Effects of Variations in Structural Parameters on Performance of Switched Parasitic Arrays

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Abstract- Influences of variations in the structural parameters (or antenna dimensions) of the five elements circular switched parasitic array (SPA) antenna at 2.4 GHz are investigated in this paper. Variations in the structural parameters are assumed to model random errors in such parameters. The effects of these variations on the performance of the SPA antenna are investigated using a numerical approach. Mutual coupling between the array elements is taken into consideration. The variations are modelled assuming both uniform and Gaussian distributions, and they are simulated using MATLAB. The simulation results demonstrated that variations in each structural parameter can either increase or decrease the SPA antenna gain and input impedance depending on the given specifications. The gain and input impedance sensitivities per unit variation in each structural parameter are computed to determine the level at which the gain and input impedance can vary for a small predefined change (or error) in the structural parameter.

Index Terms— Switched parasitic arrays (SPA); SPA antenna structural parameters; Random error modeling and analysis.

I. INTRODUCTION

Smart antennas are recommended for the performance improvements in throughput, capacity and coverage of the wireless networks [13]. However, the usual smart antennas involve complex signal processing and hence result in a power hungry system. Also, each array element is connected to the feed circuitry and requires impedance matching and phase control. These factors result in increased power consumption, complexity, the size of the array and overall system cost. They make smart antenna systems unsuitable and unaffordable for some applications, especially considering network deployment in rural areas where most network devices are battery powered.

In order to customise the design so that the antenna remains feasible in energy efficient applications, system analysis commonly starts the design process. This is typically followed by the numerical tests (e.g. in the form of simulations). Thereafter, the system is manufactured and experimental tests are performed. Throughout this development chain, there are always uncertainties and errors associated with the measurement data. There is normally more than one type of error associated with the experimental or computed data. The most common errors include the systematic and random errors [1-3].

Systematic errors are frequently associated with the uncertainty in the experimental and/or measurement instruments. Systematic errors are normally predictable, and therefore they can be compensated for [4]. On contrary, random errors are the results of random deviation of the parameters of a system from their design values [2]. This paper focuses on the effects of the latter on the performance of the circular switched parasitic array antenna in this paper.

Recently, researchers have studied the effects of random errors in linear and planar arrays, with focus on the side-lobe level [1], directivity (or gain) [2], [5], [6], and beam pointing accuracy [1]. In addition, the effects of the mutual coupling error on the input admittance of antenna arrays have been studied in [7]. However, the analysis considered a case where mutual coupling between the antenna elements is not desirable, as it is the case also in [8]. In the case of the parasitic arrays, mutual coupling is fundamental to the functioning and performance of the antenna [9], [10].

Numerous studies have shown that although such errors maybe minimal, they do have influence on the antenna performance attributes such as gain, side lobes, and beam positioning precision [4-8]. However, most studies consider a case of the linear or planar arrays consisting of only the active elements [1], [4-6].

In this work, we numerically investigate the effects of the variations in the structural parameters on the gain and input impedance of the circular SPA antenna. These variations are assumed to model uncertainties with parameters of the SPA antennae. The degree at which variations in the structural parameters of the SPA antenna alter the gain and input impedance will be shown. Also, the sensitivity measures of the attributes (gain and input impedance) of the SPA antennas could be used during the design and manufacturing of the circular SPA antennas to obtain the desired system configurations.

The rest of the paper is organized as follows. Section II outlines the basis of our formulation of random error analysis. In section III, analysis of random errors on the antenna performance attributes are introduced. Section IV presents simulation procedure, results and analysis. We conclude the study with section V.

II. BASIS OF RANDOM ERROR ANALYSES

Randomness from measurement data can be introduced by assuming that, such data consist of two components [2], [11]. The first component is considered to be the *estimated average value* of the parameter, and the second component is the *error* associated with the measurement data. These two measurement components can be combined in the linear additive model and denoted by the error equation [2]:

$$y_{ij} = \mu_j + \xi_{ij}, \tag{1}$$

where y_{ij} is the i^{th} repeated measurement of the j^{th} variable in the function, for $i = 1,2,\dots, m$ and $j = 1,2,\dots, n$. μ_i is the estimated average value of the variable y_{ij}. ξ_{ij} is the measurement error of the i^{th} replicate observation on the j^{th} variable.

Furthermore, if an assumption is made that the set of errors ξ_{ij} is independent and equally probable, then, for every single observation of the j^{th} variable, there exists a set of m error components. The result of single observation can be presented as [2]:

$$x = g(y_1, y_2, \dots, y_m), \tag{2}$$

where $g(y)$ is a particular function of interest. The entries of vector y are defined by (1), which are the repeated measurements of one specific variable. Expression (2) indicates the sampling distribution of the resulting vector x, and the randomly subjected variable y, which is in turn also subjected to random variation. Therefore, the distribution properties of x depend on both the assumptions about the nature of the function $g(y)$ and y itself [2].

III. RANDOM ERRORS IN SWITCHED PARASITIC ARRAYS

A. System model

We consider a single ring circular switched parasitic array (SPA) antenna, consisting of five dipole elements (N=5): one central active element surrounded by four parasitic elements (P = 4). The system configuration is such that only one parasitic element is open circuited while the rest are short-circuited as illustrated in Fig. 1.

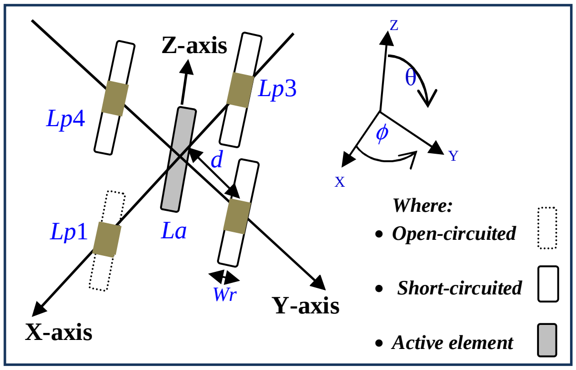

Fig. 1: Modelled SPA antenna geometry with five elements: 4 parasitic elements surrounding 1 central active element [9].

Table 1 presents the mean values of the investigated antenna system as demonstrated in Fig. 1. All parasitic elements are of equal length for the symmetrical purposes of the circular SPA antenna. Also, all elements (active and parasitic) are of the same thickness for simplicity.

Table 1: Mean values of the structural parameters of the investigated circular SPA antenna.

Parameter	***Value***
Length of active element, L_a	52 mm
Length of all parasitic elements, L_p	56 mm
Radius of all the array elements, Wr	0.8 mm
Placement of parasitic elements from active element, d	62.5 mm

B. Problem formulation

The performance of the SPA antennas is dependent on the mutual coupling amongst the array elements. To estimate the performance of the SPA antennas, firstly, analyses of the impedance matrix are carried out [9]. Examples of methods that can be used to compute the impedance matrix include an analytical technique such as the Induced Electromotive Force (EMF) method and a numerical technique such as the Method of Moments (MoM) [12].

The solution of either the induced EMF method or MoM depends on the length of the elements (L), the radius (thickness) of the elements (Wr) and spacing in between the elements (d). The operational frequency (f) also constitutes to the solution of the induced EMF method. Moreover, the antenna performance attributes are dependent on the currents in all the elements, which are in turn dependent on the structural parameters. The self and mutual impedances of the antenna array are dependent on the dimensions (structural parameters) of the array. Errors in the structural parameters of the SPA antenna will lead to errors in solutions of the Induced EMF method, and hence also in the antenna performance attributes.

Numerical variations introduced in the structural parameters signify manufacturing or experimentation or even environmental dynamics. If we distinguish between the length of the active element (L_a) and the length of all the parasitic elements (L_p), the variance in each structural parameter can be formulated based on (1). The operational frequency (f) also has influence on the performance attributes of the SPA antennas. Therefore, we would also study the performance of the SPA antenna over a defined frequency range.

Thus, variations in the studied parameters are modelled as:

$$\begin{aligned} L_a &= L_a^0 + \Delta L_a; \\ L_p &= L_p^0 + \Delta L_p; \\ d &= d^0 + \Delta d; \\ Wr &= Wr^0 + \Delta Wr; \\ f &= f^0 + \Delta f, \end{aligned} \tag{3}$$

where each parameter with superscript zero (e.g. L_a^0) represents the assumed mean values. Δ specifies the random error associated with a specified parameter.

C. Random error modelling distributions

Random errors can be described by using the probability density functions [2], [4]. The two probability distributions that are assumed and used to model the random errors in this paper are the Gaussian (normal) and uniform (rectangular) distributions [2], [4]. The normal distribution is chosen for ease of studying the spread of errors from mean values, and also for generating random numbers. However, with only the use of the Gaussian distribution, it is not easy to relate the correlation between the input and output parameters. All the values are distributed around the mean and not showing the direct correlation of the change in the input parameter versus the change in the output parameter. Therefore, the uniform distribution is chosen for the purpose of investigating the correlations between the studied parameters and attributes of the SPA antenna.

Using equation (1), the expressions for the two error distributions can be formulated. The measurement error ξ_{ij}, can be presented in such as way that it is modelled using either the Gaussian or uniform distribution. If we consider one variable at a time, for the uniform error distribution, the variable y_i is uniformly generated with the maximum deviation ± h [2]:

$$y_i = \mu + h(2U_i - 1), \tag{4}$$

where U_i is the i^{th} entry in the vector $\boldsymbol{U}$ of the variables uniformly distributed in the interval 0 to 1. An incremental step for the interval can be defined. h is the maximum possible deviation from the mean value μ. y_i is the i^{th} repeated uniformly distributed subjected variable.

Alternatively, if the measurement errors are modelled using the normal distribution, the randomly distributed subjected variable y_i is formulated as [11]:

$$y_i = \mu + \sigma R_i, \tag{5}$$

where R_i is the i^{th} value in the vector $\boldsymbol{R}$ of the normally distributed random numbers, for $i = 1,2,\ldots, m$. μ is the mean value for the varied parameter. σ is the standard deviation of measurements.

For easier comparison of the two distributions, the limit of deviations is presumed to have some relation in cases where the Gaussian and uniform distributions are used to model the same measurement data. This relation is considered here in as:

$$h \approx 4\sigma, \tag{6}$$

where h and σ are the error limits for the uniform and Gaussian distributions respectively.

Table 2 summarizes the varied parameters based on (5) for error modelling using Gaussian distribution. A table of specification for variation in the structural parameters based on uniform distribution can be obtained using Table 2 and (6).

Table 2: Gaussian distribution specifications for variation in each parameter

Structural Parameter	*Mean value, μ*	*Standard deviation, STD*
L_{ai}	52 mm	2.5
L_{pi}	56 mm	2.5
Wr_i	0.8 mm	0.195
d_i	62.5 mm	2.5
f_i	2.45 GHz	0.0013

D. Influences of the random errors on the performance of SPA antenna

The analysis for random errors on the performance of the antenna may be developed from the expression of current (both amplitude and phase) excitations [1], [4], [5]:

$$I_n = I_n^0(1+\Delta_n)e^{i\alpha_n}, \tag{7}$$

where I_n^0 is the assumed non-error current amplitude for the antenna array of N elements, for $n = 1,2,\ldots,N$. $e^{j\alpha_n}$ represents the phase in each current excitation while Δ_n indicates the associated error with the current excitation. Regardless of the type, geometry and size of the array, the radiation pattern is a function of the current distribution [4]. Thus, if there are errors in the current amplitude and phase, then all the antenna performance attributes will be affected. In this paper, the current excitation is a complex entity, and therefore the amplitude and phase excitations are not treated separately. Therefore, (7) can be represented as:

$$I_n = I_n^0 + (\Delta I)_n, \tag{8}$$

Deviations in the current excitations might be due to variations in the dimensions of the antenna as a result of manufacturing errors or environmental change. If we consider the far field of the circular switched parasitic arrays, it is a function of: the length of the elements (both active and parasitic), the current along each element and the placement of the parasitic element from the active elements as well as the vertical and azimuth angle [9]:

$$\begin{aligned} E_\theta(\theta,\phi) = I_0 \frac{\cos(kL_a\cos\theta)-\cos(kL_a)}{\sin\theta} + \\ \sum_{n=1}^{N-1} I_n \frac{\cos(kL_p\cos\theta)-\cos(kL_p)}{\sin\theta} \times \\ e^{(jkd\sin\theta\cos(\phi-\phi_n))} \end{aligned} \, . \tag{9}$$

Substituting (3) and (8) into (9), the error-inclusive representation of the radiation field of the circular SPA antenna can be obtained. Thus, all parameters that constitute to the far radiation field are assumed to have some random errors, which are introduced using either Gaussian or uniform distribution.

Based on (2), we also assume that equations used for solving the impedance matrix can be represented to accommodate the error. The relation of the current and impedance matrix can be presented in matrix form [10], [12]:

$$\mathbf{I} = \mathbf{Z}\mathbf{V}^{-1}. \tag{10}$$

where $\mathbf{Z}$ is the square impedance matrix, and $\mathbf{I}$ is the current vector. The statistical representation of (10) can be modelled as:

$$[\mathbf{I} + \Delta\mathbf{I}] = [\mathbf{Z} + \Delta\mathbf{Z}]\mathbf{V}^{-1}. \tag{11}$$

where $\Delta\mathbf{Z}$ is the matrix of the associated random errors in the impedance matrix. $\Delta\mathbf{I}$ is the vector of associated errors in the current vector. These changes lead to the change in the input impedance (Z_{in}) of the antenna. The random error associated with the input impedance can be expressed as:

$$Z_{in} = Z_{in}^{0} + \Delta Z_{in}, \tag{12}$$

where Z_{in}^{0} is the assumed non-error input impedance. ΔZ_{in} is the random error associated with the input impedance.

Antenna directivity can be defined in terms of the radiation intensity which is a function of the radiation pattern [10], [12]. If there are errors in the currents of the array elements and hence errors in the radiation pattern, the will be errors in all antenna attributes. The authors in [1], [4-6] have shown in general that the antenna directivity (or gain) can be represented as:

$$D = D^{0} + \Delta D, \tag{13}$$

where D^{0} is the computed average directivity value, assuming non errors. ΔD is the random errors associated with the average directivity. If a lossless antenna is assumed, then the directivity of the antenna would be considered as the antenna gain.

IV. SIMULATION PROCEDURE, RESULTS AND ANALYSIS

Five case studies are carried out, where in each case study, only one parameter is varied. All other structural parameters are fixed as per the specifications in Table 1, with the operating frequency being 2.4 GHz. However, for the case study of variations in the operating frequency, the frequency is varied as in Table 2. For each case study, both the uniform and Gaussian distributions are used to model the variations in the structural parameters. We assume that errors associated with the antenna gain and input impedance are as the result of the propagation of the errors (variations) in any of the structural parameters of the SPA antenna.

The results are obtained using the MATLAB simulation tool. During simulations, 1000 samples are assumed for both the uniform and Gaussian distribution modelling. Expression (3), (5) and the error-inclusive representation of (9) are used together with relevant expressions to obtain the antenna performance attributes [9], [13].

Simulation results for variations in the structural parameters (of the circular SPA antenna) are presented in this section. The results demonstrate the influence of variations in the structural parameters on the SPA antenna gain and input impedance. The mean value and standard deviation (*STD*) can be used to describe the performance of the system. Therefore, a sensitivity measure is introduced in this study for the analysis and comparison of the effects of the variations per each studied parameter. We define s*ensitivity as the change in antenna performance attribute per unit variation in the structural parameter.*

The standard deviations of the SPA antenna gain, input impedance and the structural parameters are used to define sensitivity measures in this study. The antenna attributes sensitivity per given parameter is defined as:

$$Att\_Par\_Sens = \frac{STD(Att)}{STD(Par)}, \tag{14}$$

where *Par* is a particular SPA antenna structural parameter. *Att* is the antenna attribute. *STD* is the standard deviation. The sensitivity measure is expressed in terms of the units of the antenna attribute per the measurement unit of a given structural parameter.

Although we have tried to use equal samples and limited the variations in the structural parameters to almost same interval by the use of (6), Gaussian response is random. The responses of the uniform distribution also yield non-linear curves. These explain the differences between the sensitivity measures computed using the two distributions (as illustrated in Tables 3 and 4).

Statistical results can not be generalized; however, we presume the sensitivity measure can assist in assessing the influences of the variations in the structural parameters on the SPA antenna gain and input impedance. In Table 3 to Table 5, the sensitivity measures of the SPA antenna gain and input impedance are presented. The tables are arranged in an ascending order starting with lesser sensitivity measures.

Table 3 demonstrates the influence of variations in the length of the active element, length of the parasitic elements, thickness of all elements, placement of parasitic elements from the active element and the operational frequency. The results indicate that change in the thickness of the elements is minimal since the expected error in the thickness of the elements can be of the micro-millimetre magnitude for the given specifications.

The length of the active element does not have much influence on the antenna gain as compared to the length of the parasitic elements. The SPA antenna gain does not change significantly with the change in the operational frequency. Variations in placement of the parasitic elements from the active element have an average influence on the SPA antenna gain as compared to the influence of other structural parameters.

Table 4 and Table 5 present the sensitivity measure of the SPA antenna input impedance. For accuracy, the real (*RZin*) and imaginary (*ImZin)* component of the input impedance are computed separately. Table 4 presents the sensitivity measures of the real component of the input impedance. Variations in the length of the parasitic elements have less influence as compared to the influence of variations on the active element on the SPA antenna input impedance (both real and imaginary component).

Table 3: Sensitivity measures of the circular SPA antenna gain per variations in the structural parameters.

Gain Sensitivity	***Units*** (±)	***Uniform***	***Gaussian***	***Average***	***Error between distributions***
Wr_Sens	dB/µm	0.0005	0.0004	0.00045	25%
La_Sens	dB/mm	0.0008	0.0008	0.0008	0%
f_Sens	dB/MHz	0.0041	0.0041	0.0041	0%
d_Sens	dB/mm	0.0554	0.0624	0.0589	11%
Lp_Sens	dB/mm	0.0979	0.1244	0.11115	21%

From both Table 4 and Table 5, it can be noticed that, variations in the lengths of the elements (both active and parasitic) have much influence on the antenna input impedance as compared to the other structural parameters. Generally, the results modelled using both distributions indicate that any variations in the structural parameters have some influence on the SPA antenna gain and input impedance.

Table 4: Sensitivity measures of the real component of the input impedance per variations in the structural parameters of the SPA antenna.

Gain Sensitivity	***Units*** (±)	***Uniform***	***Gaussian***	***Average***	***Error between distributions***
Wr_Sens	Ω/µm	0.0114	0.009	0.0102	26.67%
f_Sens	Ω/MHz	0.0484	0.0914	0.0699	47.05%
d_Sens	Ω/mm	2.2725	2.9643	2.6184	23.34%
Lp_Sens	Ω/mm	2.4364	4.0508	3.2436	39.85%
La_Sens	Ω/mm	4.2395	4.1488	4.1942	2.19%

Table 5: Sensitivity measures of the imaginary component of the input impedance per variations in the structural parameters of the SPA antenna.

Gain Sensitivity	***Units*** (±)	***Uniform***	***Gaussian***	***Average***	***Error between distributions***
Wr_Sens	Ω/µm	0.036	0.0287	0.0324	25.44%
f_Sens	Ω/MHz	0.183	0.1552	0.1691	17.91%
d_Sens	Ω/mm	3.6093	4.459	4.0342	19.06%
Lp_Sens	Ω/mm	3.1397	5.9884	4.5641	47.57%
La_Sens	Ω/mm	7.8686	7.7986	7.8336	0.90%

Fig. 2 and Fig. 3 illustrate the influence of variation in the thickness of the elements on the SPA antenna gain and input impedance. These are parametric plots. The results demonstrate a continual increase in antenna gain for the increase in the thickness of the elements in the interval of *Wr* >0.4 mm. However, for the given specification, Fig. 2 shows that variations in the thickness of the elements can either increase or decrease the SPA antenna gain although at a minimal level. Fig. 3 shows that variations in the thickness of the elements have minimal influence on both the real and imaginary components of the input impedance.

The resonant frequency of an antenna is determined by the antenna dimensions. The closer the operational frequency is to the resonant frequency, the better is the performance of the antenna. As the structural parameters vary, the SPA antenna turns to match or mismatch its resonant frequency. Thus, the antenna elements can be either strongly or weakly interacting with the fields. In Fig. 2, we notice that as the thickness of the elements increases, the gain of the SPA antenna becomes higher. This implies that, at the chosen operational frequency (2.4 GHz), the SPA antenna turns to match the resonance frequency of the investigated SPA antenna. Also, the minimal change in the input impedance demonstrated in Fig. 3 illustrated a better tuned antenna as the thickness of the elements increases.

Fig. 4 to Fig. 6 illustrates the results of the influences of the variations in the placement of the parasitic element from the active element. The results are modelled using the Gaussian distribution.

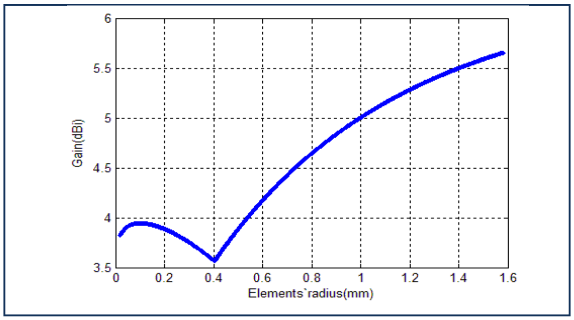

Fig. 2: Change in gain due to variations in thickness (radius) of all the SPA antenna elements, based on the uniform distribution.

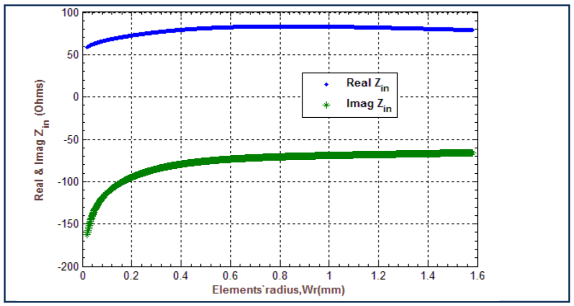

Fig. 3: Change in input impedance due to variations in thickness (radius) of all the SPA antenna elements, based on the uniform distribution.

The distributions of the SPA antenna gain and input impedance are in half form of the Gaussian distribution, as can be seen from Fig. 4, Fig. 5 and Fig. 6. This indicates the non-linearity of the uniform distribution curves (e.g. Fig. 2 and Fig. 3). Nonetheless, different values of gain and input impedance in these figures indicate that tuning the SPA antenna into and out of its resonant frequency as the structural parameters vary.

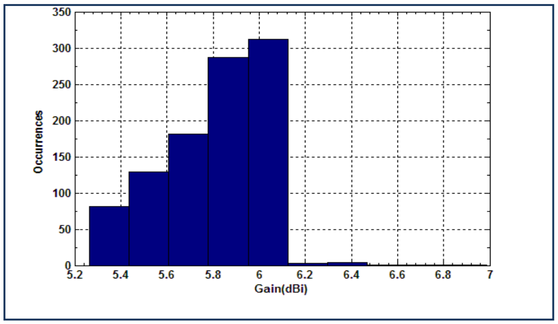

Fig. 4: Distribution of the SPA antenna gain due to variations in the placement of the parasitic elements from the active element, based on the Gaussian distribution.

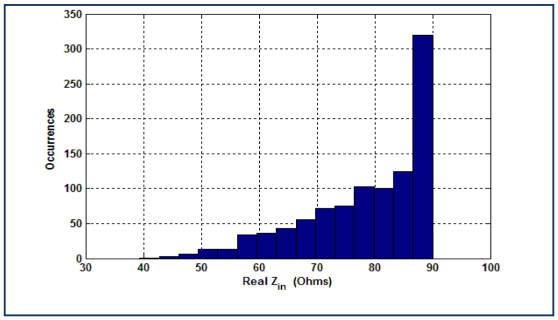

Fig. 5: Distribution of the real component of the input impedance due to variations in the placement of the parasitic elements from the active element, based on the Gaussian distribution.

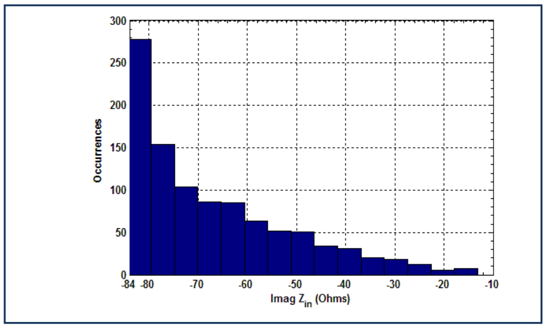

Fig. 6: Distribution of the imaginary component of the input impedance due to Variations in the placement of the parasitic elements from the active element, based on the Gaussian distribution

Although the numerically obtained statistical results may not be generalized, the result prediction for variations in each structural parameter can be summarized as follows:

- The change in the length of the active element mostly influences the change in the SPA antenna input impedance;
- The change in the length of the parasitic elements predominantly influences the change in the SPA antenna gain;
- The change in the radius (thickness) of the elements has minimal influence on both the SPA antenna gain and input impedance. This applies when considering thin wire elements and hence a change in the thickness of the elements is in the magnitude of the micrometres;
- The change in the placement of the parasitic elements from the active element contributes to the change in the SPA antenna gain and input impedance.
- The practical implication of these findings are described as follows:
    - In the SPA antennas, random errors may occur due to manufacturing errors, experimental and environmental changes.
    - Investigating effects of such errors on the structural parameters can help manage or minimise these errors and in turn improves the antenna performance.

V. CONCLUSIONS AND FUTURE WORK

The effects of variations (random errors) in the structural parameters of the circular SPA antenna have been statistically investigated. The results indicate that variations in each structural parameter have some influence on the SPA antenna gain and input impedance. Random errors can either increase or decrease the SPA antenna performance attributes with certain magnitude depending on the given specifications. The sensitivity measures for the gain and input impedance have been quantified to determine the degree at which the gain and input impedance can change per specified variations in each structural parameter.

Future work can still be done to compare the responses and performance of various SPA antenna geometries when experiencing variations in their structural parameters.

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