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Abstract. We investigate the generation and transformation of Bessel beams through linear and nonlinear optical crystals. We outline the generation of high-order vortices due to propagation of Bessel beams along the optical axis of uniaxial crystals and expand this to the nonlinear regime by outlining a new phase-matching process (full conical phase matching) in second harmonic generation of vector Bessel beams for various symmetries in uniaxial crystals. We demonstrate the principles experimentally in a uniaxial BBO crystal and find excellent agreement between the experimental and theoretical results. The results imply a coupling of the intrinsic and extrinsic optical angular momentum of the resulting fields, which may have importance in studies involving quantum entanglement of the angular momentum basis of light. [DOI: 10.1117/1.3572109]

Subject terms: Bessel beams; optical vortices; uniaxial crystals; second harmonic generation.

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1 Introduction

The transformation of polarization state of light fields while propagating in an anisotropic medium is an important topic in the development of classical crystal optics (see, for example, Refs. 1–3), and has led to a multitude of practical polarization devices. However, despite the success in applying the theory, the underlying theoretical framework is based on the limiting case of propagation of plane or quasi-plane waves.

The transition from plane waves to Bessel beams (BBs), which are also exact solutions of Maxwell equations, involves new peculiarities in the problem of the transformation of such beams during propagation in anisotropic crystals. It has been shown[4,5] that uniaxial and biaxial crystals transform simultaneously the polarization state and the spatial structure of BBs, changing the order of a dislocation of the phase front, thus changing the order of the BB after the crystal. In other words, the polarization dynamics of BBs is associated with an energy exchange between circularly polarized components of the fields propagating along the optical axis (of uniaxial or biaxial crystals). For example, it has been shown that in a uniaxial crystal, a right circularly polarized BB of the order m, converts into a left circularly polarized BB of the order $m+2$, and similarly, a left circularly polarized BB of the order m changes into a right circularly polarized BB of order $m-2$.[4,5]

High-order BBs, which are sometimes termed Bessel vortex beams, are an important class of propagation invariant fields, with an amplitude proportional to $J_m(q\rho)\exp(im\varphi)$, where J_m is the m'th order Bessel function of the first kind, ρ and φ are the radial and azimuth coordinates, and q is the transverse wave number. Because of their nondiffractive nature and a very narrow dark central region, high-order Bessel beams can be used for atom guiding over extended distances,[6–8] the focusing of cold atoms,[9] and for optical trapping and tweezing.[10,11] Single mode and superpositions[12] of such beams have been created, and the nondiffracting and self-reconstruction properties investigated and applied to simultaneous manipulation and rotation of particles in spatially separated sample cells.[13] Thus, the problem of generation and transformation of BBs of various orders is of both scientific and practical importance.

The general transformation of circularly polarized high-order BBs through uniaxial crystals has been studied previously,[4,14] and in fact, attention has shifted to studies of arbitrarily shaped optical beams. In particular, the propagation of arbitrarily shaped optical beams along the optical axis of uniaxial crystals has been considered, where it was shown that the Fourier-spectral analysis and diffraction considerations allows one to describe the full propagation and transformation of such fields within the paraxial regime.[15–19] Polarization dynamics has been studied[20] of circularly polarized paraxial Laguerre–Gaussian and Bessel Gaussian beams of higher order that propagate along the optical axis of uniaxial media. A number of features of the angular momentum exchanges between the orthogonally polarized Gaussian beams in anisotropic crystals have been investigated,[19,21–25] with emphasis on the coupling between the spin and orbital components of the angular momentum. It is now well understood that light may carry both a spin angular momentum associated with the polarization and an orbital angular momentum associated with the spatial distribution of the light.[26] This has implications in the understanding of polarized light propagating through such crystals when the incoming light is a BB: the order change reported can then be explained as a spin-orbital interaction of the light beam due to the crystal. Special attention is paid to the conversion of the spin angular momentum to orbital angular momentum, which leads to the generation in uniaxial crystals of optical vortices,[19,25] where this effect has been shown both experimentally and theoretically for circularly polarized Gaussian beams propagating along the optical axis of uniaxial crystals.[27]

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Nowadays, the generation of fields with phase singularities with the help of a form-birefringent medium[28] and photonic crystals[29,30] is of great interest. In Kurilkina *et al.*,[30] a method has been proposed of the transformation of Bessel vortices of the $(m-1)$ order into Bessel vortices of the $(m+1)$ order using one-dimensional photonic crystals (1-DPC) (with and without of a defect impurity—a layer of an uniaxial crystal). The intensity transformation of vector Bessel beams from the $(m-1)$ order to the $(m+1)$ one implies that in a beam transmitted through a multilayer system (1-DPC), the overwhelming part of photons has the orbital angular momentum, which increases by $2\hbar$ (per photon) in comparison to one of an incident Bessel beam (or corresponding charges of the vortices of the incident and transmitted beams differ by two units with respect to each other).

In this paper, we investigate the generation of high-order (vortex) BB beams through linear and nonlinear (NL) interaction in uniaxial crystals. We outline a new phase-matching condition (full conical phase matching) for the nonlinear frequency doubling of high-order BBs, where three-wave mixing of quasi-nondiffractive Bessel light beams is realized when the cone of plane waves of a Bessel beam coincides with the phase-matching cone of an uniaxial crystal. We present results for this frequency-doubling process for uniaxial crystals of hexagonal, tetragonal, and trigonal symmetry, and show experimental findings for the case of an uniaxial BBO crystal.

2 Generation and Transformation of Bessel Vortex Beams in Uniaxial Crystals

2.1 *Basic Equations*

In studies of Bessel vortex beam propagation in crystals, it is reasonable to neglect diffraction effects because the thickness of the crystal is usually small (millimeter or centimeter scale), and transverse Bessel beam size is relatively large so that the diffraction effect in the central area of the Bessel beam is not significant.[31] As a result, the analysis of the field transformation by a crystal can be performed using exact solutions of Maxwell's equations. By analogy with the ordinary (o-wave) and extraordinary (e-wave) plane waves in uniaxial crystals, Bessel beams have transverse electric field (TE)- and transverse magnetic field (TH)-polarization states. The solutions to Maxwell's equations in cylindrical coordinates (ρ, φ, z) for both TH- and TE-polarized Bessel beams propagating along the axis of a uniaxial crystal, are shown in Table 1.

Table 1 Expressions for the components of the electric E and magnetic B fields for TH- and TE-polarized BBs in uniaxial crystals.

TE-polarized BB	TH-polarized BB
$E_{o\rho} = \frac{i}{2}(J_{m+1} + J_{m-1})$	$E_{e\rho} = \frac{in_{ez}}{2n_o}(J_{m-1} - J_{m+1})$
$E_{o\varphi} = \frac{-1}{2}(J_{m-1} - J_{m+1})$	$E_{e\varphi} = -\frac{n_{ez}}{2n_o}(J_{m+1} + J_{m-1})$
$E_{oz} = 0$	$E_{ez} = \frac{n_r n_o}{\varepsilon_e} J_m$
$B_{o\rho} = \frac{n_{oz}}{2}(J_{m-1} - J_{m+1})$	$B_{e\rho} = \frac{n_o}{2}(J_{m+1} + J_{m-1})$
$B_{o\varphi} = \frac{in_{oz}}{2}(J_{m+1} + J_{m-1})$	$B_{e\varphi} = \frac{in_o}{2}(J_{m-1} - J_{m+1})$
$B_{oz} = -in_r J_m$	$B_{ez} = 0$

Here, $n_{oz} = n_o \cos(\gamma_o)$, $n_{ez} = n_e(\gamma_e)\cos(\gamma_e)$, $n_r = n_e(\gamma_e)\sin(\gamma_e) = n_0 \sin(\gamma_0) = n_1 \sin(\gamma_1)$, where $n_e(\gamma_e) = n_o n_e[n_o^2 \sin(\gamma_e)^2 + n_e^2 \cos(\gamma_e)^2]^{-1/2}$, and γ_{in} and $\gamma_{o,e}$ are the cone angles of incident TE (ordinary) and TH (extraordinary) BBs inside the crystal. Also n_o and $n_e(\gamma_e)$ are the refraction coefficients of the o- and e-waves, and n_1 is the index of refraction of the isotropic medium bordering the crystal.

Then, the vectors of the electric and magnetic fields of the TE and TH waves at the entrance face of the crystal ($z = 0$) are of the following form:

$$\vec{E}_o = \frac{i}{\sqrt{2}}\{J_{m-1}(qr)\vec{e}_+ \exp[i(m-1)\varphi] + \vec{e}_- J_{m+1}(qr)\exp[i(m+1)\varphi]\}, \quad (1)$$

$$\vec{E}_e = \frac{in_{ez}}{\sqrt{2}n_o}\{J_{m-1}(qr)\vec{e}_+ \exp[i(m-1)\varphi] - \vec{e}_- J_{m+1}(qr)\exp[i(m+1)\varphi]\}, \quad (2)$$

$$\vec{B}_o = \frac{n_{oz}}{\sqrt{2}}\{J_{m-1}(qr)\vec{e}_+ \exp[i(m-1)\varphi] - \vec{e}_- J_{m+1}(qr)\exp[i(m+1)\varphi]\}, \quad (3)$$

$$\vec{B}_e = \frac{n_o}{\sqrt{2}}\{J_{m-1}(qr)\vec{e}_+ \exp[i(m-1)\varphi] + \vec{e}_- J_{m+1}(qr)\exp[i(m+1)\varphi]\}, \quad (4)$$

where $q = k_0 n_r$ is the conicity parameter or radial wave number of the Bessel beam, and $\vec{e}_\pm = (\vec{e}_1 \pm i\vec{e}_2)/\sqrt{2}$ are unit vectors for right- and left-circular polarization.

Inside the crystal, the o- and e-modes have different velocities of propagation owing to which the spatial structure of the field changes even without diffraction.

2.2 *Calculation of Bessel Beams inside the Crystal*

We now calculate the change of the spatial field structure of the BB propagating through the crystal. Let the linear superposition of o- and e-modes, as given by Eqs. (1)–(4), of the type $\vec{E}_\pm = \vec{E}_o \pm \vec{E}_e$ be incident on the crystal face from an isotropic medium with refractive index n_1. The reflected and refracted fields can also be expressed in the form of superpositions of the o- and e-modes. At the boundary, one can calculate the transmission coefficients, $t_{o,e}$, to be

$$t_o = \frac{2n_1\cos(\gamma_1)}{n_1\cos(\gamma_1) + n_o\cos(\gamma_o)}, \quad (5)$$

$$t_e = \frac{2n_1\cos(\gamma_1)}{n_o\cos(\gamma_1) + n_1\cos(\gamma_e)n_e(\gamma_e)/n_o}. \quad (6)$$

From these expressions, one can make the following observation: calculation of the dependence of t_o and t_e on the incidence angle for a $LiNbO_3$ crystal shows that the "contrast" of the refraction coefficients, defined by $\eta(\gamma) = [t_e(\gamma)^2 - t_o(\gamma)^2][t_e(\gamma)^2 + t_o(\gamma)^2]^{-1}$ at a wavelength of $\lambda = 633$ nm does not exceed 2% for incident angles in the range from 0 to 20 deg. Consequently, when considering the angles within the BB cone typical in experiments, the difference between the t_o and t_e coefficients can be neglected.

This allows us to approximate $\cos(\gamma_i) \approx 1$ (because we may select the 0-deg option). In this case, the superposition $E_\pm$ of TH- and TE-modes of the electric field at the entrance face of crystal may be written in the simple form

$$E_+ = i J_{m-1}(qr)\vec{e}_+ \exp[i(m-1)\varphi], \tag{7}$$

$$E_- = i J_{m+1}(qr)\vec{e}_- \exp[i(m+1)\varphi], \tag{8}$$

and similarly for the magnetic field. As is seen in Eqs. (7) and (8), the polarization of the incident BB is circular. Such a beam can be obtained, for example, using an axicon.

Under these conditions, the TH and TE fields inside uniaxial crystal may be expressed as

$$\vec{E}_o(z,\rho) = \frac{t}{2}\{[J_{m-1}(q\rho) + J_{m+1}(q\rho)]i\vec{e}_\rho - [J_{m-1}(q\rho) - J_{m+1}(q\rho)]\vec{e}_\varphi\}\exp(ik_{oz}z), \tag{9}$$

$$\vec{E}_e(z,\rho) = \frac{t}{2}\{[J_{m-1}(q\rho) - J_{m+1}(q\rho)]i\vec{e}_\rho - [J_{m-1}(q\rho) + J_{m+1}(q\rho)]\vec{e}_\varphi\}\exp(ik_{ez}z), \tag{10}$$

where $k_{oz} = k_0 n_o \cos(\gamma_o)$, $k_{ez} = k_0 n_e \cos(\gamma_e)$, $\vec{e}_\rho$ and $\vec{e}_\varphi$ are the unit vectors of the cylindrical coordinate system.

Consider the case where the incident field is allowed to be right-circularly-polarized [i.e., described by the Eq. (7)]. Let $k_{oz,ez} = k_z \pm \Delta k_z/2$. Thereafter, summarizing amplitudes (9) and (10), we obtain that the transverse component of the refracted field $\vec{E}_+(z,\rho)$ is

$$\vec{E}_+(\rho,z) = \alpha\{J_{m-1}(qr)\vec{e}_+ \cos(\Delta k_z z/2)\exp[i(m-1)\varphi] + i J_{m+1}(qr)\vec{e}_- \sin(\Delta k_z z/2)\exp[i(m+1)\varphi]\}, \tag{11}$$

where $\alpha = it\exp(ik_z z)$, t is the Fresnel transmission coefficient for the normal incidence.

When a left-hand polarized beam is incident on the crystal, the transverse component of the refracted field $\vec{E}_-(z,\rho)$ is found in a similar manner and is equal to

$$\vec{E}_-(\rho,z) = \alpha\{J_{m+1}(qr)\vec{e}_- \cos(\Delta k_z z/2)\exp[i(m+1)\varphi] + i J_{m-1}(qr)\vec{e}_+ \sin(\Delta k_z z/2)\exp[i(m-1)\varphi]\}, \tag{12}$$

2.3 *Analysis of Bessel Vortex Beam Transformation*

Equations (11) and (12) were obtained in Ref. 4 and express an interesting feasibility of uniaxial crystals to transform the order of the topological charge of Bessel vortex beams. In the case of a right circularly polarized incident beam, the order of the Bessel vortex beam increases by two units and, for the left circularly polarized beam, it decreases by two units. Such a transformation takes place, as is seen from Eqs. (11) and (12), when fulfilling the condition $\sin(\Delta k_z L/2) = 1$, where L is the crystal thickness. The minimal crystal thickness is obtained from $L_{\min} = \pi/\Delta k_z$, where $\Delta k_z = k_{oz} - k_{ez}$. The relationship between $L_{\min}$ and the cone angle γ_{in} of an incident beam is shown in Fig. 1 As is seen from Fig. 1, the transformation of the order of the BB requires a crystal thickness of ~1 cm.

It is important to note that BBs of various orders have orthogonal polarizations [see Eqs. (11) and (12)], which allows one to separate them from each other easily in practice.

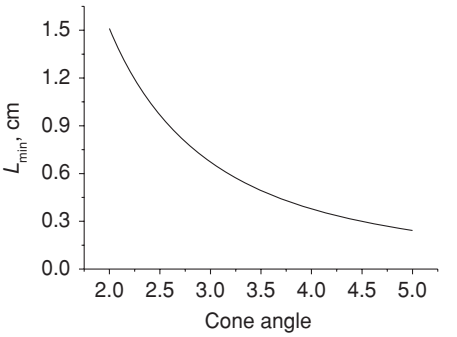

Fig. 1 Dependence of the minimal thickness of the $LiNbO_3$ crystal necessary for the full transformation of the order of Bessel function on the cone angle of an incident Bessel beam.

It can be seen from Eqs. (11) and (12) that in an arbitrary section of the crystal, the amplitude ratio is described by simple harmonic functions that allows one to form, if necessary, superpositions of two Bessel beams that differ on order by 2.

Using Eqs. (11) and (12), and considering the special case of $m = \pm 1$ for right- and left-circularly polarized incident BBs, we obtain

$$E_+(\rho,z) = \alpha[J_0(qr)\vec{e}_+ \cos(\Delta k_z z/2) + i J_2(qr)\vec{e}_- \sin(\Delta k_z z/2)\exp(2i\varphi)], \tag{13}$$

$$E_-(\rho,z) = \alpha[J_0(qr)\vec{e}_- \cos(\Delta k_z z/2) + i J_2(qr)\vec{e}_+ \sin(\Delta k_z z/2)\exp(-2i\varphi)]. \tag{14}$$

In this particular case, the transformation is of the type $J_0(q\rho) \rightarrow J_2(q\rho)\exp(\pm 2i\varphi)$, and the output beam contains a vortex of order 2. From Eq. (13), it follows that at a definite thickness of the crystal ($L = \pi/\Delta k_z$), a right circularly polarized incident J_0 beam is converted into a left circularly polarized Bessel beam J_2 (with a vortex of order 2) at the output of the uniaxial crystal. Physically, it can be related to the exchange of the angular momentum between orthogonally circularly polarized Bessel beams in an anisotropic crystal: the spin angular momentum ($+\hbar$) of the right–circularly polarized ($\vec{e}_+$) zero-order BB (J_0) is converted into the orbital angular momentum of the J_2 BB ($l = 2\hbar$) having left–circular polarization ($\vec{e}_-$) ($-\hbar$ spin angular momentum). Thus, while the overall angular momentum is conserved, orbital angular momentum is created through the generation of a high-order Bessel vortex beam.

Let us study briefly the case where the incident beam is linearly polarized. Summarizing Eqs. (13) and (14), we obtain

$$\vec{E}(\rho,z) \sim J_0(qr)\vec{e}_1 \cos(\Delta k_z z/2) + i J_2(qr)\sin(\Delta k_z z/2) \times [\vec{e}_1 \cos(2\varphi) + \vec{e}_2 \sin(2\varphi)]. \tag{15}$$

As is seen from Eq. (15), the field at $z = 0$ is linearly polarized along the x-axis parallel to the vector $\vec{e}_1$. In the case where the full transformation takes place [$\sin(\Delta k_z L/2) = 1$], from Eq. (15) we obtain

$$\vec{E}(\rho,L) \sim J_2(qr)[\vec{e}_1 \cos(2\varphi) + \vec{e}_2 \sin(2\varphi)]. \tag{16}$$

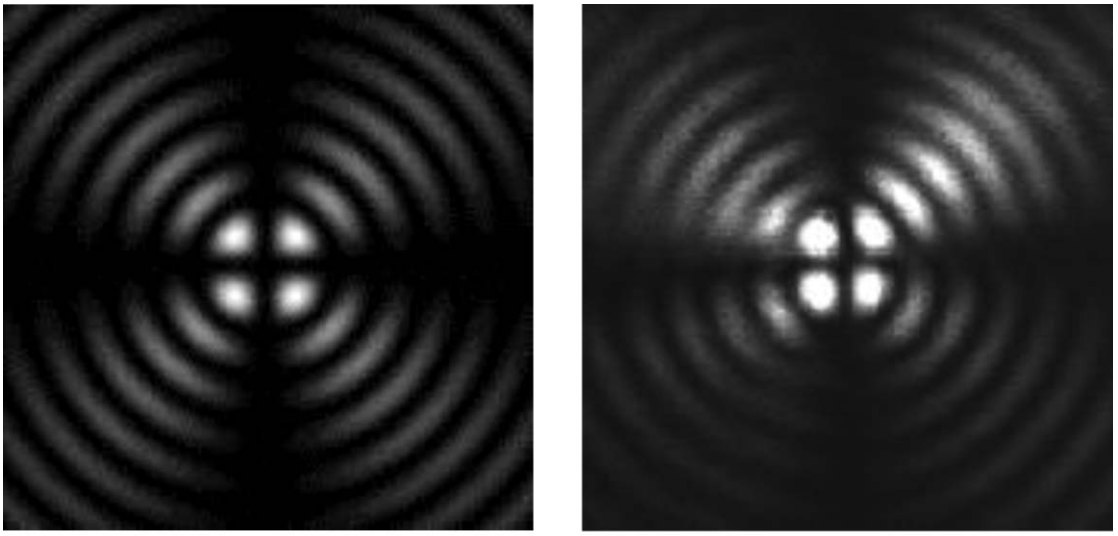

Fig. 2 (a) Calculated and (b) experimentally obtained azimuthally modulated Bessel vortex beam of the second order.

This field [Eq. (16)] is linearly polarized but with its direction of polarization rotated through an angle 2φ while the azimuthal angle changes by φ. The intensity of such a beam is proportional to $J_2(qr)^2$. In the case where at the output of the crystal the polarizer is set up with the transmission axis $y||\vec{e}_2$ (i.e., crossed with the polarization of incident beam), then the output field is expressed by

$$\vec{E}(\rho, L) \sim J_2(qr)\sin(\Delta k_z L/2)\vec{e}_2 \sin(2\varphi). \tag{17}$$

It follows from Eq. (17) that the intensity of the output field is modulated in the azimuthal angle, φ. Figure 2(a) shows the calculation of the field intensity according to Eq. (17) and the corresponding experimentally measured intensity distribution [Fig. 2(b)]. The experimental setup is described in previous work.[32] Clearly, there is very good agreement between the theory and the experimental observations.

Thus, uniaxial crystals may be considered as "mode converters" of sorts, where an input BB may be transformed into a higher order BB carrying orbital angular momentum. Such transformers are characterized by high efficiency (up to 100%), minimal distortion of the output field, and the possibility of transforming an array of input Bessel beams in parallel.

3 Generation of Bessel Vortices in the Process of Three-Wave Mixing

Bessel beams hold considerable potential in NL optics, namely, for second harmonic generation (SHG),[33–35] optical parametric generation,[36,37] and Raman conversion[38–40] to name a few. One might reasonably expect that NL optical processes open more possibilities to generate and transform Bessel vortex beams. Apart from the practical importance, for example, to create optical tweezers, there are a number of physical problems related to generalization of the spin-orbital interaction within NL optical crystals.

3.1 *Full-Conical Phase Matching*

Uniaxial crystals, discussed in Sec. 2 with respect to linear optical processes, allow one to perform the generation and transformation of vortices using NL optical interactions, particular the process of second harmonic generation as well as the processes of sum and difference frequencies generation.[32] The peculiarity of using BBs for second harmonic generation consists of the need to fulfill the transverse and longitudinal phase-matching conditions.[34,35] In the case where a BB propagates along the direction of the phase matching for plane waves or Gaussian-type beams, the simultaneous fulfillment of these conditions, strictly speaking, is unrealizable. The reason for this is that BBs are made up of plane waves traveling on a cone, and it is quite possible for the cone angle to be larger than the angular width of the phase-matching condition. The result of this is the breaking of the azimuthal symmetry of the NL interaction and, as a consequence, the intensity distribution of the second harmonic generation. As may be expected, the asymmetry increases concomitantly with the cone angle of the BB.

There exists a simple way of achieving azimuthally symmetrical generation of the second harmonic by BBs. To visualize this, it is helpful to consider the geometry (of interaction), where the direction of phase velocity of the incident Bessel beam coincides with the optical axis of the crystal (see Fig. 3). In addition, consider the case where the BB cone

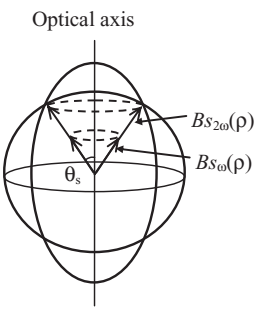

Fig. 3 Geometry of full conical phase-matched SH generation when wave vectors cone of Bessel beam at fundamental $Bs_{\omega}(\rho)$ and second harmonic frequencies $Bs_{2\omega}(\rho)$ coincides with the phase-matching cone of uniaxial crystal.

angle, γ, and cone angle, θ_s, of the phase-matching direction in an uniaxial crystal are equal.

In the scheme shown in Fig. 3, the process of *oo-e* type SHG takes place due to the collinear interaction. Thus, the second harmonic generation along the longitudinal axis is fulfilled for all azimuthal angles (or equivalent, for all waves traveling on the cone) within the region of $(0–2\pi)$. Because of the axial symmetry of interaction, the SH field is a Bessel beam with the same cone angle θ_s that provides the maximum of the overlap integral with BB of the fundamental frequency.[34] Note that the analysis of the *oe-e* type interaction is fully analogous.

3.2 *SHG of BBs in Hexagonal Symmetry Crystals*

One should note that in the analysis of Sec. 3.1 (see Fig. 3), an implicit requirement is that a BB with a large-cone angle is require for the condition of full conical phase matched SHG to be met. For example, in the case of BBO, the required angle is ~23 deg for near-infrared laser radiation (1.06 μm). For this reason, it is necessary to perform a full vectorial analysis of the SHG process, which we consider in this section.

One should note that optimal conditions are realized for SHG of high-order BBs when a circularly polarized BB is incident on the crystal. From the Maxwell equations, it follows [see Eqs. (7) and (8)] that the transverse component of the electrical field for such beams can be expressed in the form $\vec{E}_{i\perp}(\rho, z) = \vec{e}_+ J_0(q\rho)\exp(ik_z z)$.

From the boundary conditions (see Sec. 2.1), it follows that the incident beam excites in the crystal equally weighted TH- and TE-polarized BBs, denoted by indices "o" and "e", respectively. The transverse components $\vec{E}_{\perp o}$ and $\vec{E}_{\perp e}$of the electric field for these beams are expressed as [see Eqs. (1) and (2)]

$$\vec{E}_{\perp o} = i\,\frac{A_o \exp(ik_{oz}z)}{\sqrt{w_1(q)}}[J_0(q\rho)\vec{e}_+ + J_2(q\rho)\vec{e}_- \exp(2i\,\varphi)],$$

$$\vec{E}_{\perp e} = i\,\frac{A_e \exp(ik_{ez}z)}{\sqrt{w_1(q)}}\cos(\gamma_e) \times [J_0(q\rho)\vec{e}_+ - J_2(q\rho)\vec{e}_- \exp(2i\,\varphi)], \quad (18)$$

where $A_{o,e}$ are the amplitudes, $w_1(q) = 2\pi\int_0^R [J_0^2(q\rho) + J_2^2(q\rho)]\,\rho\,d\rho$ is the normalization multiplier, and R is the beam radius. Here, the longitudinal components $E_{o,z} = 0$ and $E_{e,z} = 2(\varepsilon_o/\varepsilon_e)\sin(\gamma_e)J_1(q\rho)\exp[i\varphi]$, where ε_o and ε_e are the principal values of the dielectric permittivity tensor of the crystal.

Using Eq. (18), we have calculated the nonlinear dielectric polarization $P_{2,i} = d_{ijk}E_jE_k$, where d_{ijk} are components of third-rank dielectric susceptibility tensor. In the case of *oe-e* nonlinear interaction for hexagonal symmetry crystals of point group C_6, we obtained the following:

$$\vec{P}_2(\rho, \varphi, z) = A_{10}(z)A_{1e}(z)\vec{f}_2(\rho, \varphi)\exp[i(k_{0z} + k_{ez})z], \quad (19)$$

where

$$\vec{f}_2(\rho, \varphi) = \frac{2d\sin(\gamma_e)}{w_1(q_1)}\frac{\varepsilon_o(\omega)}{\varepsilon_e(\omega)}J_1(q\rho)\{\vec{e}_+ J_0(q_1\rho)\exp[i(\varphi - \varphi_0)] - \vec{e}_- J_2(q_1\rho)\exp(3i\varphi + i\varphi_0)\},$$

$$d = \sqrt{d_{14}^2 + d_{15}^2}, \quad tg(\varphi_0) = d_{15}/d_{14}. \quad (20)$$

As is seen from Eq. (19), transfer to vectorial BBs introduces a component of the nonlinear dielectric susceptibility d_{15} in the SHG process. Its appearance is determined by the nonzero azimuthal component of the electric field of a TH-Bessel beam. Note that it is absent in an extraordinary plane wave; this peculiarity of vector BBs opens, in principle, the prospect of increasing the efficiency of the three-wave mixing process due to use of maximal values of NL coefficients.

On the basis of the form of NL polarization [Eqs. (19) and (20)], the spatial structure of the SHG TH beam can be specified. In the general case, the solution of Maxwell's equations for the *e*-wave in a crystal can be written as Eq (2). A comparison of Eqs. (19) and (20), and Eq. (2) shows that the coincidence of their azimuthal dependence takes place at $m = 2$. Consequently, the second harmonic field can be represented as

$$\vec{E}_e = \frac{iA_2(z)}{\sqrt{w_2(q_2)}}\cos(\gamma_2)[J_1(q_2\rho)\vec{e}_+\exp(i\varphi) - \vec{e}_- J_3(q_2\rho)\exp(3i\varphi)]\exp(ik_{2z}z), \quad (21)$$

where $w_2(q_2) = 2\pi\int_0^R [J_1^2(q_2\rho) + J_3^2(q_2\rho)]\,\rho\,d\rho$.

Here, the function $A_2(z)$ is described by the following equation for slowly varying amplitudes

$$\frac{\partial A_2(z)}{\partial z} = iA_{1o}A_{1e}[g_{011}(q_2) + g_{123}(q_2)]\exp(-i\,\Delta k_z z), \quad (22)$$

where $\Delta k_z = k_{2z} - k_{oz} - k_{ez}$.

The peculiarity of the SHG in the case of vector Bessel beams is a relatively complex structure of the overlap integrals that determine the so-called transverse phase matching of interacting beams. In the examined case, the overlap integrals are described by the following expressions:

$$g_{011}(q_2) = \frac{2d\varepsilon_o(\omega)tg(\gamma_e)\exp(-i\,\varphi_0)}{w_1(q_1)\sqrt{w_2(q_2)}\varepsilon_e(\omega)} \times \int_0^R J_0(q_1\rho)J_1(q_1\rho)J_1(q_2\rho)\rho\,d\rho,$$

$$g_{123}(q_2) = \frac{2d\varepsilon_o(\omega)tg(\gamma_e)\exp(i\,\varphi_0)}{w_1(q_1)\sqrt{w_2(q_2)}\varepsilon_e(\omega)} \times \int_0^R J_0(q_1\rho)J_2(q_1\rho)J_3(q_2\rho)\rho\,d\rho. \quad (23)$$

As is seen from the integrand expressions in Eq. (23), the overlap integrals $g_{011}(q_2)$ and $g_{123}(q_2)$ are responsible for the generation of the first- and third-order Bessel beams (of the SHG field), respectively, and by J_0, J_1, and J_2 of the pump fields (at the fundamental frequency). The numerical simulation of the integrals in Eq. (23) shows that they have maximum at $q_1 + q_2 = q_3$, a transverse wave number approximately the same as that for the interaction of zeroth-order Bessel beams. Note that the above relation between transverse wave numbers $q_1 + q_2 = q_3$ points to a collinear type of phase matching for plane-wave components of Bessel beams (see Fig. 3).

From the comparison of the field incident on the NL crystal and the SHG field, it follows that in this NL process there occurs a transformation of the BB order (see Fig. 4). Thus, in crystals of the C_6 class symmetry at the *oe-e* interaction

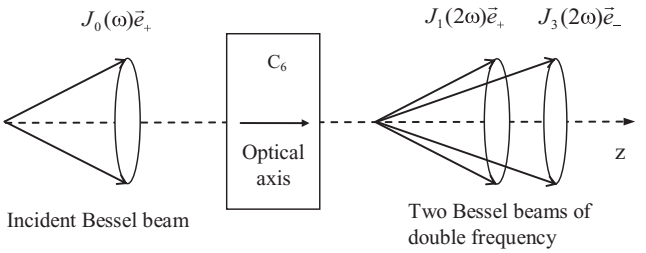

Fig. 4 Transformation of the order of Bessel function at oe-e interaction of vector Bessel beams in uniaxial crystal of C_6-symmetry.

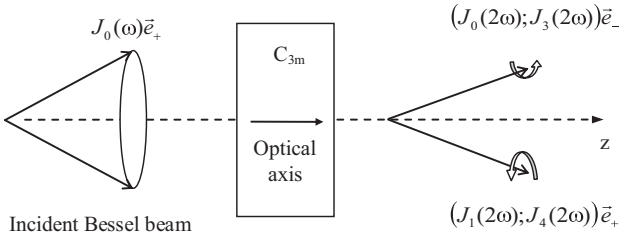

Fig. 5 Superposition of four Bessel beams of different orders generated by circularly polarized Bessel beam of the zeroth order.

there takes place the following transformation of the order of BBs (or equivalently the generation of high-order BBs): $J_0(\omega)\vec{e}_+ \rightarrow J_1(2\omega)\exp(i\varphi)\vec{e}_+ + J_3(2\omega)\exp(3i\varphi)\vec{e}_-$.

Next, let us consider the interaction of the *oo-e* type. The calculation of the NL polarization for the SHG frequency gives $p_{2x} = 0$ and $p_{2y} = 0$, while the longitudinal component is nonzero and equal to $p_{2z} = -4d_{15}A_1^2 J_0(q_1\rho) J_2(q_2\rho)\exp(2ik_{oz}z + 2i\varphi)$.

It is necessary to stress that this component of the NL polarization is absent in the case of plane waves, and its appearance is determined by the difference of polarization of the ordinary and extraordinary plane waves and the TE- and TH-polarized BBs.

From the known NL polarization, one can find the longitudinal component of the SHG field: $E_{2z} \sim J_2(q_2\rho)\exp(2i\varphi)$. The transverse component is obtained from the solution of Maxwell's equations and coincides with Eq. (21). Consequently, in the SHG process there occurs a transformation of the BB order, as illustrated in Fig. 4.

3.3 *Second Haromnic Generation of Bessel Beams in Trigonal Symmetry Crystals*

The nonlinear polarization vector for SHG of type o-ee in crystals of symmetry $3m$ point group can be represented in the form as of Eq. (19), where

$$\vec{f}_2(\rho,\varphi) = \frac{2i}{w_1(q_1)}\left\{\begin{array}{l} d_{15}\sin(\gamma_e)\dfrac{\varepsilon_o(\omega)}{\varepsilon_e(\omega)}J_1(q\rho)[\vec{e}_+ J_0(q_1\rho) \\ \times\exp[i\varphi] + \vec{e}_- J_2(q_1\rho)\exp[i3\varphi]] + \\ + d_{22}\cos(\gamma_e)\left[\vec{e}_- J_0^2(q_1\rho) + \vec{e}_+ J_2^2(q_1\rho)\right. \\ \left.\times\exp[4i\varphi]\right] \end{array}\right\}. \tag{24}$$

From Eq. (24), it is found that the SHG field is the superposition of two components:

$$\vec{E}_2(\rho,\varphi) = \vec{E}_2^+(\rho,\varphi) + \vec{E}_2^-(\rho,\varphi), \tag{25}$$

where

$$\vec{E}_2^+(\rho,\varphi) = [J_1(q_2\rho)\exp(i\varphi) + J_4(q_2\rho)\exp(4i\varphi)]\,\vec{e}_+, \tag{26}$$

$$\vec{E}_2^-(\rho,\varphi) = -[J_0(q_2\rho) + J_3(q_2\rho)\exp(3i\varphi)]\,\vec{e}_-. \tag{27}$$

As is seen, there are two components (see Fig. 5) of the second harmonic radiation: one contains the left–circularly polarized SHG field, while the other contains the right–circularly polarized SHG field. Using Eqs. (24) and (25), it is feasible to obtain equations for the slowly varying amplitudes in the usual way and to calculate the overlap integrals, but even from the from of the solution of Eq. (25) it follows that the *oe-e* type of interaction in crystals $3m$ is accompanied by a rich set of transformations of the BB order, namely, a right–circular polarized zeroth-order BB generates a superposition of high order BBs of the first and fourth orders with the polarization orthogonal to the incident beam. Moreover, an additional third-order BB is generated with the polarization coincident with that of the incident beam (see Fig. 5).

It is interesting to consider the amplitude and phase structure of the generated BB superpositions. Following Eqs. (26) and (27), the second harmonic intensity is azimuthally inhomogeneous and governed by

$$I(x,\varphi) \sim J_0^2(x) + J_1^2(x) + J_3^2(x) + J_4^2(x) + 2\,[J_0(x)J_3(x) + J_1(x)J_4(x)]\cos(3\varphi), \tag{28}$$

where $x = q_2\rho$.

Interference patterns produced by second harmonic radiation and a spherical reference wave are shown in Figs. 6 and 7. As is seen in the field depicted in Fig. 6(a), the vortex is localized in a narrow central area. In the case of interference with a BB of the third order [Fig. 6(c)], with BB of first order [Fig. 7(a)] and of the fourth order [Fig. 6(b)], there is an observable helical structure with an additional modulation. Here the axial vortex shifts from the beam axis. Near the axis, there appear m vortices, where m is the order of the BB. The presence of a BB of the zeroth order with the maximum in the center in Eq. (26) suppresses the local minimum of intensity in the beam center [Fig. 6(d)]. Mutual influence of vortices of the first and fourth order in Eq. (26) [Fig. 7(c)] destroys the helical picture [Fig. 7(b)]. The resulting intensity of the field of second harmonic is characterized by an azimuthal modulation of the third order and absence of intensity nulls.

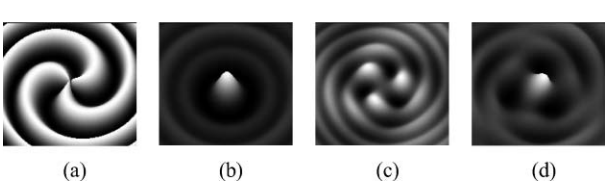

Fig. 6 Spatial structure of the left-polarized component of the second-harmonic vortex field revealed at interference with spherical reference wave: (a) interference with the field $\exp(3i\varphi)$, (b) with the zeroth-order BB in Eq. (1), (c) with the third-order BB in Eq. (1), and (d) with total field Eq. (1).

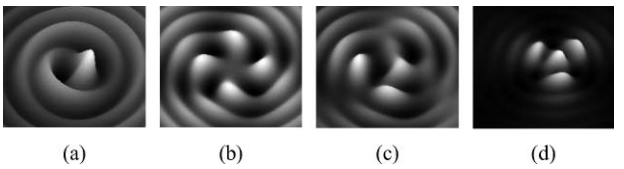

Fig. 7 Spatial structure of the right-polarized component of the second-harmonic vortex field revealed at interference with spherical reference wave: (a) interference with the first-order BB, (b) with fourth-order BB in Eq. (2), (c) with the total field in Eq. (2), and (d) spatial distribution of the intensity of the second harmonic field Eq. (2).

4 Experimental Results

We make use of BBO, a uniaxial crystal of $3m$ point group symmetry in order to test the full azimuthal phase-matching condition. The phase-matching angle was $\theta_s = 22.8$ deg for $\lambda = 1.064\ \mu$m. The crystal was cut perpendicular to the optical axis and had a thickness of 5 mm. The angular width of phase matching ($\delta\theta_s$) was equal to 0.51 mrad, which practically excludes the possibility of SHG in the traditional scheme when a BB propagates along the phase-matching direction.

In order to produce the pump BB, the necessary cone angle in air was $\gamma = 39.9$ deg. To achieve this, use was made of a refractive axicon with a base angle of 5 deg and a specially manufactured conical mirror for increasing the cone angle up to the necessary value [(Fig. 8(a)]. It should be noted that, for obtaining BBs with large cone angles, it is promising to invoke reflective axicons in a combination with a conical mirror [Fig. 8(b)].

The source was an Nd:yttrium–aluminum–garnet laser (wavelength of 1.064 μm) outputting a Gaussian beam with an angular divergence of $\theta \approx 0.8$ mrad, pulse duration of 50 ns, and energy of 20 mJ.

Figure 9(b) shows the experimentally measured intensity distribution of the SHG in the far field with the aid of a Fourier-transforming lens. For comparison, Figure 9(a) shows the calculated Fourier-spectrum field $E(\rho,\varphi)$ described by Eq. (25). The key feature of the order of generated Bessel function is seen to be the number of lobes in the intensity distribution that proves the calculation performed before. Clearly, the theoretical and experimental results are in good agreement.

5 Discussion and Conclusions

We have studied, theoretically and experimentally, the dynamics of high-order vortex generation and transformation when Bessel beams propagate or nonlinearly interact in uniaxial crystals. The theoretical description in the linear regime of vortex generation and transformation has been carried out using exact solutions of Maxwell's equations. Without taking into account the processes of diffraction, which is justified when considering BBs used in real experiments with crystals, it is possible to simplify the final equations describing the BB transformation.

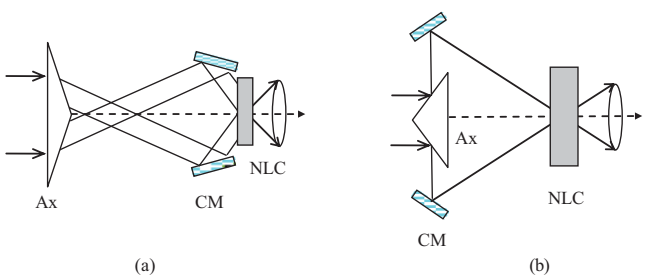

Fig. 8 Variants of experimental scheme: Ax, axicon [(a) refractive and (b) reflective]; CM, conical mirror; and NLC, nonlinear crystal (BBO).

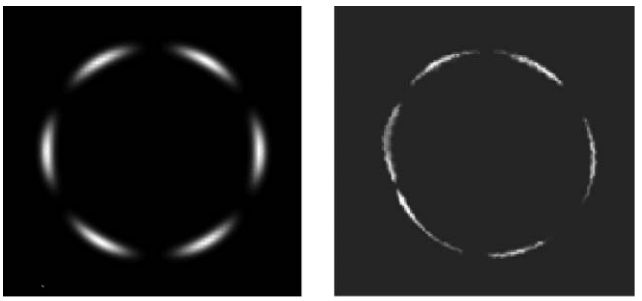

Fig. 9 Far-field intensity distribution of second harmonic, generated in BBO crystal: (a) calculation and (b) experiment.

As a result, a visual comparison can be made of the transformation of BBs and plane waves in uniaxial crystal, namely, the equation for the full transformation [$\sin(\Delta k_z L/2) = 1$] of BBs polarization coincides with the corresponding equation for a half-wave plate $\sin(\Delta kL/2) = 1$. The distinction of these equations consists in the replacement: $\Delta k_z \rightarrow \Delta k$, as a consequence of the conical nature of Bessel beams. Here, the novelty is in the fact that unlike plane waves, it is impossible to change the circular polarization of a BB into an orthogonal component while simultaneously preserving the spatial structure of the beam. This property of Bessel beams is demonstrated certainty, for example, by Eqs. (7) and (8), which describe Bessel beams obtained by superposition of TE and TH modes of the same order. On the other hand, rigorous solutions of Maxwell's equations for TE and TH Bessel modes in uniaxial crystal (see Table 1) contain combinations of Bessel functions the order of which differs by two units. Hence, it follows that the passage of the Bessel beams through the half-wave plate can be accompanied by the change of the order of Bessel function only by two units. The advantage of using "pure" Bessel beams, but not their superpositions differing by transverse wave number, is the possibility of practically full energy transformation of Bessel beams, their order change being from order m to order $m \pm 2$ (also the order of screw dislocation or topological charge), similarly to the full transformation of the polarization state of plane waves using half-wave plates. It is clear that for superpositions of BBs, which differ by transverse wave number, the full transformation of polarization as well as the order of Bessel functions is unachievable, and only a periodical oscillation is possible of the field structure in the coordinate z, as is described by others.[17] It is worth noting that the use of an analogous approach for biaxial crystals also allows one to effectively transform the order of Bessel functions by one unit.[41]

The problem of transforming the BB order in the process of nonlinear interactions of BB is much more complex, with the majority of papers in this direction considering the case when the BB propagates in the same direction of phase matching as for plane waves.[7,34,35] The complexities appear

with the description of vector BBs, which in general, given that they may propagate in an arbitrary direction in an uniaxial crystal, are not eigenmodes of the crystal. Here, the exclusion is the case of the propagation along optical axis and the SHG is realized for the condition of full conical phase matching. From the results presented here, we have outlined how to solve the problem of the generation and transformation of high-order BBs in a rigorous manner. Here, unlike the linear propagation, there appear many possibilities for the manipulation of Bessel vortex beams.

In conclusion, it is shown, theoretically and experimentally, that when a circularly polarized zeroth-order BB propagates along the optical axis of uniaxial crystals, ∼100% of its energy is converted, under certain conditions, into a second-order BB. Because of transversal invariance of the proposed crystal-based scheme, it is possible to transform several Gaussian input beams into an array of vortex beams simultaneously. The high radiation damage threshold of crystals makes it possible to use them in the generation of powerful optical vortex fields.

We have considered the frequency doubling of BBs by making use of a new full conical phase-matching condition. This scheme allows putting into practice the nonlinear frequency transformation of Bessel beams having a cone angle of several tens of degrees. Peculiarities of frequency doubling of Bessel vortices under the conditions of the full conical phase matching have been investigated for uniaxial crystals of hexagonal and trigonal symmetry. This new type of frequency doubling of Bessel vortex beams has been experimentally realized in a uniaxial BBO crystal, where the incident zero-order Bessel light beam at the fundamental frequency was directed along the optical axis of the crystal and its cone angle set equal to the conical phase matching angle.

The SHG by Bessel beams in the conditions of full conical phase matching allows one to generate Bessel vortex beams of various orders, as well as their linear superpositions. The selection of the field structure of the SHG is realized by means of the mechanism of the transverse phase matching, while the longitudinal phase matching in the scheme under study did not depend on the azimuthal angle. The particular cases of hexagonal and trigonal symmetry crystals (C_6 and $3m$ point group) differ by axial symmetry of the effective nonlinear dielectric susceptibility (d_{eff}, coefficient). In general, there is an azimuthal dependence of SHG efficiency when $d_{\text{eff}} = d_{\text{eff}}(\varphi)$. The result of this would be an additional azimuthal modulation of the second harmonic field and novel possibilities of transformation of Bessel vortices.

Let us point out some advantages of the conical phase-matching condition for BBs over traditional phase matching: (*i*) more effective components of the nonlinear susceptibility tensor can be involved in the nonlinear process and (*ii*) submicron spatial structure of the second harmonic field can be realized. In addition, the advantage of the axial-symmetric scheme is the absence of walk-off effect and, hence, the distortion of the second harmonic intensity distribution. Consequently, crystal-based transformers of the Bessel beam order, such as linear analogs, would be characterized by a high quality of the output optical signal.

We should point out that the generated vector Bessel beams are characterized by a rather small-scale ring structure. For example, in the experiments described here, the diameter of the main maximum of BB of the zeroth order was ∼0.58 μm and the diameter of the dark axial field for the first-order BB was ∼0.93 μm. Undoubtedly, such beams would be interesting for applications in microscopy and for the creation of optical tweezers.

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