Accepted version of of: W. P. du Plessis, “Weighted Thinned Linear Array Design with the Iterative FFT Technique,” *IEEE Transactions on Antennas and Propagation*, vol. 59, no. 9, pp. 3473-3477, September 2011. Published version is available online at:
http://ieeexplore.ieee.org/arnumber=5944952

Weighted Thinned Linear Array Design with the Iterative FFT Technique

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***Abstract*—A version of the iterative Fourier technique (IFT) for the design of thinned antenna arrays with weighted elements is presented. The structure of the algorithm means that it is well suited to the design of weighted thinned arrays with low current taper ratios (CTRs). A number of test problems from the literature are considered, and in each case, the IFT produces results with improved sidelobe level (SLL) at lower CTR.**

***Index Terms*—Array antennas, thinned arrays, linear arrays.**

I. Introduction

Thinned arrays are formed from normal equally-spaced filled arrays by deactivating a number of the elements. The aperture of the filled array is maintained, so the width of the main beam is comparable to that of the filled array and similar angular resolution is thus achieved. However, the reduced number of active elements means that the size, weight, cost and complexity of the antenna array, its feed network and any signal processing are reduced [1]–[5].

Thinned arrays can be designed to have identical weights for all elements leading to benefits including simplified feed networks, and identical drive for power amplifiers when the array is used for transmission [1], [6]. However, the additional degrees of freedom offered by control of the weights of the antenna elements can lead to significant improvements to the array parameters including the sidelobe level (SLL) [7], [8].

One of the key figures of merit of any array that utilises weighted element excitations is the ratio of the largest excitation magnitude to the smallest excitation magnitude – the current taper ratio (CTR). Larger CTRs are indicative of increased design complexity because of increased challenges associated with issues such as realising an appropriate feed network and higher dynamic ranges for the transmitter and receiver systems. Designing for a low CTR is thus desirable [1], [3], [4] with equally-excited arrays having the lowest possible CTR of 1.

Sparse arrays are similar to thinned arrays except that the positions of the antenna elements are not quantised. While this approach increases design freedom, potentially leading to improved array performance, periodic quantisation of the element positions has a number of advantages. Coupling between antenna elements is essentially identical when the element positions are quantised, simplifying the design. Quantisation also means that the results are valid for all frequencies below the design frequency because of the polynomial nature of the results. Furthermore, no limitation is placed on the maximum scan angle of the array when the element spacing is quantised to multiples of half a wavelength.

These points are clearly demonstrated through the use of the example of 25 elements in a linear aperture 50 wavelengths long. A number of published results for this problem are summarised in Table I. The first solution uses a cyclic difference set and represents the best value that has been obtained without resorting to iterative numerical methods. Solutions L2 and L3 and are equally-excited arrays and represent compromises between sidelobe level (SLL) and beamwidth. Solutions L4 to L8 show that considerable SLL improvements can be achieved when the elements are weighted at the cost of increasing the CTR. Solutions L9 and L10 give results whose element positions are quantised twentieths of a wavelength and achieve significantly better SLL than the other equally-excited cases, but at the cost of reducing the scan angles. Lastly, solutions L11 and L12 show that the best SLL values are achieved when the element positions are not quantised, though this improvement comes at the cost of reduced ability to scan the beam.

TABLE I
Published results for the 25-element, 50-wavelength problem.

Solution	Source	SLL (dB)	Beamwidth (u)	CTR	Length (λ)
L1	[6][1]	-11.06	0.0099	1	43.0
L2	[5]	-12.36	0.0126	1	50.0
L3	[8]	-13.06	0.0220[2,3]	1	50.0
L4	[3]	-13.51	0.0143	4.1	50.0
L5	[4]	-14.45	0.0192	6.5	50.0
L6	[9]	-14.67	0.0190	7.3[4]	50.0
L7	[8]	-14.67	0.0190	N/A	50.0
L8	[8]	-14.77	0.0204	7.1[4]	50.0
L9	[10][5,6]	-13.44	0.0125[2]	1	35.7
L10	[10][5,7]	-14.02	0.0130[2]	1	36.6
L11	[11][8]	-17.01	0.0166[2]	3.33	26.013
L12	[11][8]	-20.1	0.0185[2]	2.63	26.013

[1] Computed using the (101, 25, 6) cyclic difference set.
[2] Recomputed using cubic spline interpolation.
[3] Shoulder in the main beam.
[4] Estimated from the graphs provided.
[5] Element positions quantised to $\lambda/20$.
[6] Scan angle limited to $\pm 20°$.
[7] Scan angle limited to $\pm 10°$.
[8] Positions are not quantised, and pattern cannot be scanned.

Despite the benefits of weighted thinned arrays, the literature considering the design of such arrays is limited. Proposed design techniques utilise simulated annealing [3]–[5], [10], mixed integer linear programming [7], genetic algorithms [9], and a hyrid approach combining a genetic algorithm and a local optimiser [8].

The iterative Fourier technique (IFT) developed by Keizer [2], [12] is a version of the alternating projection technique [13] for the design of antenna arrays that exploits the fact that the excitations and pattern of an array are related by a Fourier transform pair. The IFT has been successfully applied to the synthesis of equally-excited thinned arrays and was shown to reliably produce results that are better than the best published results [2].

The extension of the IFT to the design of weighted thinned linear arrays is considered below, with the resulting algorithm being well-suited to obtaining low CTRs. Test problems from the literature are considered, and the results obtained with the IFT considerably exceed those achieved with other algorithms.

II. Description of the Algorithm

A flowchart describing the IFT is given in Fig. 1. Each of the steps is considered below and the modifications necessary for the IFT to be used for the design of weighted thinned arrays are highlighted.

Each iteration commences by generating a random excitation where each element in the allowable aperture has a value uniformly distributed between 0 and 1.

The selection of the excitations that will be used is achieved by ranking the excitations in the allowable aperture and selecting the strongest excitations. The number of excitations selected is

Manuscript received October 8, 2010; revised January 5, 2011 and February 9, 2011.

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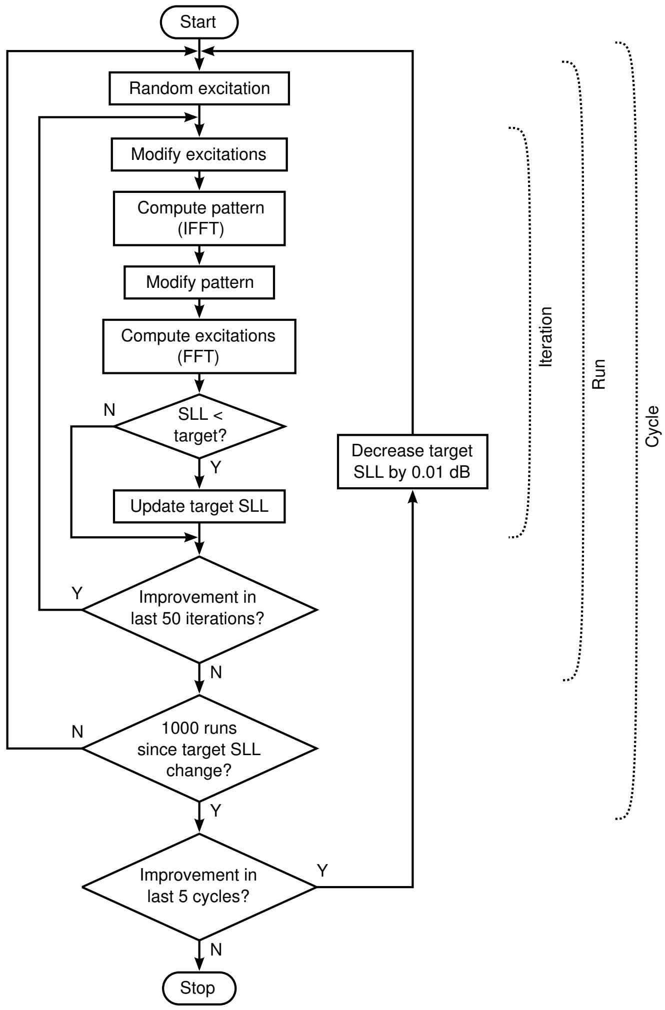
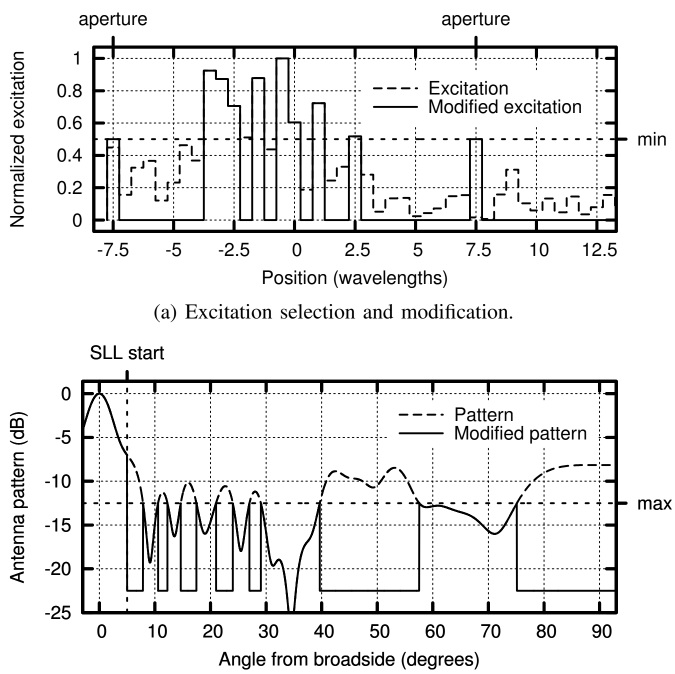


Fig. 1. Flowchart of the IFT.



(a) Excitation selection and modification.

(b) Pattern modification.

Fig. 2. Excitation selection and pattern modification in the IFT.

determined by the desired number of active elements, so the correct filling factor is implicitly achieved by the algorithm. This approach means that only the strongest elements are retained, increasing the likelihood of achieving a low CTR. The elements at the edges of the array are always retained when the array aperture length is specified, irrespective of their excitations.

The specified maximum CTR is achieved by modifying any excitations that violate the CTR requirement. The values of the selected excitations are normalised to the largest selected excitation to ensure that the largest excitation is 1. Any selected excitations with values less than $1/\mathrm{CTR}$ are set equal to $1/\mathrm{CTR}$ to ensure that the specified maximum CTR is achieved.

An example of this process is shown in Fig. 2(a) for ten elements distributed over a 30-wavelength aperture with a specified maximum CTR of 2. The outside elements and the elements with eight strongest excitations within the allowable aperture are selected as the ten active elements. The outside elements amplitudes are too low to achieve the specified CTR, so their amplitudes are increased to $1/2$.

The antenna pattern of the modified excitation is then obtained using an inverse fast Fourier transform (IFFT) by zero-padding the excitation to obtain the required number of points. This pattern is then modified by setting all pattern values in the sidelobe region whose amplitude exceeds a target SLL to some constant level below the target SLL. The phase of each point in the antenna pattern is retained during this process.

An example of the pattern-modification process is shown in Fig. 2(b). The main beam is 10° wide, the target SLL is -12.5 dB and values that exceed this level are set to an SLL of -22.5 dB.

The next candidate excitation is then computed from the modified pattern using a fast Fourier transform (FFT), and the process is repeated until the SLL does not improve for 50 iterations. This procedure allocates more tests to solutions that are improving while wasting fewer iterations on solutions that are not improving.

The algorithm checks whether the current result exceeds the best result each time a new antenna pattern is computed, and the overall best result is returned when the IFT terminates. If the achieved SLL is better than the target SLL, the target SLL is set to the nearest 0.01 dB that is better than the best achieved SLL. This approach means that the initial target SLL is not crucial because the target SLL will rapidly progress to a useful value.

After 1000 runs have been completed since the last target SLL change, the target SLL is decreased by 0.01 dB if there has been an improvement during the last five cycles. This step is necessary because the achieved SLL does not improve every time the target SLL decreases.

This procedure to automatically update the target SLL represents a significant improvement over previous versions of the IFT where the target SLL had to be specified [2], [12]. An initial target SLL of 0 dB was used to obtain all the results in Section IV, demonstrating the robustness of the algorithm. The IFT is insensitive to the new parameters introduced, and the heuristically-determined values in Fig. 1 were used to obtain all the results presented in Section IV.

III. Test Problems

Three groups of test problems were used to test the IFT, and Tables I and II summarise the published results for these problems.

The first test problem has already been described in Section I and considers 25 elements in an aperture 50 wavelengths long with the sidelobe region starting at u = 0.04. This problem will be considered in some detail because its extensive coverage in the literature allows comparisons to a large number of other algorithms.

TABLE II
ADDITIONAL TEST PROBLEMS.

Elements/ Positions	Source	SLL (dB)	Beamwidth[2] (degrees)	CTR	Length (λ)
48/64[1]	[7]	-18.76	1.812	2.76	31.5
78/200	[2]	-17.33	0.559	1	95.0[3]
132/200[1]	[2]	-22.83	0.691	1	92.5[3]
139/200	[2]	-24.30	0.652	1	98.5[3]
144/200[1]	[2]	-22.92	0.591	1	99.5[3]

[1]Symmetric array.
[2]Recomputed using cubic spline interpolation.
[3]The length is overestimated by 0.5λ in [2].

TABLE III
SOLUTIONS TO THE 25-ELEMENT, 50-WAVELENGTH TEST PROBLEM.

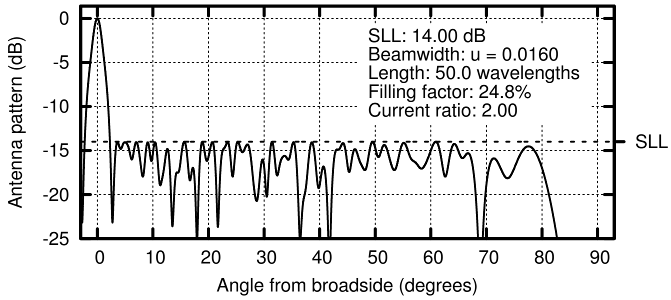
Solution	SLL (dB)	Beamwidth (u)	CTR	Length (λ)
S1	-12.72[1]	0.0145	1	50.0
S2	-14.00	0.0160	2.00	50.0
S3	-14.33	0.0199	3.00	50.0
S4	-14.47	0.0204	3.91	50.0
S5	-14.61	0.0207	4.96	50.0
S6	-14.83	0.0206	5.98	50.0

[1]Insignificant shoulder in the main beam.

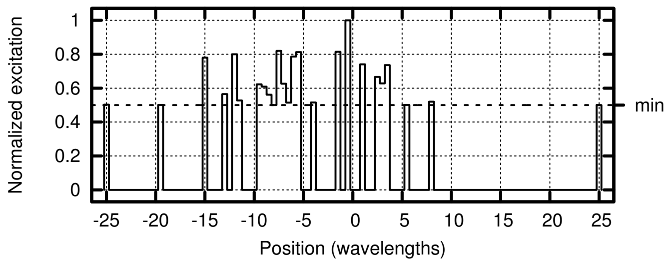
The second test problem considers the symmetric placement of 48 elements on a grid of 64 locations and allows comparisons to the mixed integer linear programming algorithm developed in [7]. The start of the sidelobe region was taken to be the same as that achieved in [7] to ensure that the main beam obtained is no broader than the published result, and the maximum CTR was set to 2.

The last four problems are similar to the problems considered in [2], except that the element excitations can be weighted. These problems are useful for testing because they have large numbers of elements and positions, and half the problems require symmetric arrays. Unlike the first two test problems, there is no requirement that the full aperture be used. The improvements that are possible by weighting the element excitations can also be demonstrated using these problems. The values for the start of the sidelobe region obtained for the equally-weighted arrays in [2] were used for the weighted arrays to ensure that the main beams of the weighted thinned arrays are no wider than for the equally-weighted case, and the maximum CTR was set to 2.

The IFFT and FFT calculations used 2048 points for the first two problems and 4096 points for the remaining four test problems, giving an average of more than 20 points per pattern root and agreeing with values used in the literature [2], [5]. At the end of each IFT cycle, the properties of the best solutions were calculated using 16 times more points to ensure that the final results are accurate. The beamwidth and SLL start angles were determined using cubic spline interpolation.

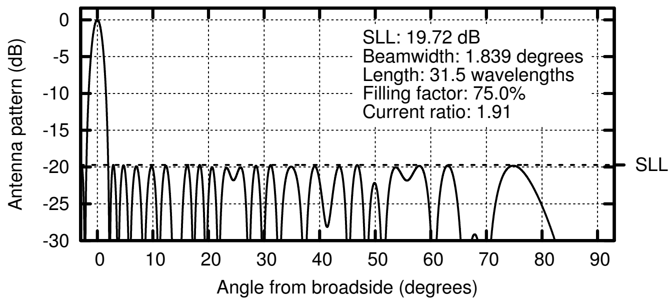


(a) Pattern.

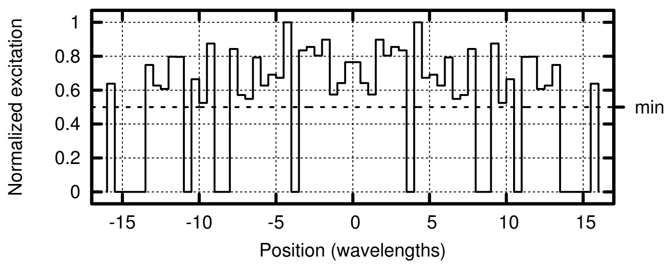


(b) Excitation.

Fig. 3. Solution S2 to the 25-element, 50-wavelength test problem.



(a) Pattern.



(b) Excitation.

Fig. 4. Solution to the 48-element, 64-position test problem from [7].

IV. ANALYSIS OF RESULTS AND DISCUSSION

The solutions to the first test problem obtained using the IFT are summarised in Table III. These solutions represent a number of compromises between SLL and CTR, and significantly improve on the published results summarised in Table I.

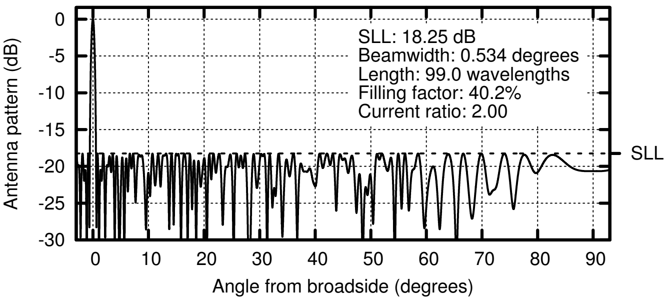
The IFT has never been applied to this problem before, so Solution S1 considers the equally-excited case. While solution L3 in Table I achieves a better SLL, its main beam is wider than that of S1 and it has a much more significant shoulder (similar to the pattern in Fig. 2(b)). It should be noted that this shoulder is an indication that the design requirements are unrealistic, but it is only recently that design algorithms have developed to the point that this limitation of the test problem has become apparent.

The remainder of the solutions to the first test problem consider weighted excitations with specified maximum CTRs varying from 2 to 6 (Solutions S2 to S6 in Table III). The patterns and excitations obtained for a maximum CTR of 2 are plotted in Fig. 3. The results obtained considerably improve on the best published results (L4 to L6 Table I) with the improvement being most marked when the CTR is low. For example, S2 in Table III has an SLL almost 0.5 dB better than L4 in Table I while more than halving the CTR.

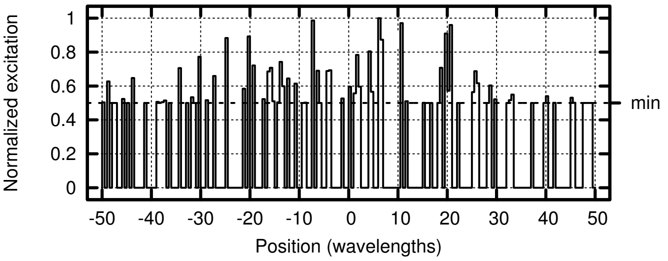
The result for the second test problem is summarised in the first line of Table IV and plotted in Fig. 4. The IFT result substantially improves on the published result and achieves a lower SLL at a smaller CTR while maintaining essentially the same beamwidth.

TABLE IV
SOLUTIONS TO THE ADDITIONAL TEST PROBLEMS FROM [7] AND [2].

Elements/ Positions	SLL (dB)	Beamwidth (degrees)	SLL start (degrees)	CTR	Length (λ)
48/64	-19.72	1.839	1.958	1.91	31.5
78/200	-18.25	0.534	0.556	2.00	99.0
132/200	-24.05	0.676	0.783	2.00	99.5
139/200	-25.71	0.639	0.755	2.00	95.5
144/200	-24.64	0.581	0.671	1.98	99.5



(a) Pattern.



(b) Excitation.

Fig. 5. Solution to the 78-element, 200-position test problem from [2].

The results for the 200-position test problems are summarised in the last four lines of Table IV, and the solution to the 78/200 problem is plotted in Fig. 5. The use of weighted excitations leads to SLL improvements of more than 1 dB over the equally-excited cases for all but the 78/200 problem. This is achieved while marginally reducing the beamwidths and maintaining CTR values of no more than 2.

The limited improvement in the 78/200 case is due to the fact that the beamwidth specification is narrower than for the other 200-position problems, yet fewer elements are available. Furthermore, the beamwidth specification is based on an equally-excited array so it is likely that the specifications favour equal or nearly-equal excitations. This hypothesis is supported by the way the excitation in Fig. 5(b) resembles an equally-excited array.

It is also possible to design weighted thinned arrays by applying IFT once to determine the positions of the active elements by designing an equally-excited array, and then applying the IFT a second time to determine the active-element weighting. The results obtained using this approach are summarised in Table V where the active-element positions are determined by the difference-set solution L1 from Table I in first line, S1 from Table III in the second line and the solutions from [2] in Table II for the rest.

Only the last solution in Table V shows an improvement over the results in Tables III and IV, and even then, by less than 0.1 dB. Given the additional complexity required by the two-stage approach and the small number of known difference sets, the active-element positions and weights should be determined simultaneously as described in Section II.

TABLE V
RESULTS OBTAINED USING A TWO-STAGE IFT PROCESS.

Elements/ Positions	SLL (dB)	Beamwidth	SLL start	CTR	Length (λ)
25/101[1]	-12.79	0.0102	0.0189	2.00	50.0
25/101	-13.81	0.0158	0.0395	2.00	50.0
78/200	-17.95	0.544°	0.559°	2.00	95.0
132/200	-23.99	0.671°	0.783°	2.00	92.5
139/200	-25.56	0.639°	0.755°	2.00	98.5
144/200	-24.73	0.580°	0.671°	2.00	99.5

[1]Element positions determined by the (101, 25, 6) difference set.

V. CONCLUSION

A modification of the IFT for the design of thinned arrays with weighted elements is presented. The structure of this algorithm makes it ideal for the design of arrays with low CTR values because the strongest excitations are selected at each iteration.

Results for a number of test problems from the literature are presented, and in each case, the IFT produces substantially better SLL levels at lower CTR values. The use of weighted elements with CTR values of only 2 was also shown to produce substantial SLL improvements (more than 1 dB in the majority of cases) over otherwise identically-specified equally-excited thinned arrays.

ACKNOWLEDGEMENT

The author would like to thank Will P. M. N. Keizer for his willingness to explain the subtleties of the IFT, and the anonymous reviewers for their valuable comments and suggestions.

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