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Solid isotropic waveguides are frequently used for generation, transmission, and mechanical-vibration amplification, for example, in acoustic converters. The theoretical investigation of acoustic, mechanical, and electromagnetic waveguides is conventionally based on using second-order wave equations [1–3]. This approach is justified for describing the wave-propagation effects in relatively thin and long solid rods. As Rayleigh showed [4], the error caused by neglecting the effects of the transverse motion of the rod is proportional to the square of the ratio between the characteristic cross-section radius and the length of the rod. In [5] for more exact analysis of the longitudinal vibrations of a relatively thick and short rod, we used the more accurate Rayleigh model, which took into account the effects of inertia of the transverse cross section. It was shown that the eigenvalues for harmonic vibrations of the rod with constant and variable cross sections are limited from above. Therefore, the Rayleigh model has a restricted field of application. In this work, for analyzing the longitudinal vibrations of a conic rod, we used the Rayleigh–Bishop model [6, 7], which generalizes the Rayleigh model and takes into account both lateral displacements and the shear stress in the transverse cross section. The rod vibrations are described by the linear partial differential equation with the variable coefficients containing the mixed fourth-order derivative. The free vibrations of cylindrical and conic rods are considered. It is shown that the classical model of longitudinal vibrations of the rod described by the second-order wave equation can substantially overestimate the frequencies of the rod free of vibrations in comparison with the Rayleigh–Bishop model. It should be noted that transverse vibrations of a rod described by a linear partial differential equation of the fourth order were considered in many works (see, for example, [8–11]).