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Characterization and Properties of Breakwater Structures modelled by a Physics Engine

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Abstract

We examine the question of stability of breakwater structures using the simulation software PhysX. This "physics engine" can be used to construct various packings of breakwater elements, each of which can be used to analyze the damage when these structures are exposed to (wave) forces of varying magnitude. We find that the damage is roughly proportional to the force squared, and that there is not a clear transition point where one can talk of the collapse of the structure. A range of packings of breakwater structures and a variety of numerical analyses will be discussed in the paper. Analogies to the analysis of granular media are noted.

1 Introduction

In order to protect harbours and coastlines engineers often rely on breakwater structures made out of armour units. Hence it is important to understand the dependence of the stability of these structures on various underlying parameters and factors. First,the material properties (dynamic and static friction, restitution, density) and the shape and geometry of the breakwater units play an important role in the overall stability. Since there are different types of units, such as dolosse, Antifer cubes, Core-loc and Xbloc units, it is important to distinguish between these different cases in analyzing stability. Second, the nature of the supporting layer plays a role in the stability of the structure. The supporting layer can be characterized by its geometry (slope, length, width, curvature) and by its physical characteristics (rubble specifications, frictional and restitution properties). Finally, the stability is influenced by the manner of packing of the units on the slope and the number of layers placed on top of each other. In the actual packing by crane operators, subjective factors enter as well, as the crane operator will use their momentary judgment to correct or adjust the placement of units, particularly as they cannot see the armour unit once it is under water.

With such an overwhelming number of factors and possibilities to vary the nature of the structures, a rigorous analysis of the dependence of stability on these factors is far from trivial. However, with some physics engine software tools, it is possible to vary individual factors and to study their effect on the stability. Also, these numerical tools allow one to inspect the inside of these structures, which would normally be inaccessible to harbour engineers. We have embarked on such a program, and are using the physics engine PhysX to carry out our simulations and technical analyses.

Before describing our exploitation of PhysX in more detail let us look at some other model studies of breakwater structures. Recent successes in breakwater armour unit modelling are reported by Latham et al. [1] using FEMDEM (Finite Element Method-Discrete Element Method) based on the methods of Munjiza [2] and Xiang et al. [3]. The group has evaluated the packing of units from free fall and the slopes generated by the removal of one supporting wall, and has developed numerical pull-out tests similar to those used in the field (see e.g. [4]). FEMDEM methods have the advantage of allowing the inspection of force chains transmitting stress through the packing (demonstrated for two-dimensional packing of ellipses in [1]). The FEMDEM code has been coupled to the fluid dynamics code Fluidity and preliminary results have been presented. The action of porous breakwaters in dissipating energy through turbulence has been demonstrated by Dentale [5]. Static breakwater units have been arranged in regular and random matrices on representative rubble mounds, and a Fractional area-Volume of Fluid method validated for surf zone hydrodynamics [6] has been used to quantify turbulent dissipation in the pores as well as laying the foundation for a full motion equation approach. Game engine kinematics models are being incorporated into fluid-rigid body interactions by Grobler et al. [7], as reported at this conference.

We have extended PhysX to carry out specific tasks and to provide desired information at various points of the simulation. The extended PhysX tool allows us to build a breakwater structure by lowering each individual unit sequentially according to a predetermined scheme. In the real world the units are lowered by a crane while held by a clamp, sling or other devices. In PhysX they are lowered at constant speed and then released (subjected to gravity) as soon as contact is made with the structure already in place. The software automatically imposes proper contact dynamics, while respecting the physical characteristics of the units. A nearly complete specification of the breakwater structure can be provided by the software at various stages of the simulation. For example, the porosity and coordination number of the structures can be calculated. The porosity of substructures (e.g of different layers through the structure) can also be provided. Another property, namely the “look” of the structure has also been investigated. This property (e.g. random or tiled) is used by harbour engineers as an important determinant of the stability of breakwater structures.

While it is fairly straightforward to compute these properties of structures, it is more difficult to characterize the stability of a certain structure theoretically. The practical definition of harbour engineers is to examine changes in the structure after a storm, characterizing a certain amount (percentage) of change as damage. While our software can evaluate microscopic changes, a suitable definition of macroscopic change, which applies to all types of structures, is not so obvious. Furthermore, the physics engine alone cannot model real waves, making it more difficult to implement the practical definition. As a substitute, our software can model a number of forces that may mimic waves: forces on one unit, forces on all units, or shaking the support layer as a whole.

In order to advance our theoretical knowledge and understanding of the stability of breakwater structures in a systematic way, we have limited ourselves to a simple breakwater structure of Antifer cubes with a smooth sloping underlayer with a toe. The main purpose is to come to a sensible theoretical definition of stability that can be used in physics engines. Once such an operational definition is developed, one can predict the stability of a greater variety of breakwater structures for different sets of physical unit characteristics.

2 PhysX experiments

In order to examine the response of breakwater structures to suitable wave forces, we constructed various packings of breakwater structures and exposed them to a particular set of forces scaled by an overall magnitude. We first describe the packings.

2.1 Nature of the breakwater packing

In order to enhance the practical relevance of our analyses we used a packing for Antifer cubes suggested by v.d. Meer [private communication, v.d. Meer, 2008/11/05 and 2009/6/15]. The Antifers have flat sides which makes it easy for them to slide down the slopes. The Antifer cubes, as used in this simulation, have a volume of 16 m^3, and are smaller at the top than at the bottom. The bottom square has a length of 2.674 m, while the top has a length of 2.475 m. The height of the Antifer is 2.574 m. The four sides of manufactured Antifers have small grooves of width 0.658 m and depth 0.232 m, running from top to bottom. We have approximated these grooves in the model by small rectangular grooves of comparable cross-sectional area. The total volume without grooves is about 17 m^3, with the grooves reducing it to 16 m^3. The further physical characteristics of the structures are as follows. The density of the Antifer cube is 2350 kg/m3, its restitution is 0.01, its static and dynamic friction are both .75. For the rubble properties we choose the same physical parameters. PhysX also uses linear and angular damping (both 0.9) to make the response of solid structures more realistic.

For the Antifer cubes there is a variety of placing grids available and there is no standard. The packing we have used are based on a parallelogram placing grid, whereby the Antifers are packed under an angle of about 60^0 against the toe and consist of two layers. The slopes considered are between 1:2 and 1:2.5. The longitudinal length between the grid points is 4.011 m, while the transversal length is 2.754 m. Other strategies were also considered, for example a longitudinal length of 3.343 m and a transversal length of 3.316 m, which leads to a similar density. We used a slope at the low end of the range (1:2). The resulting idealized packing in shown in Fig. 1. The number of Antifer cubes used in this packing is 319. The bed on which the Antifers are placed is 90 m wide and 27 m deep. The toe of the bed has a height of 1.5 m. Since the crane operator has little control over the angular orientation of the Antifer cube (the idealized packing assumes that they are parallel to the sea side), we have subsequently allowed for random rotations (aroud the vertical axis) of the units. Since PhysX checks for contact dynamics, it will reorient a unit when it hits another unit when it is placed. Hence, the final structure depends on the order in which the units are placed. They are placed from the bottom to the top along the (approximately) 60^0 line, after which the placement of the next line is initiated. When all units in the first layer have been placed, the second layer is placed on top in the same sequence. Fig. 2 shows the effect of the random angular orientation and the consequences of the rearrangement of the units.

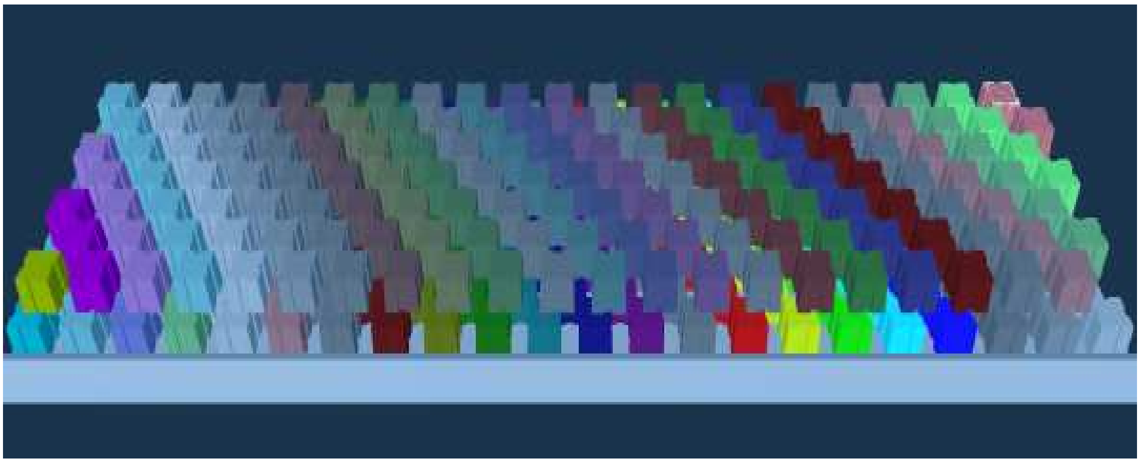

Figure 1: Idealized packing grid as described in the text

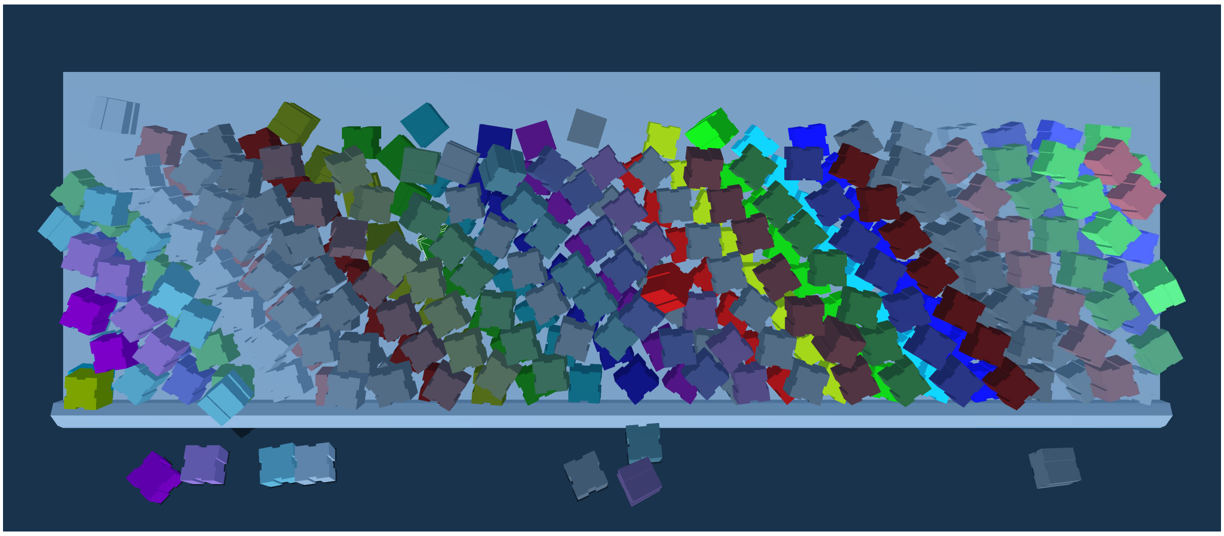

Figure 2: Grid after allowing random rotations of the Antifer cubes

2.2 Wave forces as modelled in PhysX

We have defined a particular force model for the impounding waves to determine the response of the breakwater structure to waves. The oscillating magnitude and direction of wave forces is mimicked by equating the forces to the local amplitude of a wave profile typical of shallow waves. The model is not meant to mimic waves accurately, since the purpose of the present study is just to come to a proper definition of stability in the context of harbour engineering. We use a simple shallow wave assumption which correlates the wave speed (celerity) to the square root of the depth d: $c^2 = g\, d$ [8]. Assuming the slope is characterized by an angle θ_0, and starts at the toe with $z = 0$, we get an effective wave number:

$$k_z(z) = \frac{\omega}{\sqrt{g}} \frac{2}{(d_0 - z \tan\theta_0)^{1/2}}, \tag{1}$$

where ω is the frequency of the wave. As stated we link the force exerted on the Antifer cube to the wave profile at the same point in the grid (not a very physical assumption, but useful to mimic the variation in the wave forces over the wave length of the wave) and only consider force components in the z-direction (inside the slope). We then arrive at the following force model:

$$\begin{aligned} \vec{F} &= \vec{i}_z F \sin\left[\omega t - k_z(z)(z - z_0)\right] \\ &= \vec{i}_z F \sin\left[\omega t + \frac{2\omega}{\sqrt{g}\tan\theta_0}\left(\sqrt{d_0 - z\tan\theta_0} - \sqrt{d_0}\right)\right], \end{aligned} \tag{2}$$

where we set the arbitrary phase parameter z_0 to $d_0/\tan\theta_0$, and we added the term $\sqrt{d_0}$ to the spatial part of the argument, to ensure that this phase is always zero for $z = 0$. The parameter d_0 is the height of the water at the sea shore (at $z = 0$). In our calculations we used a frequency of 1/2 Hz ($\omega = \pi$), and adjusted the parameter in front of the square root. To make the effect of this force more realistic, we only activate it when it comes into contact with a surface, and then only when it is pushes the surface rather than pulls it. Negative forces are only activated when they act from the back on an exposed surface. This procedure can be realized in PhysX by the use of ray-optics, as this allows us to determine when a ray is blocked by a solid surface. The combination of different elements in the modelling of the wave forces, illustrated the difficulty in representing the wave forces efficiently in PhysX. However, as stated before, for the present exercise we are just interested in a force model whose strength can be varied by a single parameter (the strength F in Eq.(2)) and whose damage can be analyzed in terms of this single parameter.

In future work we will try to improve the physical nature of this model by comparing it to finite element calculations which are carried out in a related project. However, in view of our current objectives, there is little incentive to perfect the wave model at present.

2.3 Definition of damage or displacement in our calculations

After placing the Antifer cubes according to a certain grid (the random features can be reproduced by recording the seed which was chosen up front), the PhysX program can be used to characterize the whole structure in detail via a so-called checkpoint file. All the positions and orientations of the Antifer cubes are specified in such a file. We can now apply the force model discussed in the previous section and record its effect on the structure. We record the "damage" by calculating the average displacement for the N elements:

$$D = \frac{1}{N}\sum_{i=1}^{N} |\vec{x}_i - \vec{x}'_i|. \tag{3}$$

Other measures are also possible. For example, we could look at the average displacement in one direction:

$$D_z = \frac{1}{N}\sum_{i=1}^{N} (x_{iz} - x'_{iz}), \tag{4}$$

or alternatively:

$$D'_z = \frac{1}{N}\sum_{i=1}^{N} |x_{iz} - x'_{iz}|\,. \tag{5}$$

Euclidean measures are also possible. The elements we are considering are limited to the middle half of the breakwater structure (excluding the outside quarter portions). This is to eliminate boundary effects, as the units on the side can easily fall off, which would lead to an enormous change in the displacement measure, thereby upsetting the analysis of the response to forces.

We have concentrated on the first measure Eq.(3), but the directional displacements could also be useful to analyze the nature of the damage. The relevant directions are: x along the shore, with positive x towards the left; z moving into the slope, with $z = 0$ the start of the slope, and y up vertically. The slope is characterized by $y = z \tan \theta_0$. The damage in the direction of the slope is of particular interest as, we can expect that D_z will be negative, because under the actions of forces the Antifer cubes are expected to slide down the slope. On the other hand, D_x should average out to zero (unless there is a clear right-left asymmetry in the packing). Hence, in this case it is more natural to use D'_x to characterize the movements of the units from their original x-position.

3 Stability definitions and tests as modelled in PhysX

The aim of our study is to develop a suitable definition of stability of the breakwater structure, or what is more or less equivalent: a definition of damage. The definitions of harbour engineers are usually based on experience and not very precise. We hope that our model calculations will allow a more precise and quantitative definition of these important parameters. The PhD thesis of van der Meer [9] discusses stability and damage from the point of view of a coastal engineer. He introduces a clearly defined damage level, S, by coupling the cross sectional eroded area, A, to the nominal diameter of the armour stones, D_{n50}: $S = A/D_{n50}^2$. For the “no damage” criterion of Hudson [10], S is taken generally to be between 1 and 3. The lower and upper damage levels, that is the onset of damage and failure (filter layer visible), were analyzed in his thesis. Generally the start of damage is characterized by $S = 2 - 3$. In the case of PhysX an area criterion is difficult to implement. A possible advantage of the area definition is that it may be easier to correlate to the damaged “look” of a structure.

The definition of damage by the US Army Corps of Engineers [11] is similar to the one used by van der Meer. However, an additional factor is introduced: $S = A/\Delta D_{n50}^2$, where Δ is the difference in specific gravity between the seawater and the armour stone. The empirical damage equation developed by Melby and Kobayashi [12] is

$$\bar{S}(t) = 0.25 N_s^5 \qquad N_s = \frac{H_s}{\Delta D_{n50}} \tag{6}$$

This equation has been simplified from [12], by assuming zero damage at the beginning of a single wave and considering damage at the end of the wave. The overbar denotes the predicted quantity. The significant wave height, H_s, equals the average height of the highest one-third of waves in a wave train and is measured trough to crest. For the single monochromatic pulse with height amplitude a we may take the trough to crest height to be $2a$.

In the building industry stability plays an important role. Therefore, it is also of interest to note the definition of stability in this field: the ability of a structure to maintain equilibrium and to resist displacement or overbalancing [13]. Practical tests for stability in the building field are: transverse flexure, horizontal load resistance, strength of connections of various kinds;

anchorage of structure to the base; response to simulated wind loading.

In other fields, such as road research, damage, or the extent thereof, is often correlated with performance. Hence, changes are only judged as damage if they lead to a considerable reduction of the performance of the structure considered.

In order to test stability in our model calculations we exert forces on the breakwater structures and record the response of the structure. We characterize this response in terms of the average displacement of the units, comparing the initial center-of-mess against the final one. Both the model of the force and the way we record the response were discussed in the previous section. We now present our results.

In Fig. 3 we display the deviation as a function of applied force. We have analysed the displacement for a particular packing, indicated by the label 150x103. The resulting values of D, as defined by Eq.(3), are shown as blue points. Since we expected a slow response to the increasing force for small forces, followed by a fairly abrupt increase when the forces reach a critical value (the "breaking point"), we fitted the points by two curves for the lower and upper section. Originally, we used lines to represent these two sections, however, this appeared unsatisfactory. We then introduced separate second order polynomials for the upper and lower section. The break point was found dynamically by minimizing the overall χ-squared.

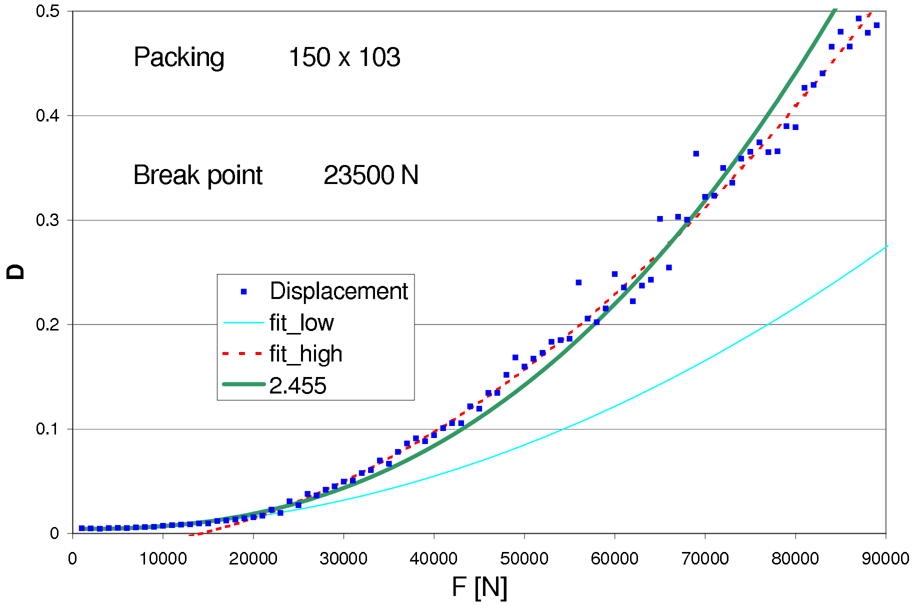

Figure 3: Deviation as a function of applied force

For the current case we find a break point at a force of 23500 (no units specified). However, the break is not very prominent, and in many other cases studied there is no break at all. However, what is interesting is that the power curve (fitted to find the optimal power) gives a power curve with exponent 2.45 with a near perfect description of the response (we have used a small constant offset for small forces, because of the idiosyncrasies of the game engine PhysX, which may initiate changes in the component structures even if the forces are physically insignificant). Therefore the response of breakwater structures to increasing forces does not really seem to be captured by breakpoints and sudden collapse of the structure, but more by a gradual deterioration of the structure, which accelerates according to the given power law. We should caution, however, that the discrete structures studied often give irregular results for different structures and that a general conclusion, like the one given above, is bound to fail for certain cases and only has an approximate value. Also, the fit indicated is a relative fit, in which the relative errors in the deviations (with respect tot he fit) are summed. If one minimizes the absolute χ-squared, which emphasize the fit of the top of the curve, then the optimization becomes less discriminative, and the power fit becomes less convincing.

The power behaviour witnessed here, which is close to an exponent of 2.5, is confirmed by the theoretical analysis given below. Estimates of the force exerted by a passing wave of amplitude a are given in the basic theory [14]:

$$F = \frac{1}{2}\rho g a^2 \tag{7}$$

This estimate is for deep water; for shallow water an additional factor is added,

$$F = \frac{1}{2}\rho g a^2 (1 + 2kh/\sin 2kh). \tag{8}$$

For present purposes we are interested in a rough estimate of dependence and will use the deep-water approximation. For the PhysX case, therefore, we may estimate

$$F = \frac{1}{8}\rho g H^2. \tag{9}$$

Substituting in Eq.(6) we obtain

$$\bar{S}(t) = 0.25\frac{H_s^5}{(\Delta D_{n50})^5} = 0.25\frac{8F^{5/2}}{(\sqrt{\rho g}\Delta D_{n50})^5}. \tag{10}$$

a relationship which leads us to expect a dependence of surface damage area on $F^{5/2}$. This agrees closely, with the PhysX result obtained before. One may object that the theoretical argument is based on damaged areas, rather than on average displacement. However, we believe that there is not a big difference in the two notions, as displacement will also lead to a shift of areas, and thus to a similar notion of damage. In fact, mathematically it is easier to characterize damage by individual displacement of elements, than by the inaccurate notion of eroded area. In the following we will analyze specific defects which develop when forces are exerted in the breakwater structures.

4 The appearance of arches and cracks

In the physical model halls, breakwaters which have been damaged by simulated storm action exhibit volumes extending over many typical armour unit lengths in which no armour units are in contact. As wave action progresses, units are dislodged from the upper part of the volume, cross the volume, and impact on the units below.

In PhysX simulations described above, a similar gap is sometimes observed, as we see in Fig. 4. Two typical structures are seen: one which extends horizontally for about 4 typical armour unit lengths, and one a crack, or region of no contact between neighbouring units, which extends in an approximately diagonal direction characteristic of the breakwater construction. In both cases, the gap may be confined to the top layer of the structure. We will refer to these cases as

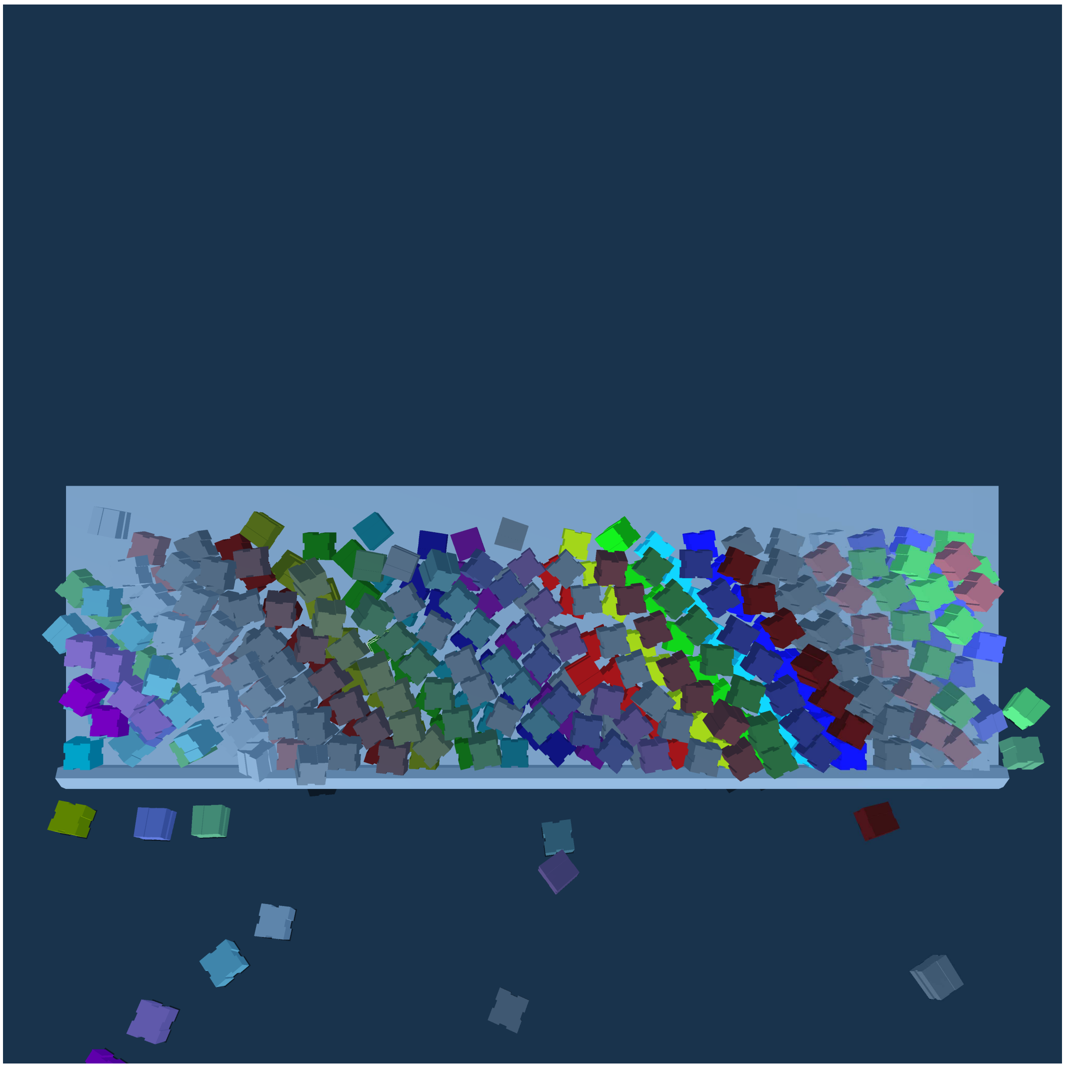

Figure 4: The development of cracks in breakwater structures

a horizontal gap and a diagonal gap for the purposes of discussion. We investigate the nature of these gaps with the intent of, in future, providing recommendations on increasing breakwater stability.

4.1 Breakwaters as granular media

In view of this behaviour it is tempting to describe the assembly of breakwater units as a granular medium. The inter-particle forces are contact forces, responsible for support and sometimes for interlocking, and friction. The interactions between particles in a physical granular medium are characterised by the material properties, such as the coefficient of (normal and tangential) restitution and the coefficients of friction between the units, and between the slope and the units. In quasi-static granular assemblies, many studies have been made of sand piles and rubble mounds. The flow of granular media under gravity presents a multitude of phenomena. One of these is the formation of local spontaneous density structures [15]. Density waves are responsible for clogging in hoppers and for the progressive decompaction observed in hoppers, and a subset includes arches and cracks. The distribution of static stress in the pile can be characterised by contact networks. It has been shown by Radja et al. [16] that contacts in granular media can be classified into strong networks, or force chains, which sustain the load on the system, and weak networks, with more contacts, and which in many systems support the force chains. The strong force chains are present along the principal directions of stress while the weak contacts are usually normal to the force chains.

An arch in a granular assembly is defined as a material layer capable of bearing the load arising from the material above [17]. Arches hold up the flow of material from above, and transmit the load to walls or other parts of the structure. The appearance of the armour units on the upper side of a horizontal gap suggests that an arch is distributing compressive stresses down units on either side to the toe, and units below the arch may slide away without disturbing the structure above. Stresses are also, of course, transmitted to the slope. When no wave perturbation is applied, the slope $\cot \alpha$ and static friction μ_s are such that isolated static units do not slide, and the role of the units in the arch is not that of supporting higher units in the assembly. No water line is modelled in this simulation. When the wave forces are applied down the slope, however, the arch assumes a support role. When wave forces are applied up the slope, we suggest that force chains could be identified which are distributing stresses up the slope, and that under these conditions the presence of a sea wall at the top of the slope provides the plane to which the compression is transmitted. An investigation of the time-dependant compressive stresses and force chains in these interlocking armour units is warranted. For this purpose the building of a FEMDEM model founded on the work of Latham et al. [1] is underway.

4.2 Diagonal gaps or cracks

The progressive decompaction of spherical particles falling from a suddenly opened pipe has been observed in physical and numerical experiments [15]. When the walls perturb the flow because of frictional stresses, cracks oriented diagonally with respect to gravity appear between zones of decompacting, falling particles, and stagnating, temporarily supported particles. When surfaces are strongly coupled by friction, the particles can be shown to be rotating in alternate directions in layers normal to the cracks. The cracks separate zones as the array of particles breaks into pieces. Similar cracks are observed as a container of spherical particles is vibrated from the bottom.

It may be hypothesised that the diagonal breakwater gaps are comparable to these cracks as the array of armour units disintegrates under the time-varying forces. Strong surface coupling of the particles is provided by both friction and the contact forces of interlocking shapes. Again, the phenomenon can be investigated by following force chains as described above.

4.3 Stability

Both of these phenomena may provide insight into the granular motion of armour units. It has been established that the appearance of arches and cracks depends on the packing history of the pile or hopper. The packing algorithms for breakwaters are well-established and documented, and classic algorithms have been used for our examples. In placing the units, a distribution of initial orientations has been assumed, introducing a stochastic nature to the packing, and providing the local variations which in which lie the origins of gaps and cracks.

Note that the forces to which the breakwater is subjected are both the downhill component of gravity and the oscillating wave perturbation. The latter is not applied uniformly to the assembly but has a variation across the units (through the ray casting model). However, when there is a component of the resultant force uphill, the uphill contact network is an important factor in breakwater stability. We are led to a requirement for contact networks that work in either direction, uphill or downhill, and an examination of the role of the upper sea wall.

Arches and cracks signal advanced damage in a breakwater. In considering how to improve breakwater stability, we are led to ask how to redistribute both the strong force chains and the supporting weak networks before the onset of these macroscopic damage indications. Using the tools of the following section, it may be possible to establish whether packing algorithms and parameters - such as the distribution of initial orientations - can affect the stability of the assembly.

5 Conclusion

We have demonstrated the power of PhysX in analyzing the consequences of exerting wave forces on breakwater structures. This study suggests that these consequences are not characterized by discrete break or failure points, but are rather characterized by a monotonically increasing effect of the forces, characterized by a power law with exponent between 2 and 2.5. Because of the discrete nature of the breakwater structures, different structures will each have their specific characteristics, and the indicated power law should thus only be a rough guide. Accepting this power law, one then must characterize the stability of a structure in terms of the coefficient in front, with a small coefficient indicating higher stability. The existing basic theory already gives a hint how to reduce this coefficient to a single remaining dimensionless parameter, by factorizing out various physical dimensionfull factors. We show that the study of cracks and arches in damaged breakwater structures may benefit from analyzing these structures as examples of granular media.

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