The Quality Factor of Aberrated Gaussian Laser Beams

29 September 2010

SAIP Conference (27 September - 1 October 2010)

CSIR Convention Centre, Pretoria

Presentation by Cosmas Mafusire

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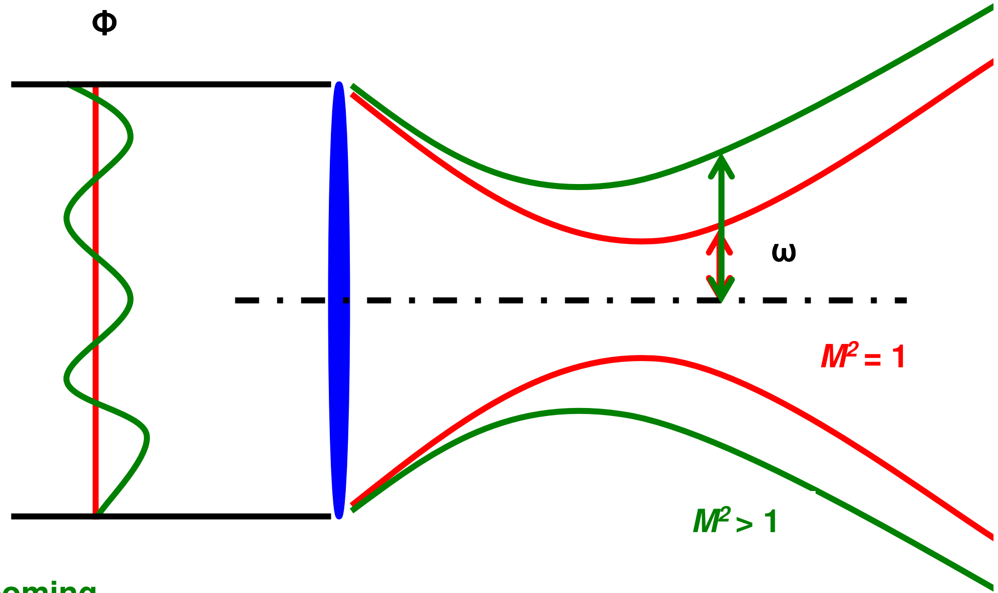
Aberrations and Beam Quality Factor



Incoming aberrated wavefront

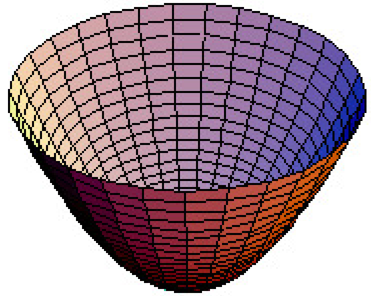
Zernike Polynomials

$$R_n^{|m|}(\rho) = \sum_{k=0}^{\frac{n-|m|}{2}} \frac{(-1)^k (n-k)!\rho^{n-2k}}{k!\left(\frac{n+|m|}{2}-k\right)!\left(\frac{n-|m|}{2}-k\right)!}$$

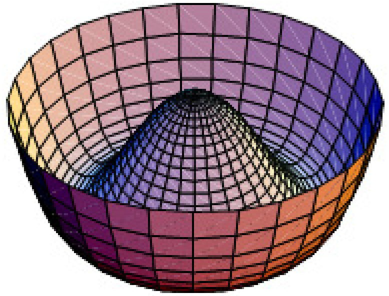
$$n \ge 0 \quad and \quad n-|m| \text{ is even}$$

$$U_n^{|m|}(\rho,\theta) = R_n^{|m|}(\rho)\cos|m|\theta, \quad m \ge 0$$
$$V_n^{|m|}(\rho,\theta) = R_n^{|m|}(\rho)\sin|m|\theta, \quad m < 0$$

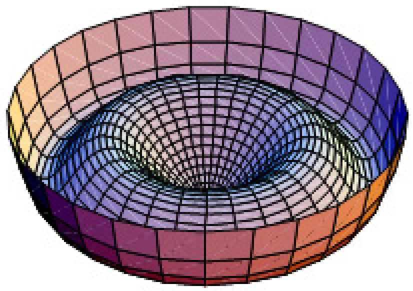
$$\phi(\rho,\theta) = 2\pi\sum_{n=0}^{\infty} A_n^0 R_n^0(\rho) +$$
$$2\pi\sum_{n=1}^{\infty}\sum_{m=1}^{n} R_n^{|m|}(\rho)\left[A_n^{|m|}\cos|m|\theta + B_n^{|m|}\sin|m|\theta\right]$$



Defocus



Primary Spherical Aberration



Secondary Spherical Aberration

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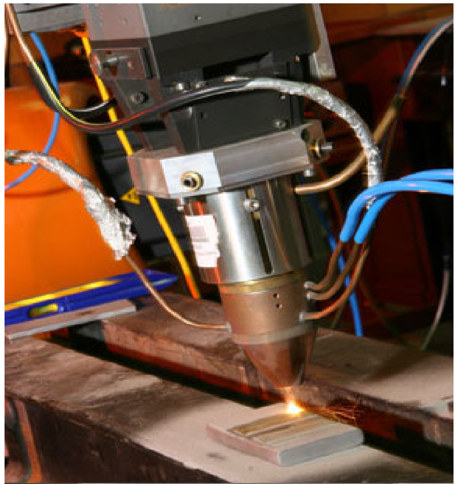
Zernike Primary Aberrations

n	m	Description and symbol	Polynomial
0	0	Piston, A_0^0	1
1	-1	y-Tilt, B_1^1	$\sqrt{2}\rho \sin\theta$
1	1	x-Tilt, A_1^1	$\sqrt{2}\rho \cos\theta$
2	-2	y-Astigmatism, B_2^2	$\sqrt{6}\rho^2 \sin 2\theta$
2	0	Defocus, A_2^0	$\sqrt{3}(2\rho^2 - 1)$
2	2	x-Astigmatism, A_2^2	$\sqrt{6}\rho^2 \cos 2\theta$
3	-3	y-Triangular Astigmatism, B_3^3	$\sqrt{8}\rho^3 \sin 3\theta$
3	-1	y-Primary Coma, B_3^1	$\sqrt{8}(3\rho^3 - 2\rho)\sin\theta$
3	1	x-Primary Coma, A_3^1	$\sqrt{8}(3\rho^3 - 2\rho)\cos\theta$
3	3	x-Triangular Astigmatism, A_3^3	$\sqrt{8}\rho^3 \cos 3\theta$
4	0	Spherical Aberration, A_4^0	$\sqrt{5}(6\rho^2 - 6\rho^2 + 1)$

Truncation - Intensity and Transmitted Power

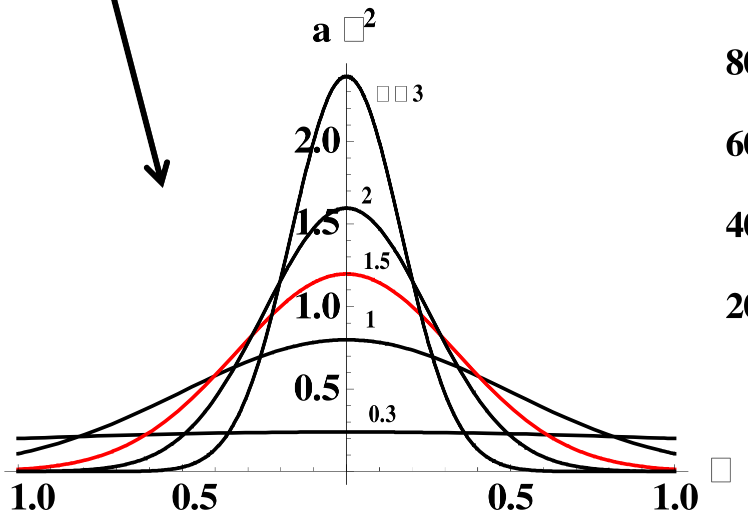


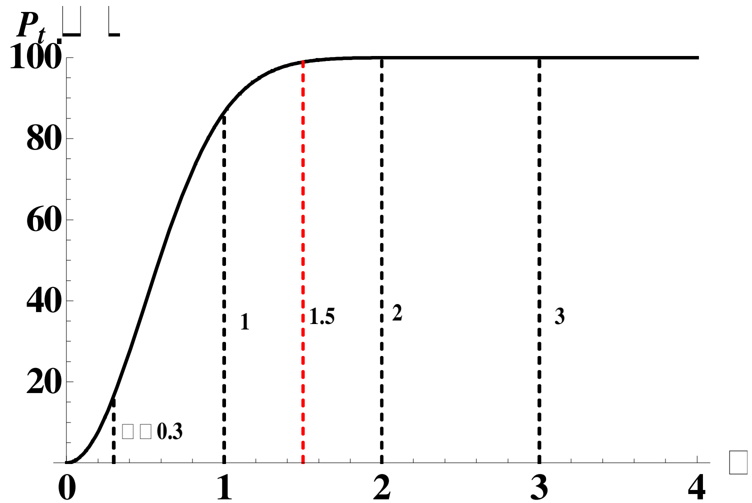
$$E(\rho,\theta)=\left(\frac{2}{\pi\omega^2}\right)^{\frac{1}{4}}\exp\left(-\frac{a^2\rho^2}{\omega^2}\right)\exp(i\phi(\rho,\theta))$$

$$\phi(\rho,\theta)=2\pi\sum_{n=0}^{4}A_n^0R_n^0(\rho)+2\pi\sum_{n=1}^{3}\sum_{m=1}^{n}R_n^{|m|}(\rho)\left[A_n^{|m|}\cos|m|\theta+B_n^{|m|}\sin|m|\theta\right]$$

$$\psi^2(\rho,\theta)=\left(\frac{2\gamma^2}{\pi a^2}\right)^{\frac{1}{2}}\exp\left(-2\gamma^2\rho^2\right)\quad P_T=\frac{a^2\int_0^{2\pi}\int_0^1\rho d\rho d\theta\psi^2}{a^2\int_0^{2\pi}\int_0^{\infty}\rho d\rho d\theta\psi^2}\rightarrow P_T=1-\exp(-2\gamma^2)$$

$\gamma=a/\omega$ Truncation Parameter





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Beam Quality Factor – General Hard Aperture

$$M^4 = \frac{\pi^2}{\lambda^2}\left(\omega^2\theta^2 - \left(\frac{\omega^2}{R}\right)^2\right)$$

where M^2 is the square root of M^4

B_2^2
A_3^1
B_3^1
A_3^3
B_3^3
A_4^0
γ

$$M_{xx}^4 = \frac{1}{\pi(e^{2\gamma^2}-1)\gamma^8}(24\pi^3(B_2^{2\,2}(e^{2\gamma^2}-1)(e^{2\gamma^2}-1-2\gamma^2)^2\gamma^4 + 3(B_3^1(e^{2\gamma^2}-1)(e^{2\gamma^2}-1-2\gamma^2)$$
$$(e^{2\gamma^2}-1-2(\gamma^4+\gamma^2))\gamma^2 + A_3^{1\,2}(e^{2\gamma^2}-1-2\gamma^2)(5e^{4\gamma^2}+2(\gamma^4+\gamma^2)-2e^{2\gamma^2}(9\gamma^4+\gamma^2+5)+5)\gamma^2$$
$$+2(B_3^1B_3^3+A_3^1A_3^3)(e^{2\gamma^2}-1)(e^{2\gamma^2}-1-2\gamma^2)(e^{2\gamma^2}-1-2(\gamma^4+\gamma^2))\gamma^2$$
$$+(B_3^{3\,2}+A_3^{3\,2})(e^{2\gamma^2}-1)^2(e^{2\gamma^2}-1-2\gamma^2)(e^{2\gamma^2}-1-2(\gamma^4+\gamma^2))\gamma^2 + 20A_4^{0\,2}(2\gamma^2(\gamma^2+2)-e^{4\gamma^2}$$
$$-2e^{2\gamma^2}(2\gamma^6-\gamma^4+2\gamma^2+1)+1)))) + (e^{2\gamma^2}-1)(e^{2\gamma^2}-1-2\gamma^2)(\pi(e^{2\gamma^2}-1-2\gamma^2)+32)\gamma^8)$$

$$M_{yy}^4 = \frac{1}{\pi(e^{2\gamma^2}-1)\gamma^8}(24\pi^3(B_2^{2\,2}(e^{2\gamma^2}-1)(e^{2\gamma^2}-1-2\gamma^2)^2\gamma^4 + 3(B_3^1(e^{2\gamma^2}-1-2\gamma^2)(5e^{4\gamma^2}+2(\gamma^4+\gamma^2$$
$$-2e^{2\gamma^2}(9\gamma^4+\gamma^2+5)+5)\gamma^2 - 2B_3^1B_3^3(e^{2\gamma^2}-1)(e^{2\gamma^2}-1-2\gamma^2)(e^{2\gamma^2}-1-2(\gamma^4+\gamma^2))\gamma^2$$
$$+(e^{2\gamma^2}-1)(e^{4\gamma^2}((B_3^{3\,2}+(A_3^1-A_3^3)^2)\gamma^2+20A_4^{0\,2})+\gamma^2(40A_4^{0\,2}(\gamma^2+2)+(B_3^{3\,2}+(A_3^1-A_3^3)^2)$$
$$(4\gamma^6+6\gamma^4+4\gamma^2+1))-2e^{2\gamma^2}((B_3^{3\,2}+(A_3^1-A_3^3)^2)(\gamma^3+\gamma)+20A_4^{0\,2}(2\gamma^6-\gamma^4+2\gamma^2+1))+20A_4^{0\,2})))$$
$$+(e^{2\gamma^2}-1)(e^{2\gamma^2}-1-2\gamma^2)(\pi(e^{2\gamma^2}-1-2\gamma^2)+32)\gamma^8)$$

Beam quality factor independent of tilt, defocus and x-astigmatism

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Beam Quality Factor – Special Cases

- Soft Aperture ($\gamma \geq 2$)

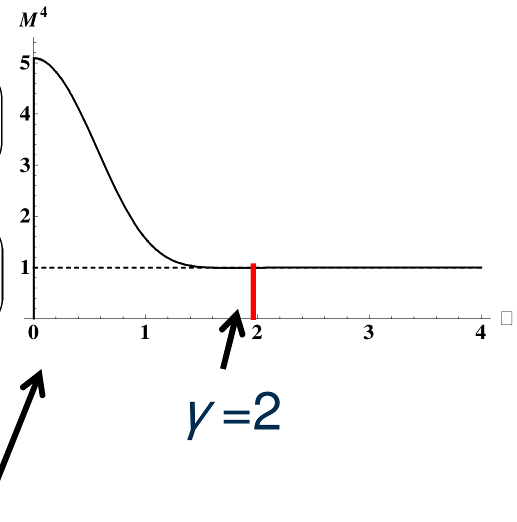
$$M_{xx}^4 = 1+24\pi^2\gamma^{-4}\left(B_2^{2\,2}\gamma^4 + 3\left(5A_3^{1\,2} + 2A_3^1A_3^3 + A_3^{3\,2} + \left(B_3^{1\,2} - B_3^{3\,2}\right)\right)\gamma^2 + 60A_4^{0\,2}\right)$$

$$M_{yy}^4 = 1+24\pi^2\gamma^{-4}\left(B_2^{2\,2}\gamma^4 + 3\left(5B_3^{1\,2} - 2B_3^1B_3^3 + B_3^{3\,2} + \left(A_3^{1\,2} + A_3^{3\,2}\right)\right)\gamma^2 + 60A_4^{0\,2}\right)$$

- Uniform Beam ($\gamma \to 0$)

$$M_{xx}^4 = \frac{16}{\pi} + 8\left(\pi^2\left(3B_2^{2\,2} + 20A_4^{0\,2} + 6\left(3A_3^{1\,2} + 2A_3^1A_3^3 + \left(B_3^1 + B_3^3\right)^2 + A_3^{3\,2}\right)\right)\right)$$

$$M_{yy}^4 = \frac{16}{\pi} + 8\left(\pi^2\left(3B_2^{2\,2} + 20A_4^{0\,2} + 6\left(3B_3^{1\,2} - 2B_3^1B_3^3 + \left(A_3^1 + A_3^3\right)^2 + B_3^{3\,2}\right)\right)\right)$$



- Aberration-less Gaussian Beam

$$M^4 = M_{xx}^4 = M_{yy}^4 = 1 + \frac{4}{\pi(e^{2\gamma^2}-1)^2}((\pi-16)\gamma^2 - 8 + \pi\gamma^4 + e^{2\gamma^2}(8-\pi\gamma^2))$$

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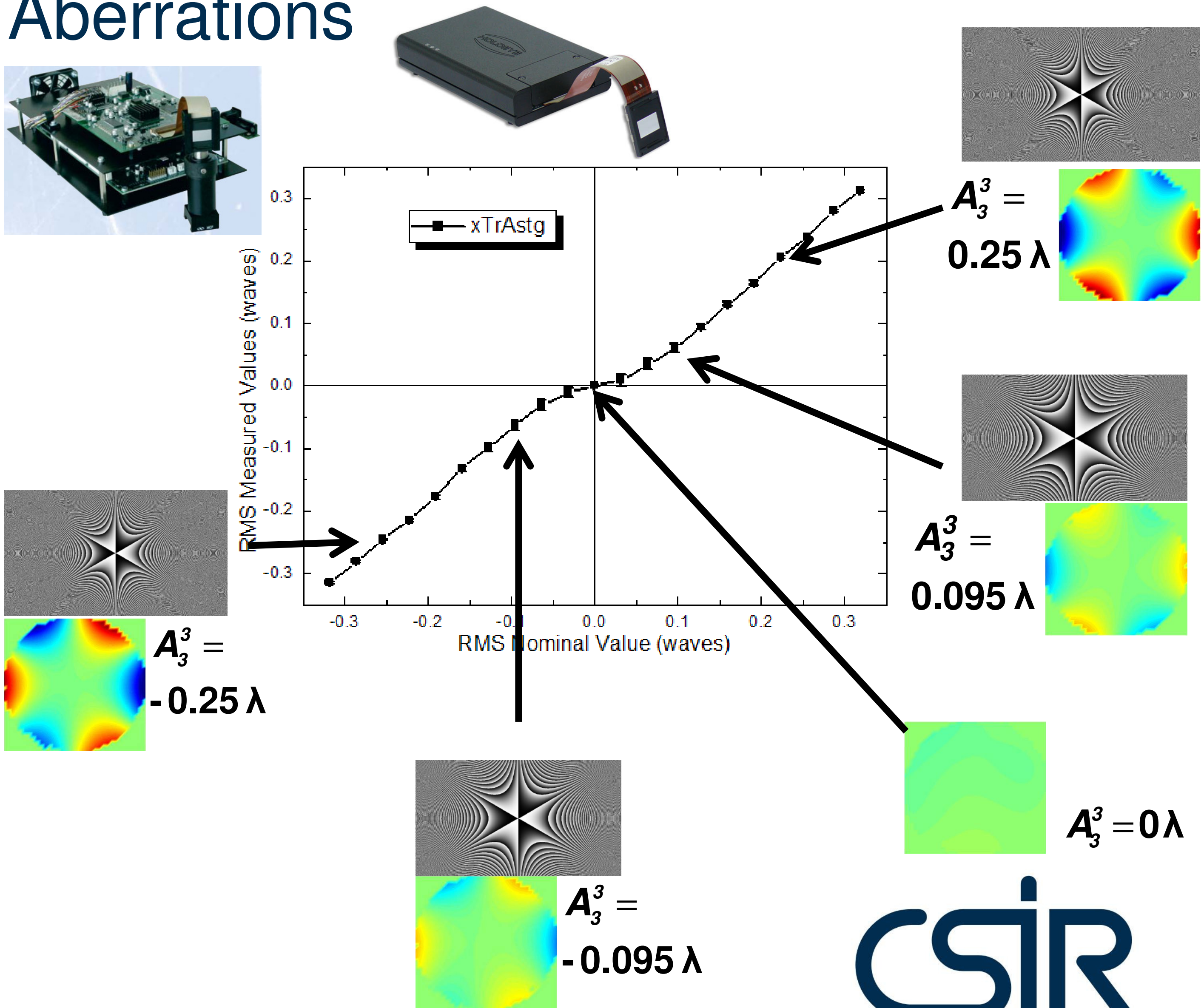
Spatial Light Modulator – Programming Aberrations



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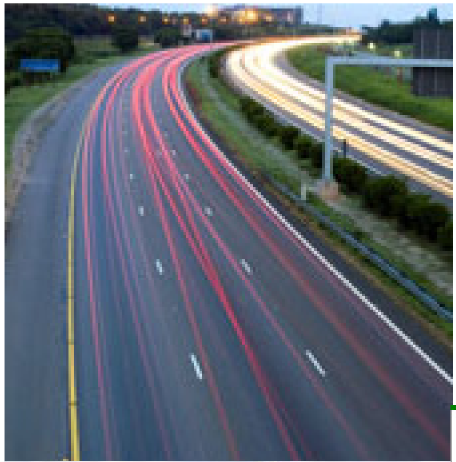
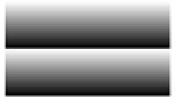
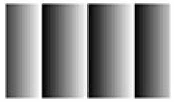
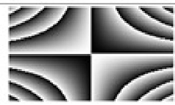
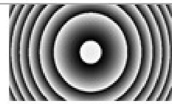
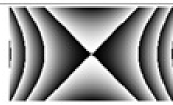
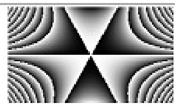
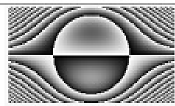
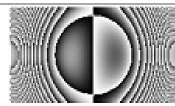
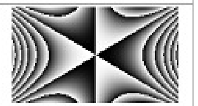
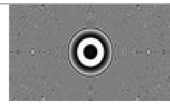


Spatial Light Modulators – Primary Aberration Phase Screens

n	*m*						
	-3	-2	-1	0	1	2	3
1							
2							
3							
4							

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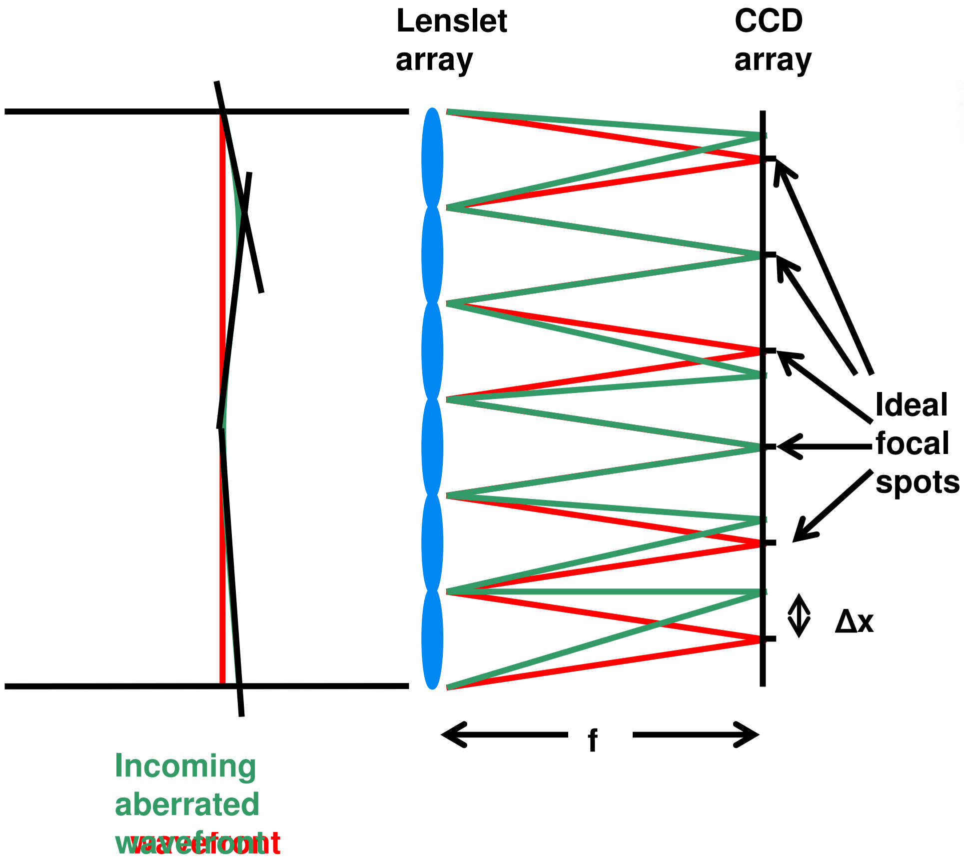
Shack-Hartmann Wavefront Sensor – Observing Phase Changes



$$\frac{d\phi(x,y)}{dx} = \frac{\Delta x}{f}$$

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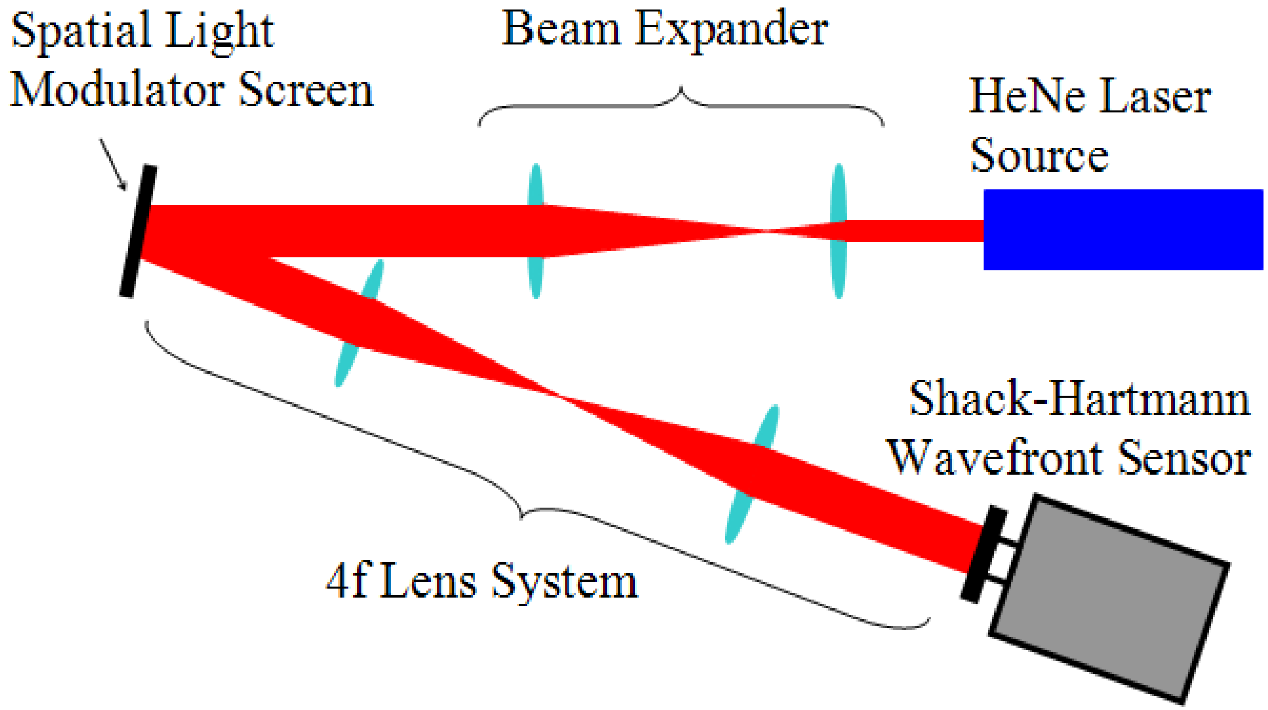
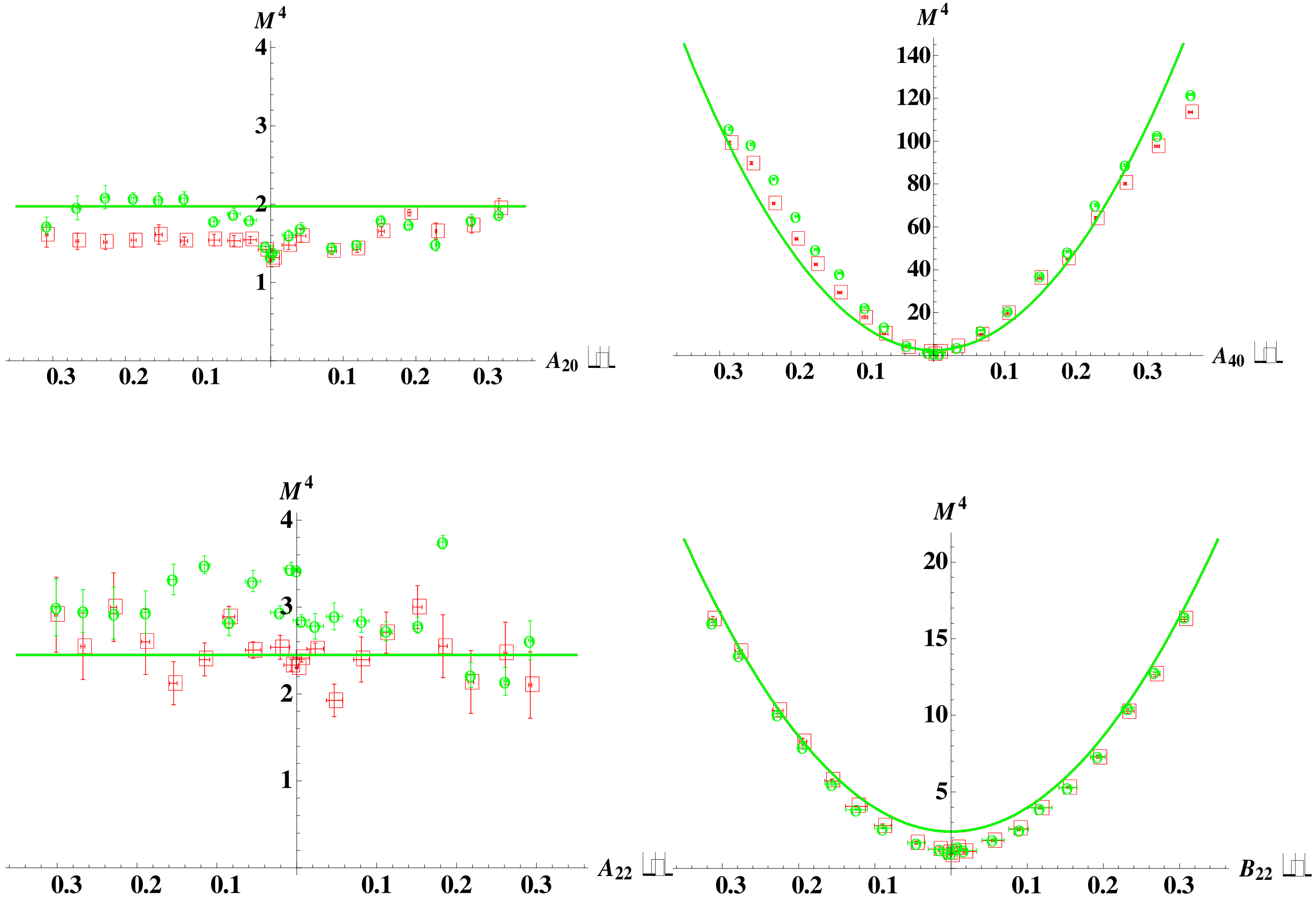
Experimental Investigation



a =1.44 mm ω = 1.89 mm γ = 0.763

Model and Experimental Results



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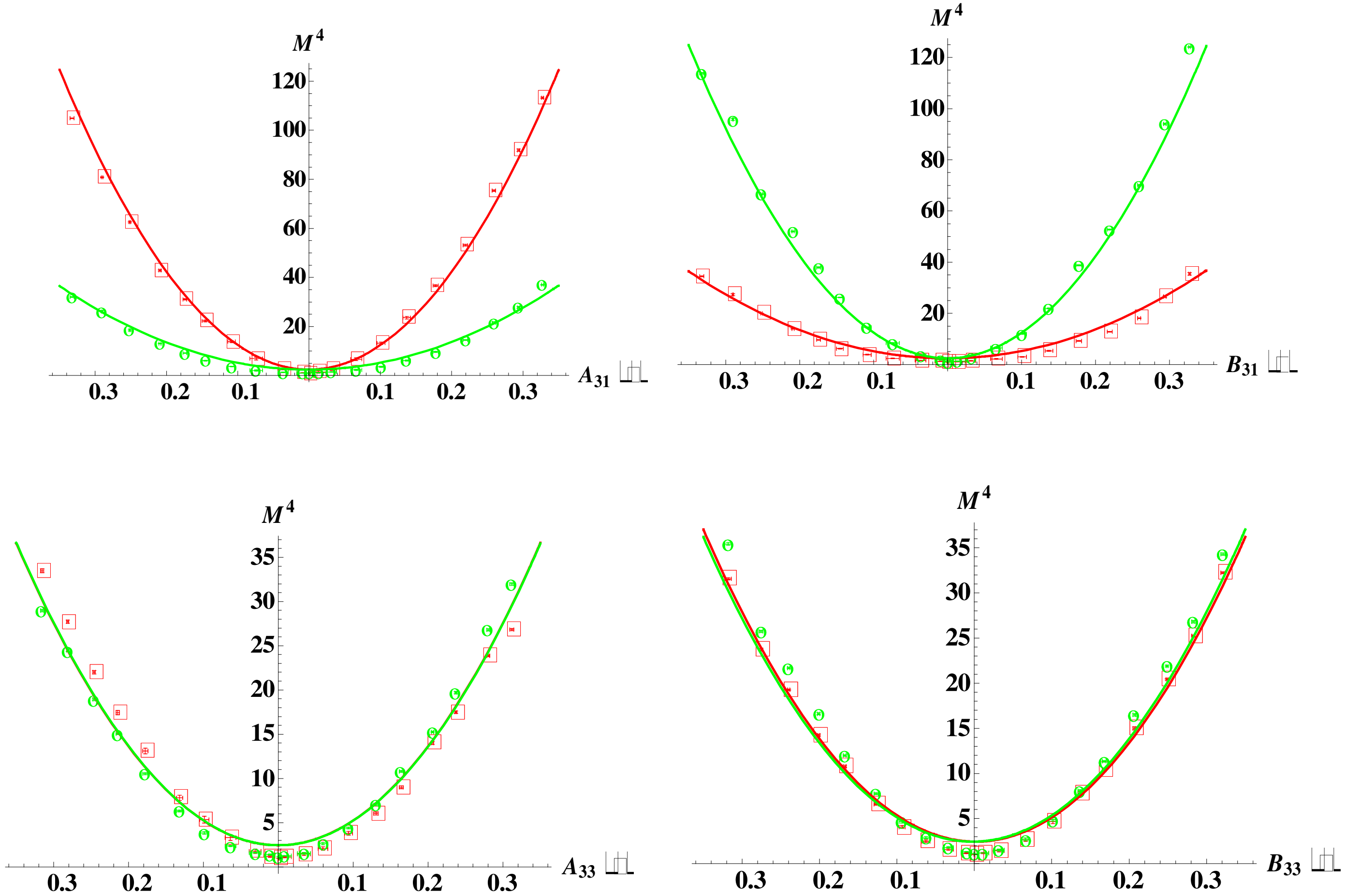
Model and Experimental Results



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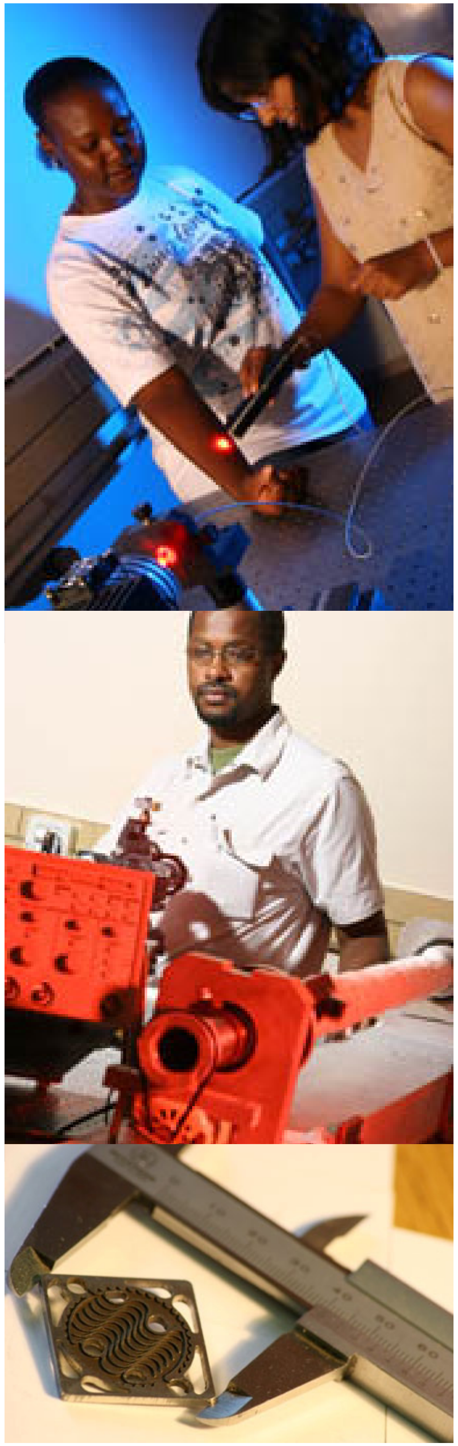
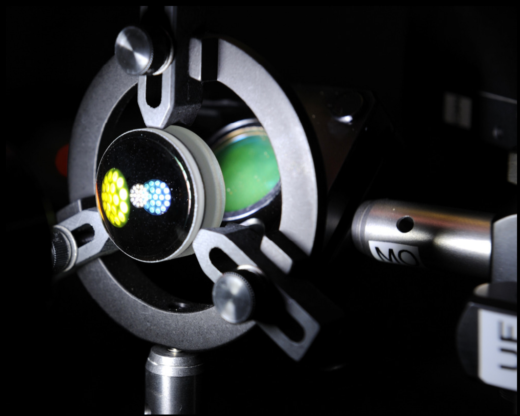


Conclusion

- Laser beam quality depends on
  - *y*-Astigmatism
  - *y*-Coma
  - *x*-Coma
  - *y*-Triangular astigmatism
  - *x*-Triangular astigmatism
  - Spherical aberration

- The aperture size can be adjusted such that one can extend the model to a small aperture (uniform beam) and a large one (soft aperture).

- A simple experimental investigation can be used to verify the model.



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