implementation is easy,
- It looks a bit like ALE but isn't (conceptual framework; boundary conditions)
- In the theory, relative frame equations are useful in guiding us in what to look for.
- The code is in the process of validation:
  - Some cases have been done, mostly in near-linear regimes
  - Experimental cases are hard to find
  - The relative theory can assist in guiding us in what to look for.

Further work

- Validation, validation, validation
- Find proofs:
  - Is entropy conserved in frame transformation?
  - Do Rankine-Hugoniot relations transform?
- Boundary conditions on accelerating walls
  - Boundary layer formation

Further work II

- Turbulence
  - How do we deal with numerical turbulence models under significant acceleration?
  - Is it appropriate to apply classic turbulence models even in the absolute frame?
- Perturbations
  - Rossby waves
  - Orr-Sommerfeld and transition
- Applications
  - Missile manoeuvre
  - Flow dynamics.

¿questions?

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