Amplitude damping of Laguerre-Gaussian modes

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Abstract: We present an amplitude damping channel for Laguerre-Gaussian modes. Our channel is tested experimentally for a Laguerre-Gaussian mode, having an azimuthal index $l = 1$, illustrating that it decays to a Gaussian mode in good agreement with the theoretical model for amplitude damping. Since we are able to characterize the action of such a channel on orbital angular momentum states, we propose using it to investigate the dynamics of entanglement.

OCIS codes: (050.0050) Diffraction and gratings; (050.4865) Optical vortices; (270.0270) Quantum optics.

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1. Introduction

Amplitude damping is a quantum operation which describes the energy dissipation from an excited system, such as an atom or a spin system, to an environment [1]. Under the action of an amplitude damping channel, the ground state, $|0\rangle$, is left invariant, but the excited state, $|1\rangle$, will either remain invariant or it will decay to the ground state, by losing a quantum of energy to the environment. The probability that the excited state remains unchanged or that it decays to the ground state is given by $1\text{-}p$ and p, respectively, illustrating that the amplitude of the

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excited state has been ‘damped’. This quantum operation is a fundamental tool in classifying the behaviour of many quantum systems that incur the loss of energy, from describing the evolution of an atom that spontaneously emits a photon to how the state of a photon evolves in an optical system due to scattering and attenuation. The amplitude damping channel is an elementary quantum operation which is used to model quantum noise in understanding the dynamics of open quantum systems [1].

For the case of a two-level system, the state change under the action of amplitude damping is defined by the following transformation

$$|\psi\rangle\langle\psi| = M_0|\psi\rangle\langle\psi|M_0^* + M_1|\psi\rangle\langle\psi|M_1^*, \qquad (1)$$

where M_0 and M_1 are the Krauss operators defined as

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \qquad (2)$$

and

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{p} \end{pmatrix}, \qquad (3)$$

The initial state of the two-level system is written as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad (4)$$

where $|0\rangle$ and $|1\rangle$ represent the ground state and excited state of the system, respectively and α and β denote complex amplitudes with $|\alpha|^2 + |\beta|^2 = 1$.

By making use of the Kraus operators, M_0 and M_1, and coupling the system appropriately to an environment with two orthogonal states $|K = 0\rangle_E$ and $|K = 1\rangle_E$, the unitary time evolution operator, U_{SE}, of the system, S, and the environment, E, produces the following transformations

$$|0\rangle_S|K=0\rangle_E \xrightarrow{U_{SE}} |0\rangle_S|K=0\rangle_E, \qquad (5)$$

$$|1\rangle_S|K=0\rangle_E \xrightarrow{U_{SE}} \sqrt{p}|0\rangle_S|K=1\rangle_E + \sqrt{1-p}|1\rangle_S|K=0\rangle_E, \qquad (6)$$

where K represents an environment observable, such as a path along which a photon propagates.

Equation (5) illustrates that the ground state of the system remains invariant and an excitation of the system is not absorbed by the environment and consequently the environment too remains in the ground state. In the transformation given in Eq. (6), an excitation of the system transforms into an excitation shared in a superposition between the system and the environment. The excitation of the system either decays to the ground state or it remains as an excitation with probabilities p and $1\text{-}p$, respectively, resulting in the environment either acquiring an excitation or remaining in the ground state with probabilities p and $1\text{-}p$, respectively.

Combining Eqs. (4), (5) and (6) and adapting them for the case of a two-level OAM system, the overall transformation for such a channel is described as

$$\alpha|l=0\rangle^A + \beta|l=1\rangle^A \rightarrow \alpha|l=0\rangle^A + \beta\left(\sqrt{1-p}|l=1\rangle^A + \sqrt{p}|l=0\rangle^B\right), \qquad (7)$$

$|l = 0\rangle$ and $|l = 1\rangle$ are state vectors representing a single photon in the LG_0 mode (ground state) and LG_1 mode (excited state), respectively, and the upper indices A and B refer to the two states of the environment, the ground and excited states, respectively. Later it will be illustrated that the two environments are represented as two optical paths. Discarding the environment leaves the system in a statistical mixture of being excited with probability *p* or de-excited with probability 1-*p*, which is the characteristic trait of amplitude damping.

It is now well known that the Laguerre-Gaussian (LG_{lp}) laser modes carry OAM of $l\hbar$ per photon [2,3]. Since the OAM is determined only by the azimuthal phase dependence, $\exp(il\varphi)$, of these modes, we neglect the radial index *p* for the rest of the paper. The generation of LG_l beams (a variety of which are depicted in Fig. 1(a) - 1(d)) has advanced from cylindrical lens mode converters [2] to spiral phase plates (SPP) [4] and the most frequently used 'fork' holograms [5]. Recently these 'fork' holograms have been realized with the use of spatial light modulators (SLM) [6]. Since the discovery of light beams carrying OAM many new research areas have emerged in the field of optical angular momentum, from transferring OAM to matter in optical tweezers [7] to investigating the conservation and entanglement of OAM in parametric down conversion [8]. In investigating the dynamics of entangled OAM states one requires components that manipulate and measure OAM states. The 'fork' holograms that can be used to generate OAM in the various diffraction orders [5] were also used by Mair *et al* to sort and infer OAM states [8].

But how to implement amplitude damping for the OAM states of light? Recently it has been shown that LG_l beams of odd and even orders of *l* could be sorted in a Mach-Zehnder interferometer incorporating Dove prisms in each arm [9]. In this paper we show that by adapting this device it may be used for a new application: an amplitude damping channel for the azimuthal modes of LG_l beams.

Due to the fact that photons with OAM represent multiple-level systems, storing and processing information in OAM states of photons promise high storage and parallel computation capacity per photon. However, many tasks in quantum information processing depend upon entanglement, resulting in the need to investigate the evolution of entanglement due to interactions with an environment [10]. For example, examining how the entanglement of OAM states decoheres through a turbulent medium has already been pursued [11], but there is value in outlining how to execute "text-book" quantum channels for the OAM states of light, in part due to the ability to analytically calculate the expected concurrence of the system after the channel. Recently a controlled-not gate [12] has been realized through the use of suitable optical elements, and here we show for the first time how an amplitude damping channel may be implement, using known optical systems.

2. Concept of the channel

The diagrammatic setup for the amplitude damping channel for LG_l modes, which is depicted in Fig. 1(e), is based on a Mach-Zehnder interferometer with a Dove prism in each arm. Such an optical system has already been shown to be useful for the sorting of modes [9] as well as determining the intrinsic versus extrinsic nature of OAM in arbitrary beams [13]. Here we outline how this versatile device may also be used for a new application (amplitude damping) as yet unreported in the literature. For the benefit of the reader, we outline the principle of the device and highlight its use as an amplitude damping channel.

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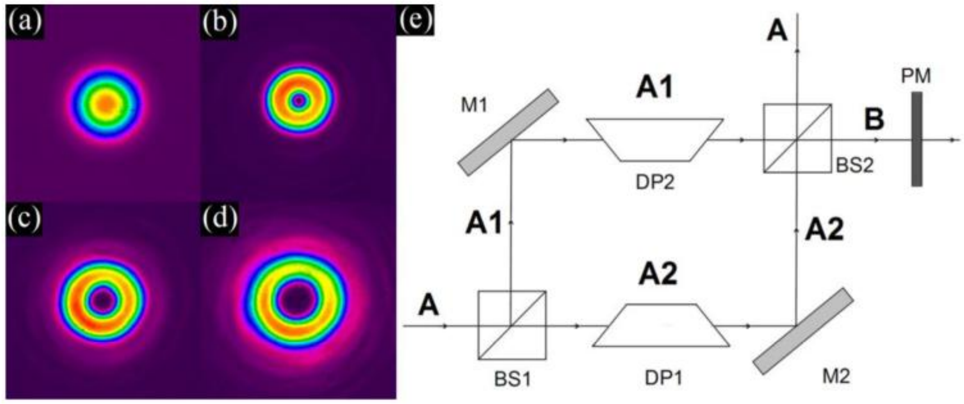

Fig. 1. (a) – (d) Intensity profiles of various LG_l modes for (a) $l = 0$, (b) $l = 1$, (c) $l = 2$, and (d) $l = 3$. (e) Schematic of the amplitude damping channel (BS: beam-splitter, M: mirror, DP: Dove prism, PM: phase mask).

A Mach-Zehnder interferometer can be aligned appropriately so that constructive interference will occur in either output paths *A* or *B*, where *A* and *B* represent the two states of the environment, the ground and excited states, respectively. By placing a Dove prism, which flips the transverse cross section of a transmitted beam [14], in each arm of the interferometer, a relative phase shift between the two arms is introduced. More specifically, this relative phase shift, $\Delta\phi$, is proportional to both the helicity, l, of the incoming LG_l beam and the relative angle, θ, between the two Dove prisms: $\Delta\phi = 2l\theta$. In varying the relative angle, θ, between the two Dove prisms, the LG_l beam will either exit solely in path *A* or path *B* or in a weighted superposition of both paths *A* and *B*. One is able to control how much of the initial LG_l mode will exit into the two environments, *A* (the ground state of the environment) and *B* (the excited state of the environment). A phase mask, which decreases the azimuthal mode index by 1, is placed in path *B* resulting in the 'excited' LG_l mode decaying to an 'unexcited' LG_{l-1} mode, thus performing the action of amplitude damping.

In following the LG_l mode through arms *A*1 and *A*2 of the interferometer to output path *A*, the additional phase picked up by the beam from each of the components is given by

$$\varphi_{A1\to A} = \underset{BS1}{\pi} + \underset{M1}{\pi} + \underset{DP2}{2l\theta} + \underset{BS2}{\frac{2\pi}{\lambda}t} + \frac{2\pi}{\lambda}d_1 \tag{8}$$

and

$$\varphi_{A2\to A} = \underset{BS1}{\frac{2\pi}{\lambda}t} + \underset{M2}{\pi} + \underset{BS2}{\pi} + \frac{2\pi}{\lambda}d_2, \tag{9}$$

respectively. $d1$ and $d2$ are the path lengths of the two arms, *A*1 and *A*2, respectively and t is the optical path length of the beam-splitter. The phase difference in output path *A* is consequently given by

$$\Delta\varphi_A = 2l\theta, \tag{10}$$

since the interferometer is constructed such that the path lengths of the two arms are equal.

Similarly, it can be shown that the phase difference in output path *B* is

$$\Delta\varphi_B = \pi + 2l\theta, \tag{11}$$

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where the additional phase shift of π is due to the reflection in the first beam-splitter. The amplitude of the field for a LG_l mode emerging after the second beam-splitter in path *A* and path *B* is described as

$$U^A = \frac{U_0}{2}\left(e^{i(l\phi+2l\theta)} + e^{il\phi}\right) = U_0 e^{il\phi} e^{il\theta} \cos l\theta \tag{12}$$

and

$$U^B = \frac{U_0}{2}\left(e^{i(l\phi+2l\theta)} - e^{il\phi}\right) = U_0 e^{il\phi} e^{il\theta} i \sin l\theta, \tag{13}$$

respectively. *U* denotes the amplitude of the field and the negative sign in Eq. (13) for the field emerging in path *B* from arm *A*2 is due to an additional phase shift of π, evident in Eq. (11).

In the case of propagating a LG_1 mode through the interferometer there is a non-vanishing relative phase difference ($\Delta\phi = 2\theta$) between the two arms of the interferometer, leading to partially constructive and destructive interference of the LG_1 mode in both paths *A* and *B*

$$\left|l=1\right\rangle^A \to \cos\theta\left|l=1\right\rangle^A + \sin\theta\left|l=1\right\rangle^B, \tag{14}$$

As we are only interested in the intensity of the field, as this gives the probability of the single photon state, we have neglected the phase components and will do so for the rest of the paper. The intensity of the LG_1 mode exiting in the two paths, *A* and *B*, is proportional to $\cos^2\theta$ and $\sin^2\theta$, respectively and consequently the probabilities of the LG_1 mode existing in the two paths follow the same trend. The incoming LG_1 mode exits in a superposition of paths *A* and *B* with probabilities $\cos^2\theta$ and $\sin^2\theta$, respectively and a phase mask, in path *B*, decreases the azimuthal mode index, *l*, by 1, converting the 'exited' LG_1 mode to an 'unexcited' Gaussian mode

$$\left|l=1\right\rangle^A \to \cos\theta\left|l=1\right\rangle^A + \sin\theta\left|l=0\right\rangle^B. \tag{15}$$

When a LG_0 mode enters the device there is no azimuthal phase dependence in such a mode, the field is unaffected by the Dove prisms and no phase difference occurs between the two arms; the result is that the Gaussian mode exits in path *A*. The transformation of the Gaussian mode through the interferometer is denoted as

$$\left|l=0\right\rangle^A \to \left|l=0\right\rangle^A. \tag{16}$$

Combining Eqs. (15) and (16), the general equation for the amplitude damping of OAM, given in Eq. (7), is obtained, where $\sqrt{1-p} = \cos\theta$ and $\sqrt{p} = \sin\theta$

$$\alpha\left|l=0\right\rangle^A + \beta\left|l=1\right\rangle^A \to \alpha\left|l=0\right\rangle^A + \beta\left(\cos\theta\left|l=1\right\rangle^A + \sin\theta\left|l=0\right\rangle^B\right). \tag{17}$$

From the definition of amplitude damping which states that the ground state will remain invariant while the excited state decays to the ground state with a probability *p*, it is sufficient to experimentally investigate each case individually in order to verify the action of our amplitude damping channel.

3. Experimental methodology and results

A HeNe laser (λ = 632.8 nm) was directed onto a phase-only SLM (HoloEye PLUTO VIS SLM with 1920 × 1080 pixels of pitch 8 μm and calibrated for a 2π phase shift at λ = 632.8 nm). The mode of the field was prepared before it entered the channel by programming an appropriate phase pattern. The two exiting paths of the interferometer were monitored for various angles, θ, between the two Dove prisms using a CCD camera (Spiricon LW130).

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To confirm the correct operation of the interferometer, we first repeated the sorting experiment of Ref [9]. When the relative angle between the Dove prisms was set to $\theta = \pi/2$, the interferometer sorted the incoming field into even LG_l modes (path *A*) and odd LG_l modes (path *B*). We can confirm this result for incoming LG_l modes having indices $l = 0$ to 4, as shown in Fig. 2(a). In the special case that $l = 0$ (Gaussian mode) the interferometer only produces an output in path *A* (Fig. 2(b)). This experiment is instructive when considering the errors in the system due to imperfect alignment and environmental fluctuations. The variance in the data set of Fig. 2(b) for path *A* (where we should expect 100% transmission) and for path *B* (where we should expect no transmission) is given as 0.88 ± 0.06 and 0.15 ± 0.08, respectively. In performing the measurements depicted in Fig. 3, for each occurrence for which the Dove prism was rotated, constructive and destructive interference was first achieved in paths *A* and *B*, respectively, for the case of an incoming Gaussian beam. This ensured that the interferometer was correctly aligned before the LG_1 mode entered the channel.

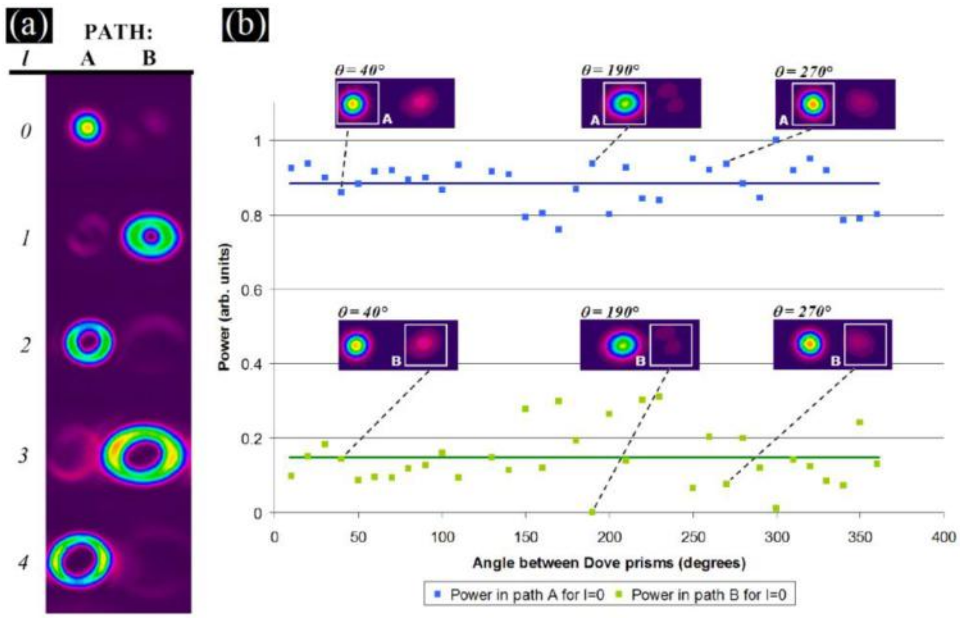

Fig. 2. (a) The interferometer 'sorting' various LG_l modes. The even LG_l modes ($l = 0$, 2 and 4) exit in path *A* and the odd LG_l modes exit in path *B* as expected [9]. (b) Plot of the power of the beam in path *A* (light blue points) and path *B* (light green points) for various values of θ. The dark blue line (dark green line) is the mean of all the measured powers in path *A* (path *B*).

To test the amplitude damping channel concept, the incoming mode was set to LG_1 and the angle between the two Dove prisms varied from 0° through to 360°. The results are shown graphically in Fig. 3. In Fig. 3(a) it is evident that when the relative angle between the two Dove prisms is an even multiple of 90°, maximum transmission of the LG_1 mode occurs, which is in agreement with the 'LG_l sorting' nature of this device. At an angle of an even multiple of 90°, the reverse occurs in path *B* (shown in Fig. 3(b)), where minimum transmission of the LG_1 mode occurs. When the relative angle between the two Dove prisms is an odd multiple of 90°, minimum transmission of the LG_1 mode occurs in path *A* and maximum transmission in path *B*. For angles varying between 0° and 90° (and multiples of these angles) the LG_1 mode exits in both paths *A* and *B*. As the angle increases from 0° to 90° the transmission of the LG_1 mode in path *A* decreases, but consequently increases in path *B*. In our theoretical model, the probability that the LG_1 mode exits in paths *A* and *B* is given by $\cos^2\theta$ and $\sin^2\theta$, respectively (depicted in Eq. (17)) and it is evident from Figs. 3(a) and 3(b) that our measured data are in very good agreement with the predicted model. The errors given in the measurements in Fig. 3 are the standard deviations of the measurements in Fig. 2(b), for paths *A* (0.06) and *B* (0.08).

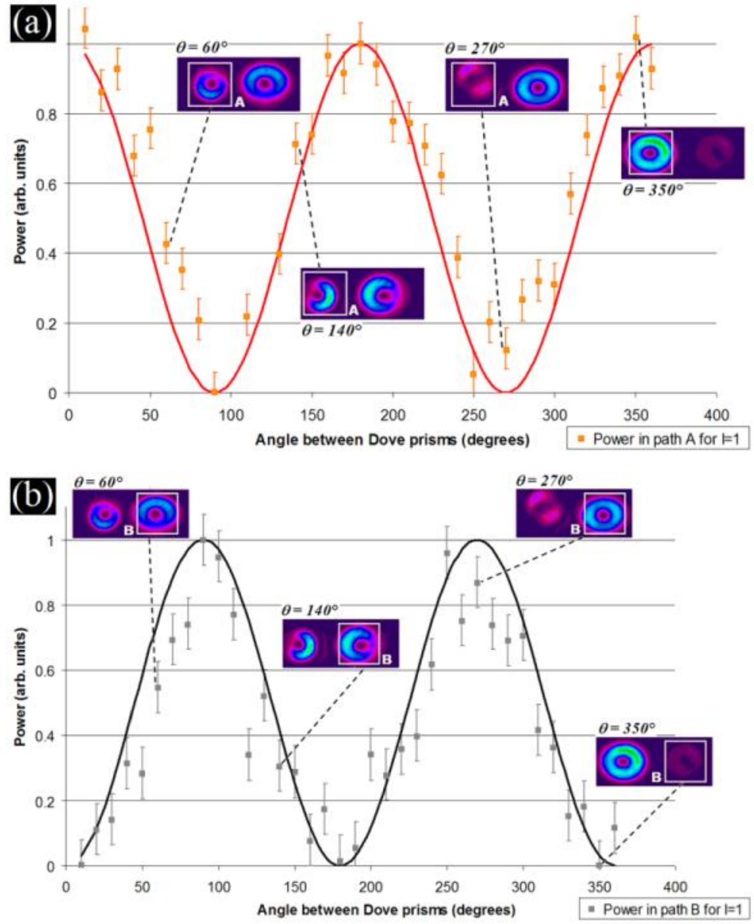

Fig. 3. Plot of the power of the beam in path *A* ((a); orange points) and path *B* ((b); grey points) for various values of θ. The measured results follow the theoretical curves: $P_A \sim \cos^2\theta$, ((a); red curve) and $P_B \sim \sin^2\theta$ ((b), black curve).

4. Conclusion

We have outlined how the concept of "amplitude damping" in quantum systems may be implemented for the OAM states of light using a standard optical system comprising an interferometer with Dove prisms in both arms, and have verified this experimentally using the LG_0 and LG_1 laser modes. We have shown excellent agreement between theory and experiment, and believe this is the first time such a concept has been outlined in the literature. With this idea, one is able to mimic the well known quantum operation where an excitation (or in this case OAM) is lost to the environment, a key testing bed for the interaction of entangled states with an environment. The advantage of this particular concept – amplitude damping – is that the entanglement decay can be predicted analytically, while the concurrence of the entanglement may easily be measured both before and after the channel with standard state tomography techniques, thus allowing for a quantitative comparison of theory and experiment in the decoherence of entanglement due to interactions with a noisy environment.

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