DEVELOPMENT OF A CONTROL MODEL FOR A FOUR WHEEL MECANUM VEHICLE

M. de Villiers[1], Prof. G. Bright[2]
[1] Council for Scientific and Industrial Research
Pretoria, South Africa
e-mail1: mfdevilliers@csir.co.za

[2] University of KwaZulu Natal
Durban, South Africa
e-mail2: brightg@ukzn.co.za

ABSTRACT

This paper presents the development of a control model for Mecanum wheels. Available control models used in robotics are based on a simplification which defines the contact point of the wheel on the ground as the point in the centre of the wheel, which does not vary. This limits the smoothness of motion of a vehicle employing these wheels and impacts the efficiency of locomotion of vehicles using Mecanum wheels. The control model proposed accounts for the fact that the contact point in fact changes position down the axle of the wheel as the angle roller moves on the ground. Taking this into consideration makes control of the wheels more predictable and accurate.

Keywords: Mecanum wheel, Kinematics and Dynamics Modelling

1. INTRODUCTION

Modeling of a Mecanum wheel vehicle for the purposes of robotic control has been investigated in other works [1], [2], [3]. These works are based on the simplification that defines the contact point of the wheel with the ground to be positioned on the circumference of the wheel, but in the centre of the wheel's width.

When the wheel in motion is examined closely it is observed that the contact point moves from one side of the wheel to the other as the contact point moves down the angled roller. This fact is not mentioned at all in [2], and while it is mentioned in [1], it is not added to the mathematical analysis of the problem. A more recent paper [4] pointed this out, but the analysis of the model is different to the one proposed in this paper.

2. MODELLING THE PLATFORM

A Mecanum wheel is a wheel with a number of rollers arranged around a hub at a predefined angle θ, usually ±45°, to the rotational plane of the wheel as can be seen in Fig.1.

2.1 Wheel Forces

When the wheel rotates there is a force applied down the roller axis, generated by the torque applied by the motor shaft, this force acts at the same angle θ to the wheel plane. The rollers rotate freely about their respective axes as they make contact with the ground.

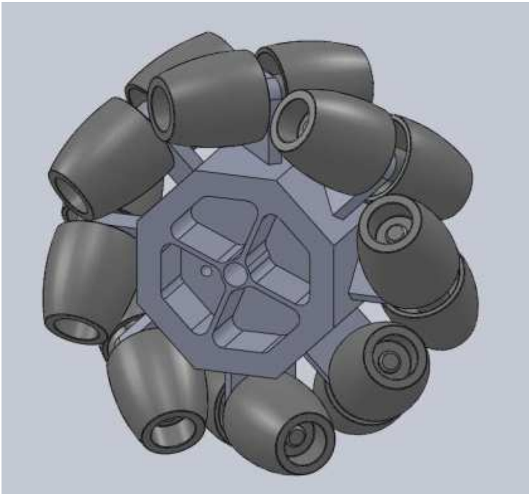

Figure 1: Mecanum Wheel

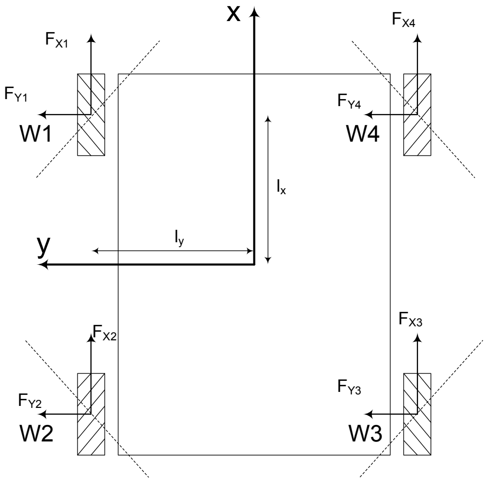

Figure 2: Forces acting on a Mecanum wheel vehicle

In Fig.2 a local co-ordinate system is defined with two orthogonal axes, *x* and *y*. Positive rotation is defined as anticlockwise around the origin. The wheels are made and attached such that the forces produced by the wheels act in the directions of the dotted lines in Fig. 2. These wheel forces can then be split into *x* and *y* components and summed to get the following equations (1)(2) defining the total force applied to the vehicle, the subscript T is applied to show that these are total forces:

$$F_{Tx}\boldsymbol{i} = \sum_{w=1}^{4} F_{xw}\boldsymbol{i} \tag{1}$$

$$F_{Ty}\boldsymbol{j} = \sum_{w=1}^{4} F_{yw}\boldsymbol{j} \tag{2}$$

Where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the *x* and *y* directions respectively and *w* represents the wheel number. If there is pure translation, i.e. no rotational motion, then the direction of motion, which makes an angle α with the *x*-axis can be shown to be,

$$\alpha = arctan\frac{F_{Ty}}{F_{Tx}}$$

If there is rotational motion then moments are taken around the vehicle centre at (0,0), which results in an equation for torque, τ, which will result in rotation of the vehicle. The resulting torque can be expressed with the following equation,

$$\tau = (-F_{x1} - F_{x2} + F_{x3} + F_{x4})l_y + \left((F_{y1} - F_{y2} - F_{y3} + F_{y4})\right) l_x$$

Where l_x and l_y are half the wheel base and track respectively and are the radii at which the forces are applied by the wheels.

For the dynamic case Newton's Second Law allows the following equations to be applied:

$$m\ddot{x} = F_x - c_x\dot{x}$$

$$m\ddot{y} = F_y - c_y\dot{y}$$

$$I_o\ddot{\phi} = \tau - c_z\dot{\phi}$$

Where $c_{x,y,z}$ are the viscous damping factors for motion in the three dimensions, m is the mass of the vehicle and $I_o$is the moment of inertia of the vehicle. Because this is not a spring system, there is no displacement term and in the steady state case all accelerations are zero:

$$\dot{\hat{x}} = \frac{F_x}{c_x} = \frac{F_{x1} + F_{x2} + F_{x3} + F_{x4}}{c_x}$$

$$\dot{\hat{y}} = \frac{F_y}{c_y} = \frac{F_{y1} + F_{y2} + F_{y3} + F_{y4}}{c_y}$$

$$\dot{\hat{\phi}} = \frac{\tau}{c_z} = \frac{(-F_{x1} - F_{x2} + F_{x3} + F_{x4})l_y + (F_{y1} - F_{y2} - F_{y3} + F_{y4})l_x}{c_z}$$

In the steady state case the velocities are shown with a ˆ to differentiate them from the case when the vehicle is accelerating.
This implies that in the steady state the vehicle velocity and, by implication, the wheel velocities, are proportional to the forces applied by each wheel:

$$V \propto \sum_{w=1}^{4} F_w \qquad (3)$$

and so, because this is a rigid body, for translational motion

$$V = V_w \qquad w = 1 \rightarrow 4 \qquad (4)$$

2.2 Wheel Velocity

Consider Fig. 3, the dotted lines represent the path taken by the contact point of the wheel with the ground when the vehicle is moving straight forward, note that this contact point moves back and forth along the wheel axis. The horizontal dotted lines are discontinuities where the contact point transfers from one roller to the next, however for the purposes of calculation the vector origin is put halfway along this line. The diagonal dotted line is in the same direction as the roller axis.

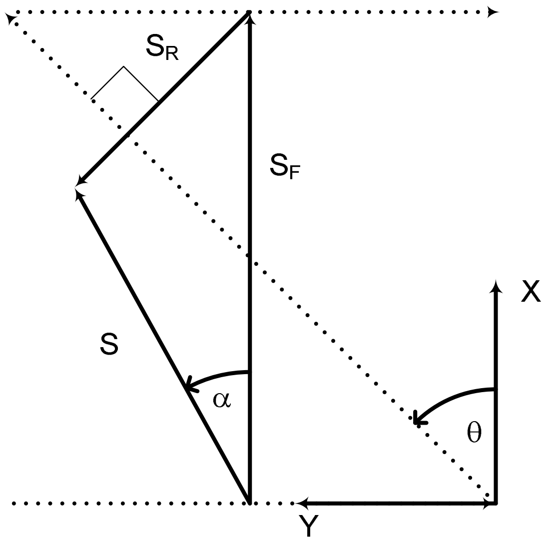

Figure 3. Wheel displacement vectors

The vertical solid line represents the displacement vector due only to wheel rotation S_F, in the positive *x*-direction. The solid line S_R represents the displacement due to rolling, in the direction orthogonal to the roller axis. *S* represents the resultant displacement of the wheel.

The angle α is the direction of the resultant displacement vector, and is measured from the *x*-axis, θ is the angle that the roller makes with the *x*-axis. The above displacement vectors translate to the velocity vectors when differentiated with respect to time, resulting in Fig. 4.

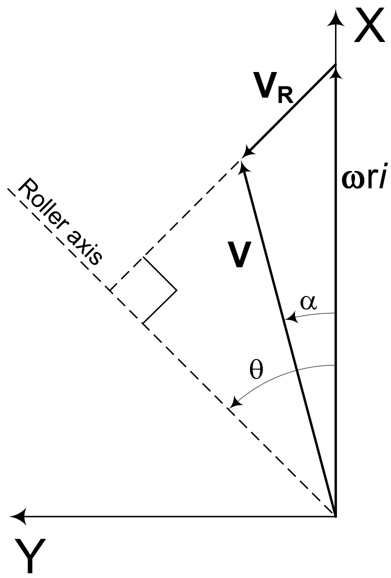

Figure 4. Wheel velocity vectors

In Fig. 4, ω is the wheel rotational velocity, positive rotation results in a positive velocity vector $\omega r\boldsymbol{i}$ along the $\boldsymbol{X}$ axis, r is the wheel radius and $\boldsymbol{i}$ is a unit vector in the $\boldsymbol{X}$ direction. Consider that the components along the roller axis of the $\omega r\boldsymbol{i}$ and $\boldsymbol{V}$ vectors are equal, hence

$$\omega r \cos\theta \quad = \mathrm{V}\cos(\theta - \alpha) \qquad (5)$$

$$\Rightarrow \omega r \quad = V\frac{\cos(\theta-\alpha)}{\cos\theta} \qquad \theta \neq \frac{\pi}{2} + n\pi; n \in \mathbb{Z} \qquad (6)$$

When $\theta = \frac{\pi}{2} + n\pi$, the rotation of the wheel results in no translational motion. When $\theta = n\pi$ the wheels become “standard” 90° omni-wheels, but in the configuration of a Mecanum vehicle this will allow only forward motion, omni-directionality is lost. Additionally any external force in the *Y* direction will result in motion in that direction as the rollers will allow free motion in those directions.
Equation (6) results in an equation for the magnitude of the velocity in the direction α:

$$V = \omega r \frac{\cos\theta}{\cos(\theta-\alpha)} \qquad (\theta - \alpha) \neq \frac{\pi}{2} + n\pi; n \in \mathbb{Z} \qquad (7)$$

When $(\theta - \alpha) = \frac{\pi}{2}$, the rotational speed of the wheel must be zero, but any value of translational motion is valid as this is driven by other wheels on the vehicle.

2.3 Wheel Velocities for Translational Motion

The equations for the forces acting on the vehicle and the vehicle velocity can be used to calculate the wheel velocities (ω) required to move the vehicle in any particular direction α at any particular speed V. Assume that there is enough motor torque to overcome friction in the drive train and rolling friction and that the vehicle is in steady state. It is known from (3) that $V_w \propto F_T$ and from (4) that, $V_w = V$, because this is a rigid body. Using (6) we get an equation for wheel rotational velocity ω_w:

$$\omega_w = \frac{V\cos(\theta_w - \alpha)}{r_w \cos\theta_w} \qquad \theta \neq \frac{\pi}{2} + n\pi; n \in \mathbb{Z} \qquad (8)$$

2.4 Wheel Velocities for Rotational Motion

It is now possible to define equations for rotation of the vehicle around any point in the plane. In this case the angle α defining the direction of motion of an individual wheel is no longer the same. It varies depending on the position of the centre of rotation. For (8) to be applied an additional calculation must be made to determine α_w, $w = 1 \rightarrow 4$.

At an arbitrary point in time it is desired that the velocity of the origin of the vehicle be $V = iV_x + jV_y$ and the rotational velocity be $\dot{\phi}$ around the resulting Instantaneous Centre of Rotation (ICR)[5]. The radius to the ICR in Fig.5 is then:

$$R = \frac{V}{\dot{\phi}} \qquad (9)$$

and

$$\gamma = \operatorname{atan}\frac{V_y}{V_x} \qquad (10)$$

which allows us to define the position of the ICR:

$$X_I = -R\sin\gamma \qquad (11)$$

$$Y_I = R\cos\gamma \qquad (12)$$

Consider Fig. 5, we again define a local reference frame with its origin at the centre of the vehicle. For the purposes of the geometric calculation we now fix the contact point of the wheel with the ground at the centre of the wheel’s width.

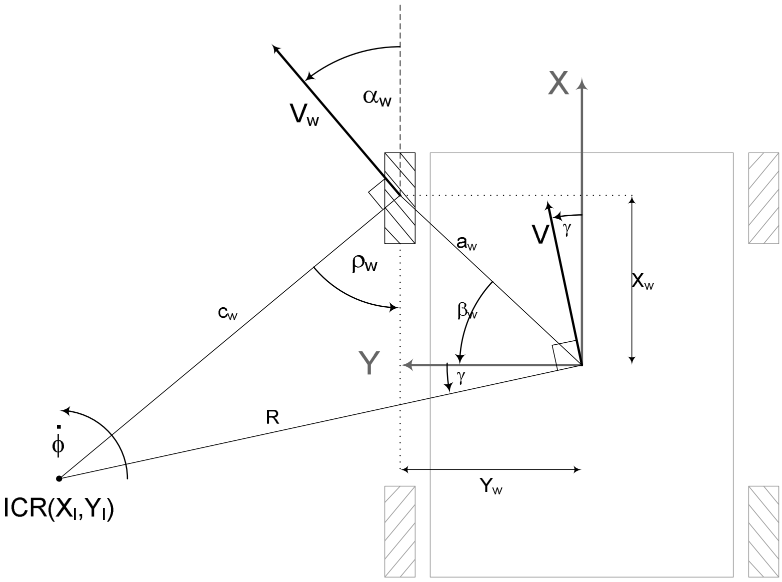

Figure 5. Vehicle and ICR Geometry

Each wheel is then defined by its position X_w and Y_w and the orientation of its rollers θ. Let β_w be the angle from the Y-axis to the ground contact point of wheel w, then

$$\beta_w \quad = \arctan \frac{X_w}{Y_w} \tag{13}$$

Let a_w be the distance from the origin to the contact point of wheel w, then

$$a_w \quad = \sqrt{X_w^2 + Y_w^2} \tag{14}$$

In the case of a Mecanum vehicle all a_w should be equal and constant as the geometry of the vehicle is fixed and symmetrical. β_w can be pre-calculated and will remain constant. Using (9),(10) ,(13) and (14) it is now possible to calculate the distance from the wheel contact point to the ICR. Let c_w be this distance

$$c_w \quad = \sqrt{a_w^2 + R^2 - 2a_w R\cos(\beta_w + \gamma)} \tag{15}$$

The resultant wheel velocity V_w must act perpendicular to the line c_w which makes an angle ρ_w with the X-axis, note that in Fig. 5 ρ_w is negative, which follows since $X_I - X_w$ will be negative

$$\rho_w \quad = \arctan\left(\frac{Y_I - Y_w}{X_I - X_w}\right) \tag{16}$$

$$\Rightarrow \alpha_w \quad = \frac{\pi}{2} + \rho_w \tag{17}$$

Given (9), the instantaneous resultant velocity of the wheel, V_w, orthogonal to the line c_w, can be shown to have a magnitude of.

$$|V_w| \quad = c_w|\dot{\phi}| \tag{18}$$

Equation (18) can now be substituted in (8) to calculate ω_w, resulting in

$$\omega_w \quad = \frac{c_w\dot{\phi}\cos(\theta_w - \alpha_w)}{r\cos\theta_w} \qquad \theta \neq \frac{\pi}{2} + n\pi; n \in \mathbb{Z} \tag{19}$$

Which is a general equation giving wheel velocity ω_w in terms of the position of the ICR (X_I, Y_I) and rotational velocity around this point, ultimately in terms of velocity $\boldsymbol{V}$ and rotational velocity $\dot{\phi}$. The application of this equation follows a matrix approach similar to [6].

3. TESTING THE MODEL

3.1 Single Wheel Tests

A test platform was designed to run tests on a single Mecanum wheel. The wheel motion was constrained on a belted treadmill, which was in turn mounted on a turn table as shown in Fig. 6.

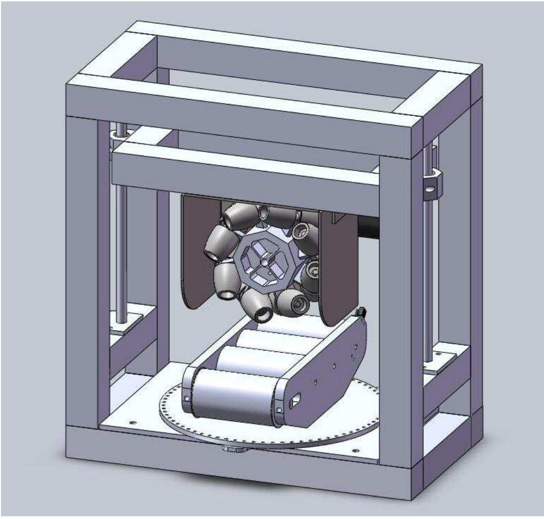

Figure 6. Mecanum Wheel Test Platform

The tester is equipped with quadrature encoders on the wheel and the treadmill to measure relative speeds. Fig 7. shows theoretical vehicle (treadmill) speeds at varying wheel speeds (ω) and angles (α) as calculated by (7). At -45° and 135° the predicted vehicle speeds tend towards infinity as the equation reaches an asymptote. This corresponds to when the vehicle is moving at 45°and the wheel rotational velocity is zero, but the wheel translational velocity is nonzero. Some of the data points approaching this asymptote have been removed in Fig 7. making it clearer to view.

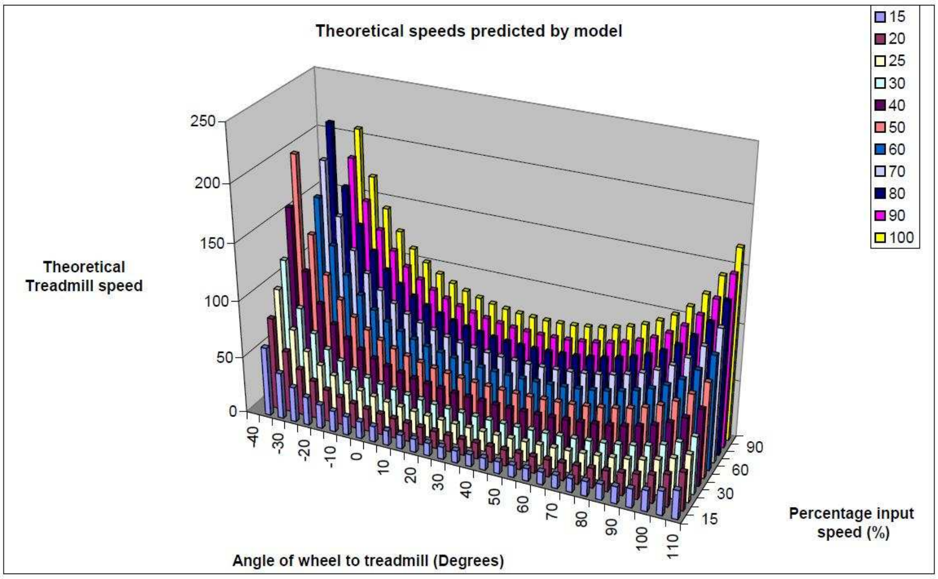

Figure 7. Theoretical wheel speeds

Tests were performed at various speeds up to an arbitrary maximum speed determined by the available torque of the motor.

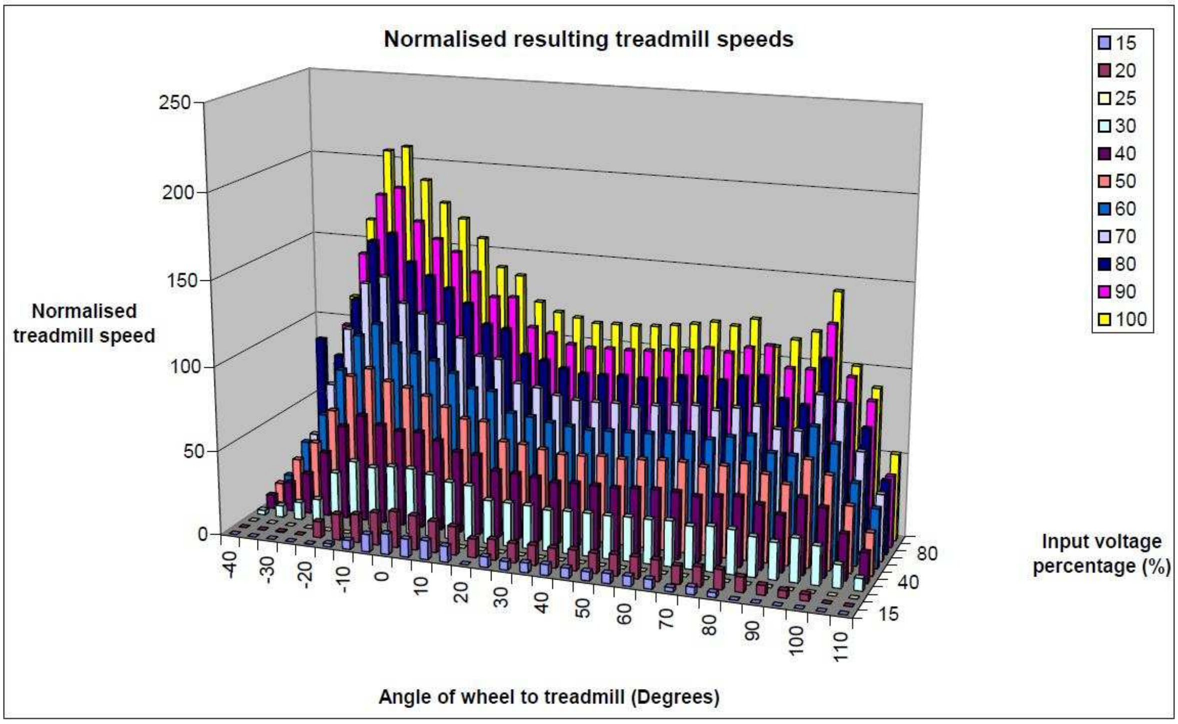

Figure 8. Test Results

While the results shown in Fig. 8 do not match the predictions exactly there is some correlation. The primary reason for the mismatch is a known problem with the equipment. As the wheel-treadmill angle is changed, the wheel exerts more and more force sideways on the belt, this results in the belt rubbing against the end plates of the treadmill. This problem requires modification of the test equipment; where-after more tests will be conducted as part of an ongoing research project.

3.2 Vehicle tests

A locus described in [7], shown in Fig 9. was used to test the application of (19) on a custom Mecanum vehicle. The test described was performed with no feedback as yet; all wheel motors are controlled with open loop controllers.

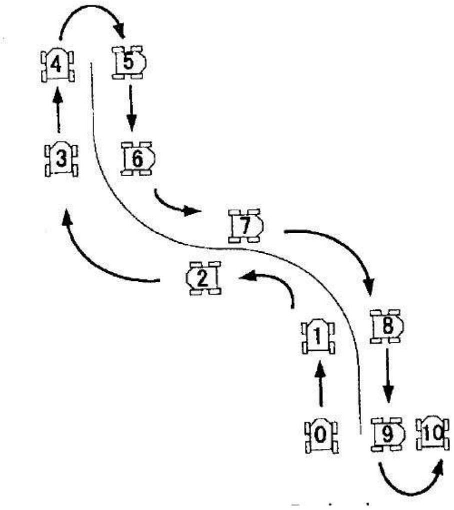

Figure 9. Benchmark locus

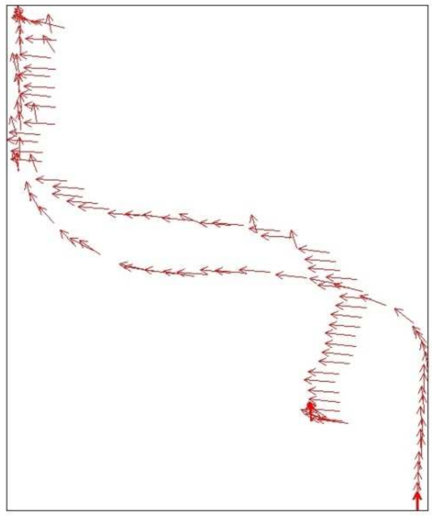

Figure 10. Laser Scanner results

Position and direction data was captured using a Sick LMS200 Laser Scanner. A flat board was mounted longitudinally on the platform to provide directional information. Data was captured from the laser scanner as the platform performed the pre-programmed locus. The locus begins at the bottom right hand corner of Fig. 10. All the motion required to demonstrate omni-directionality can be seen. Once feedback control is implemented it should follow the locus more closely.

4. CONCLUSION

A mathematical model relating the rotational velocity of a Mecanum wheel to its instantaneous velocity has been derived, taking into account that the position of the ground/wheel contact point moves within the width of the wheel as the wheel turns.
Tests were conducted to show that the model works for a single wheel and a vehicle using four wheels.

5. FUTURE WORK

More tests will be run on the single wheel tester to eliminate some of the errors seen in the current results.
Feedback control will be implemented on the vehicle to allow it to follow a path more closely.

6. ACKNOWLEDGEMENTS

Many thanks to Russ Ether for his valuable criticism of the content, and specifically the equations found, in this paper.

7. REFERENCES

[1] N. Tlale and M. de Villiers, "Kinematics and dynamics modelling of a mecanum wheeled mobile platform," *15th International Conference on Mechatronics and Machine Vision in Practice, M2VIP'08,* Auckland: 2008, pp. 657-662.

[2] R.B. Ferriere L., "Design of omnimobile robot wheels," *Proceedings - IEEE International Conference on Robotics and Automation*, Minneapolis, MN, USA: IEEE, Piscataway, NJ, United States, 1996, pp. 3664-3670.

[3] J.B. Song and K.S. Byun, "Design and control of a four-wheeled omnidirectional mobile robot with steerable omnidirectional wheels," *Journal of Robotic Systems*, vol. 21, 2004, pp. 193-208.

[4] A. Gfrerrer, "Geometry and kinematics of the Mecanum wheel," *COMPUTER AIDED GEOMETRIC DESIGN*, vol. 25, 2008, pp. 784-791.

[5] N.I. Siegwart R, *Introduction to Autonomous Mobile Robots*, MIT Press, Cambridge, Massechusetts, USA, 2004.

[6] A. Shimada, S. Yajima, P. Viboonchaicheep, and K. Samura, "Mecanum-wheel vehicle systems based on position corrective control," *IECON 2005: Thirty-First Annual Conference of the IEEE Industrial Electronics Society, Vols 1-3,* L.G. Franquelo, A. Malinowski, M.Y. Chow, and H.L. Hess, 2005, pp. 2077-2082.

[7] S.A. Viboonchaicheep P., "Position Rectification Control for Mecanum Wheeled Omni-directional Vehicles," *IECON Proceedings (Industrial Electronics Conference)*, Roanoke, VA: 2003, pp. 854-859.