DETECTING 3-D ROTATIONAL MOTION AND EXTRACTING TARGET INFORMATION FROM THE PRINCIPAL COMPONENT ANALYSIS OF SCATTERER RANGE HISTORIES

Willie Nel*, Duncan Stanton#, Mohammed Yunus Abdul Gaffar+
Department of Radar and Electronic Warfare, DPSS, CSIR
Pretoria, South Africa
*wajnel, #dstanton, +ynagaffar @ csir.co.za

***Abstract*—ISAR imagery of ships are complicated by the 3-D motion of the target, which causes blurring in the imagery. A technique is proposed which could help detect such motion and prove useful to both analyse the 3-D motion as well as possibly help to estimate the 3-D position of scatterers as a by-product of the analysis. The technique is based on principal component analysis of accurate scatterer range histories and is shown only in simulation. Future research should focus on practical application.**

Keywords-ISAR; 3-D Motion; Principal Component Analysis; Radar Imaging; Range history

I. INTRODUCTION

Inverse Synthetic Aperture Radar (ISAR) is a well known radar signal processing technique for generating images of moving targets [3]. The technique relies on the relative rotational motion of the target which produces range-Doppler histories that can be converted into a 2-D image. Many conventional motion compensation techniques assume that the target possesses rotational motion around a fixed axis at a constant rate of rotation during the coherent processing interval (CPI) [8] [14] [6]. This assumption leads to severely blurred images when the target experiences 3-D rotational motion during the CPI. However, small ships at sea produce few CPI's during a typical sample that conform to the 2-D rotational motion assumption [13]. For this reason, detection algorithms are needed to find intervals with a low degree of 3-D rotational motion so that conventional motion compensation algorithms can be used to form focused ISAR imagery. Alternatively, 3-D imaging algorithms can be used to form a 3-D ISAR image of target using its 3-D rotational motion [5]. If such detection algorithms can provide insight into target movement directly from the range history data it could be an advantage i.t.o. computational complexity.

Some of the ISAR motion compensation algorithms include a multiple scatterer range tracking step, which is one of the signal processing steps that is required [2] [1]. This is done to estimate the targets radial motion and correct for the translational component of the target. An autofocus technique is then applied to perform phase correction along the cross-range dimension, before a Fourier Transform is used to generate an ISAR image.

The current literature shows that there are individual algorithms to detect the presence of 3-D rotational motion [5] and to generate a 3-D ISAR image using 3-D rotational motion [9] [10]. These algorithms assume that it is possible to accurately track the range or phase history of multiple scatterers from measured high range resolution (HRR) profiles; in [9] it is also shown that it is possible to accurately track multiple scatterers in HRR data even when scatterers overlap in range. The work by Li et al. [5] shows that range histories of scatterers can be used to detect the presence of 3-D rotational motion by examining the linear dependence of these range histories under a small angle assumption. If scatterers are found to be linearly dependent on one another, then it follows that 3-D rotational motion was not present in the CPI. A geometric invariant technique for generating 3-D ISAR images of ground moving targets was proposed by Stuff in [9]. However, this method involves many matrix calculations; as a result, it is computationally expensive.

This paper proposes a technique that jointly detects the existence of 3-D rotational motion and extracts 3-D information of the target of interest. It is hypothesised that the rotational motion of a target about more than one orthogonal axis, causes 3-D scatterers to exhibit unique range history components that can be broken into multiple orthogonal components. The proposed method performs principle component analysis (PCA) on the range history of multiple scatterers to estimate the amount of 3-D rotational motion in a CPI and to extract information about the relative position of scatterers in 3-D space. An advantage associated with the proposed technique is that it is computationally less expensive than the method described in [9]. In addition, the proposed algorithm can be seen to be an extension of the Li et al. technique in [5] as it is also applicable when the target exhibits large rotation angles over a longer CPI and the small angle approximation is no longer valid.

To the best of the authors' knowledge, this technique has not been previously shown or documented in the field of ISAR. It makes novel contributions by (1) Showing that a single technique can jointly detect the presence of 3-D rotational motion and provide information relating to the relative position of scatterers in 3-D space; and (2) Relating the outputs of the algorithm to a quantitative measure of the degree of 3-D rotational motion. This measure can be used to assess whether the CPI is better suited to 2-D or 3-D ISAR imaging techniques.

The proposed technique could be useful both to the fields of RCS measurement of cooperative vessels and to the classification of non-cooperative vessels. All navies require their radar cross section (RCS) measurements of their platforms in order to identify the scattering hotspots. This technique may be used to extract the relative positions of scatterers of a ship, in order to pinpoint the relative location of unknown scatterers to known scatterers. In the non-cooperative case, the relative location of scatterers can be used as one of the features for classification.

The paper is structured as follows: Section II describes the system model that is considered and defines some nomenclature; Section III discusses the effects of 3-D rotational motion on the linear dependence of scatterer range histories. Section IV discusses PCA. Section V provides the motivation for using PCA to analyse of the range history of scatterers. Section VI discusses the use of PCA to evaluate the existence of 3-D motion in the data and finally Section VII shows a set of simulation results (purely theoretical as well as based on measured motion data) that is used to confirm (but not yet mathematically prove) the hypothesis.

II. SYSTEM MODEL

Figure 1 illustrates the system model that is considered. In this paper we assume that the translation motion of the target has already been range compensated. The target is allowed to possess arbitrary rotational motion around the Y and Z axes, denoted by φ(t) and θ(t) respectively. The radar is positioned at a distance of R along the –X axis.

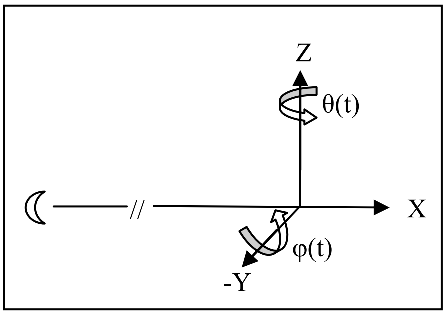


Figure 1. *System model showing the radar, the target and the coordinate system*

III. THE EFFECT OF ROTATIONAL MOTION ON THE LINEAR DEPENDENCE OF SCATTERERS' RANGE HISTORIES

According to [5] the non-planar 3-D rotational motion of the target cannot be compensated by current rotational motion compensation algorithms such that the phase of each scatterer in the CPI is a linearly varying phase history. Such motion thus results in target blurring if a FFT based ISAR image is formed, and it is not possible to compensate all of the scatterers simultaneously in a single 2-D ISAR image. (For another analysis of the effects of 3-D motion see the work reported in [11]).

The argument presented in [5] shows that, under the assumption of 2-D rotational motion the phase histories (and therefore range histories) of prominent point targets in the HRR data, will show a linear relationship (i.e. be linearly dependent on each other). The cross range coordinate or *y* component of the target is shown to be the constant of dependence. The linear relationship is shown to be dependent on the scatterer cross range (or *y*) coordinate.

It is also shown that for the case of 3-D rotational motion, the linear dependence of the phase (and range) histories will no longer hold, and the target range histories now depend on both the *y* and *z* coordinates of the scatterers, as well as the angles through which the target has rotated in three-space.

Not noted in [5] is that, for the case where the small angle approximation does not hold, scatterer phase histories will not be linearly dependent, since they will depend on *sin* and *cosine* terms, which by their very nature are orthogonal (and by definition not linearly dependent).

In this paper we propose to use PCA as a tool to analyse the linear dependence of the scatterers' range histories. The next section discusses PCA in more detail.

IV. PRINCIPAL COMPONENT ANALYSIS

Principal Component Analysis (also termed the Karhunen-Loeve Transform or Hotelling Transform) is a technique that estimates an optimal set of orthonormal basis vectors that spans the observed data. The first principal component is optimal in the sense that it accounts for the most variance (power) in the data, the second for the variance orthogonal to that, and so on.

Given an *M* by *N* matrix $\mathbf{X}$ of which *M* different variables are observed *N* times (*N* samples), the goal of PCA can be summarized as follows:

Find some orthonormal matrix **P** that diagonalises the covariance matrix of X formed as $\mathbf{X} = \mathbf{PY}$, i.e. find a matrix **P** such that the covariance $\mathbf{C}_X = (N-1)^{-1}\mathbf{XX}^T$ of **X** is diagonalised. The rows of **P** are the eigenvectors of $\mathbf{C}_X$, which also corresponds to the principal components of $\mathbf{X}$.

The rank of the covariance matrix of a set of data determines the number of linearly independent components in the dataset [7] [4]. For a dataset of M different variables, a covariance matrix with a rank $r < M$ indicates that there are r number of vectors that span the signal subspace and $(M - r)$ vectors that span the noise subspace.

In [12], it is reported that the eigenvectors associated with distinct eigenvalues of a real symmetric matrix are orthogonal, and the number of distinct eigenvalues indicate the rank of the matrix. Since the covariance matrix $\mathbf{C}_X$ is real symmetric, the number of distinct (non-zero) eigenvalues can be used to estimate its rank.

Each unique eigenvalue therefore indicates an associated eigenvector basis required to span the signal subspace of the data.

The technique of principle component analysis is applied widely in the field of statistics to find, in order of importance, those components that best accounts for the variance in the data. The principal components corresponding to the smallest (M – r) eigenvalues are considered to represent noise.

V. Applying PCA on Scatterer Range Histories

This section describes the proposed technique for detecting 3-D rotational motion and extracting information relating to the relative position of scatterers.

A. Estimating the number of significant eigenvalues

Assume that we are able to track M scatterers of the target and extract N range values for each scatterer. Thus, we can define a row vector $\mathbf{R}_m$ with N elements for the m^{th} scatterer, where $1 \le m \le M$.

The principle components of the range histories of M scatterers are calculated using the following steps:

a) Estimate the mean of the range history for all M scatterers. The mean of the m^{th} scatterer's range history, denoted by $\bar{\mathbf{R}}_m$, is given by:

$$\bar{\mathbf{R}}_m = \frac{1}{N}\sum_{n=1}^{N}\mathbf{R}_m \quad (1)$$

b) Let $\mathbf{X}_m$ denote the range history of the the m^{th} scatterer with a mean of zero (i.e. the input vector $\mathbf{X}_m$ is zero mean for each observed variable);

$$\mathbf{X}_m = \mathbf{R}_m - \bar{\mathbf{R}}_m \quad (2)$$

c) Let $\mathbf{X}$ denote the zero mean matrix of range histories:

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix} \quad (3)$$

d) Calculate the covariance matrix of X, which is denoted by $\mathbf{C}_X$:

$$\mathbf{C}_X = \frac{1}{N-1}\mathbf{X}\mathbf{X}^T \quad (4)$$

e) Perform the eigenvector decomposition on the covariance matrix to calculate the eigenvectors and eigenvalues of $\mathbf{C}_X$;

$$\text{EVD}\left(\mathbf{C}_X\right) = \mathbf{P}^T\mathbf{D}\mathbf{P} \quad (5)$$

where the rows of the matrix $\mathbf{P}$ contain the eigenvectors of $\mathbf{C}_X$ and the diagonal elements of the matrix $\mathbf{D}$, denoted by $\{d_1, d_2 \ldots d_M\}$ in descending order, contain their corresponding eigenvalues.

f) Determine the significant eigenvalues. In this paper we propose fitting a linear model to the least significant eigenvalues (in dBs) using linear regression and detecting eigenvalues that deviate from this model by more than a threshold k.

B. Relationship between the eigenvalues and target motion

The distinct eigenvalues as produced by PCA provides us with the ability of detecting 2-D or 3-D motion during the CPI, since it relates to the number of linearly independent range history components. It is hypothesized that the number of significant eigenvalues relates to the complexity of the target motion as described below:

1) Only one significant eigenvalue

If there exists only a single unique (and non-zero) eigenvalue, then the data space is spanned by a one dimensional basis, and all the components are linearly dependent. In this case, it is postulated that there exists little information about the cross range dimension in the data (i.e. the 2-D ISAR image will most likely not exhibit good cross range resolution).

2) Two significant eigenvalues

If there exists 2 unique, non-zero, eigenvalues then the data spans two orthogonal bases, and thus contains enough information to produce a 2-D ISAR image, that will have significant range and cross range information. This will hold true even under the case of significant 2-D rotation, such that the small angle approximations are invalidated.

3) Three or more significant eigenvalues

If there are 3 unique significant eigenvalues, then the range histories contain information that spans a 3-D subspace, and the motion most likely contained significant 3-D components. In this case, it is postulated that there should be enough information to produce a 3-D image of the target from the single aperture. Depending on the significance of the third component, 2-D ISAR processing of the data will result in blurring, and the amount of blurring will be related to the significance of the third eigenvalue.

C. Extracting information relating to the relative position of scatterers

Assume that the covariance matrix $\mathbf{C}_X$ exhibits s significant eigenvalues. We hypothesize that the three eigenvectors associated with the three largest eigenvalues contain information relating to the relative position of the M scatterers in three-space. If s is equal to 3 then a 3-D plot of the scattering centres can be produced by simply using the first eigenvector as the x-coordinate, the second eigenvector as the y-coordinate and the third eigenvector as the z-coordinate. In the case of only two significant eigenvectors, the z-coordinate will contain noise.

The next section shows through simulation that the above claims seems valid under many conditions. A rigorous mathematical analysis of these claims has not yet been performed, and will be pursued in a future publication.

VI. SIMULATION

A. Simulation Setup

A point scatterer based model was used to simulate the range histories of targets during the CPI for various types of motion. The target, shown in Figure 2, represents a boat with a mast (shown as blue circles), and some added corner reflectors (shown as red asterisks). The corner reflector positions were added for comparison with measured radar data of this target, but the results shown in this paper will be based solely on simulation.

The radar is positioned at range of 20 km, and the simulations are run at X-Band (10 GHz) using a waveform bandwidth of 420 MHz, a CPI of 640ms and 64 HRR profiles per CPI.

The range histories that form the input to the proposed technique are based on the accurate simulated range of each scatterer during the CPI. A Gaussian noise component with standard deviation σ is added as a range tracking error to these histories to make the results more realistic[1].

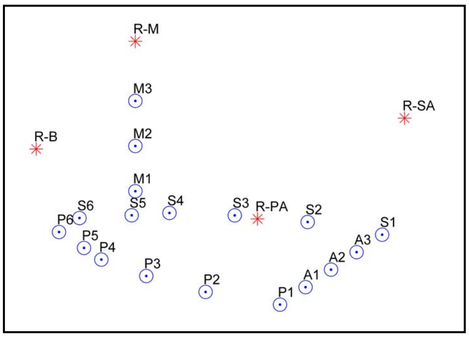


Figure 2. *3-D view of point scatterer boat target*

B. Simulation Results using simulated motion data

This section presents results of the simulations performed to illustrate the outputs of the proposed technique under different motion conditions. In the simulations the threshold for detecting significant eigenvalues was set to *k=6dB*.

Case 1*: 2-D motion with only a single significant eigenvalue*

The target is rotating around only the z-axis, producing a top view image (θ(t) = t (deg), φ(t) = 0). Figure 3 shows the largest 10 eigenvalues as **normalised by the first**, in a log scale. Hence the first eigenvalue is at 0 dB.

It is clear from the Figure 3 that there is only 1 significant eigenvalue. The ISAR image associated with this motion is shown in Figure 4. It is clear that the motion is not significant enough to produce an image with a good cross range resolution.

[1] Admittedly, this circumvents the problem of estimating scatterer ranges from the coherent HRR profiles. However, as shown in [9], there are techniques that can perform this task quite accurately under certain conditions.

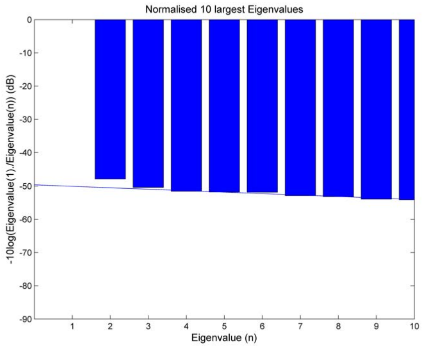


Figure 3. *Normalised eigenvalues for Case 1*

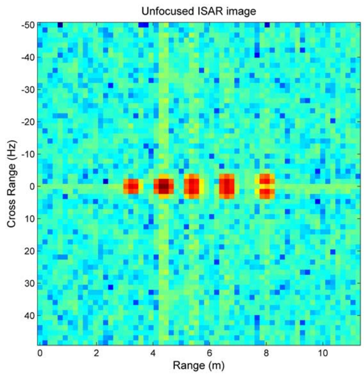


Figure 4. *Simulated ISAR image for Case 1*

Case 2*: 2-D motion with two significant eigenvalues*

In this case, the target is now experiencing significant 2-D rotation. The rate of rotation was chosen such that the image should have approximately equal range and cross range resolution. To achieve this, θ(t) = 7.64t (deg), φ(t) = 0.

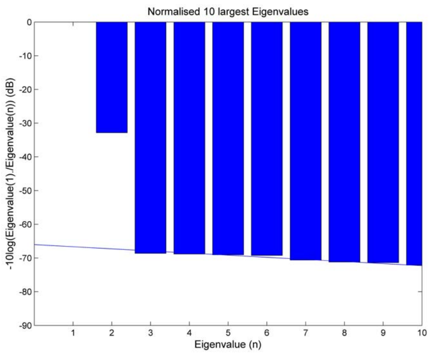


Figure 5. *Normalised eigenvalues for Case 2*

Figure 5 shows the eigenvalues from this case; Figure 6 shows the corresponding ISAR image. It is clear that there now are two significant eigenvalues, and the corresponding image confirms this, and as expected the target is imaged with approximately equal range and cross-range resolutions.

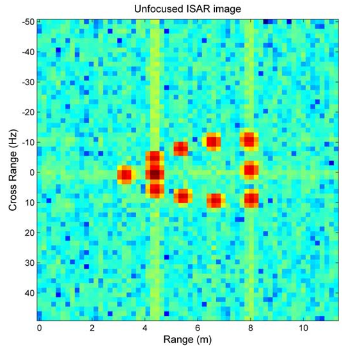


Figure 6. *Simulated ISAR image for Case 2.*

Case 3*: Non-significant 3-D motion*

In this case, the target is now experiencing some insignificant 3-D rotation. The rate of linear rotation is still chosen such that the image should have approximately equal range and cross range resolution. However, the rotation around the Y axis now is causing some 3-D motion. $\theta(t) = 7.64t$ (deg), $\varphi(t) = 2t$. The two motion components however are still linearly dependent.

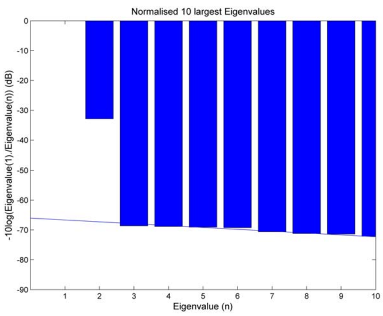


Figure 7. *Normalised Eigenvalues for Case 3*

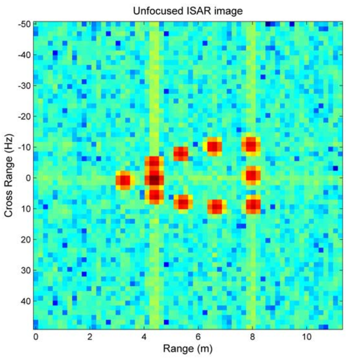


Figure 8. *Simulated ISAR image for Case 3*

Figure 7 shows the eigenvalues from Case 3; Figure 8 shows the corresponding ISAR image. It is clear that there are still only two significant eigenvalues, and the corresponding image confirms this, and as expected the target is imaged with approximately equal range and cross-range resolutions without significant blurring.

Case 4*: Significant 3-D motion (3 eigenvalues)*

The target now experiences significant 3-D motion. Figure 9 shows the eigenvalues, and it is clear that there are now 3 eigenvalues that are significant. Figure 10 shows the corresponding ISAR image which is severely blurred due to the 3-D motion. Figure 11 shows the relative 3-D positions of the point scatterers as extracted from the first three eigenvectors. The angular motion of theta and phi was set to $\theta(t) = 7.64t$ (deg) and $\varphi(t) = 20(t - 0.3)^2$ (deg).

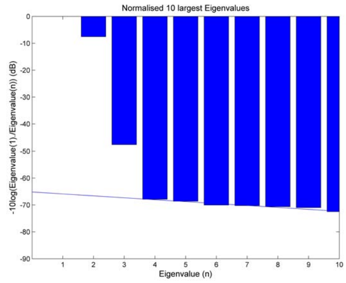


Figure 9. *Normalised Eigenvalues for Case 4*

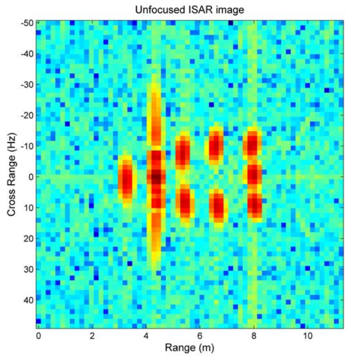


Figure 10. *Simulated ISAR image for Case 4*

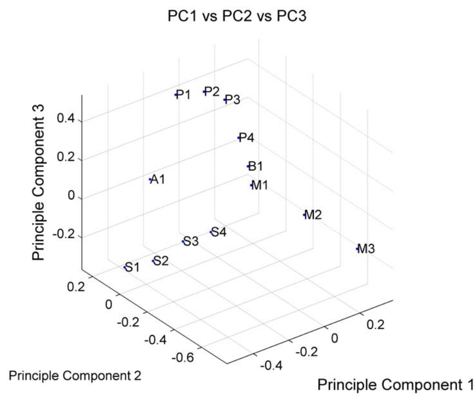


Figure 11. *Relative 3-D positions of scatterers as extracted from the eigenvectors for Case 4*

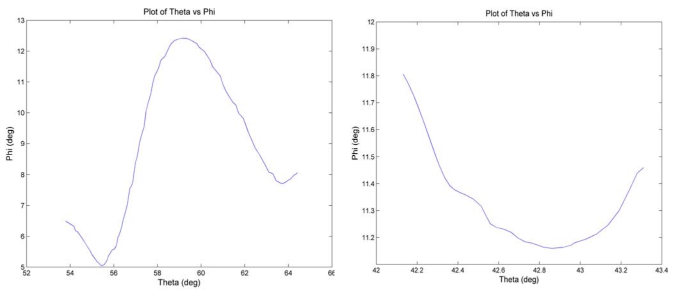
From Figure 11 it is clear that the 3-D structure of the target can be recognised, and that scatterers remain in the same approximate relative location to each other, even though the complete target is rotated at some random position in space.

C. Simulation results using measured motion data

Lastly, we will show how the analysis fares on range history data generated using measured motion data from an INS / GPS system for a boat of size similar to that of the simulated target. The data for the instrumented boat comes from an inbound run, where the target is experiencing both a change in heading and also significant pitch and roll motion. In this case, the simulation is fed only with the heading data for $\theta(t)$ and the pitch data for $\varphi(t)$, since these are the two rotations that could cause significant range histories.

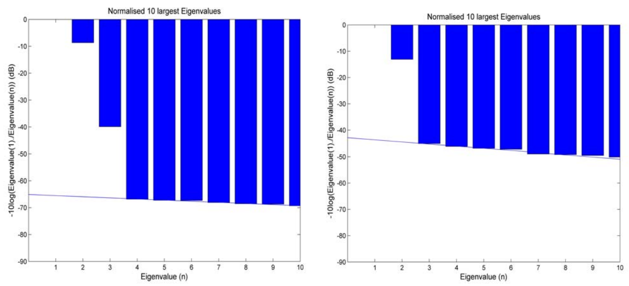
Figure 12-14 shows the results of the simulation for two cases. Figure 12-14 (a) shows motion, as well as the eigenvalues and corresponding simulated image for the case where the boat is experiencing significant 3-D motion. From the eigenvalues it is clear that the target is experiencing 3-D motion and thus the image is completely blurred.

Figure 12-14 (b) shows an imaging interval for the case where the 3rd eigenvalue is not significant and the first basis function is estimated to be a good approximation of uniform linear rotation. It is clear that in this case, the image is not blurred anymore and the shape of the boat becomes recognisable.



(a) (b)

Figure 12. *Measured motion data containing 3-D motion*

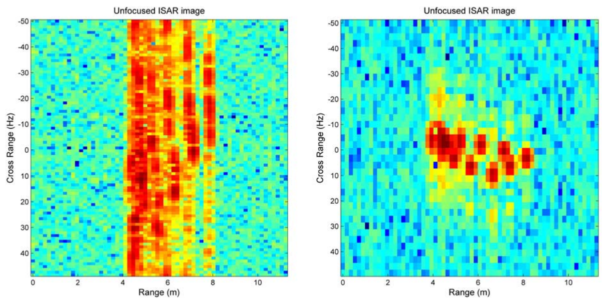


(a) (b)

Figure 13. *Eigenvalues for real motion data containg 3-D motion*

VII. CONCLUSIONS AND FUTURE WORK

Principal component analysis of the range histories of scatterers shows promise as a technique to both detect the presence of 3-D target motion, as well as to extract information about the relative positions of scatterers. This paper shows in simulation that the technique could prove to be quite useful.



(a) (b)

Figure 14. *ISAR image for real motion data containg 3-D motion*

However more research is required to characterise the technique as well as to understand its limitations. It is proposed that this be done in a follow on publication.

REFERENCES

[1] B. D. Steinberg and H. M. Subbaram, "Microwave Imaging Techniques", New York: John Wiley and Sons, 1991.

[2] C. Yuan and D. Casasent, "Composite filters for inverse synthetic aperture radar," Proceedings of the SPIE, vol. 41, pp. 94-104, Jan. 2002.

[3] D. R.Wehner, "High Resolution Radar", Norwood, MA: Artech House, 2nd edition, 1995

[4] J. E. Jackson. "A User's Guide to Principal Components", p. 31, John Wiley and Sons, 1991

[5] J. Li, H. Ling. V.C. Chen, "An Algorithm to Detect the Presence of 3-D Target Motion from ISAR Data", Multidimensional Systems and Signal Processing, Volume 14, Numbers 1-3, January 2003 , pp. 223-240(18), Springer

[6] J. M. Munoz-Ferreras and F. Perez-Martinez, "Uniform rotational motion compensation for inverse synthetic aperture radar with non-cooperative targets," IEEE Proc. Radar, Sonar Navig., vol. 148, pp. 160-166, June 2001

[7] J. Shlens, "A Tutorial on Principal Component Analysis", Technical Report, 2007-12-06, Systems Neurobiology Laboratory, Salk Insitute for Biological Studies.

[8] J. Wang and D. Kasilingam, "Global range alignment for ISAR", IEEE Trans. Aerosp. Electron. Syst., vol. 39, pp. 351-357, Jan. 2003

[9] M. A. Stuff, "Three-Dimensional Analysis of Moving Target Radar Signals: Methods and Implications for ATR and Feature-Aided Tracking", SPIE Proc. Algorithms for Synthetic Aperture Radar Imagery VI, vol. 3721, Apr. 1999, pp. 485-496.

[10] M. A. Stuff, P. Sanchez, M Biancalana, "Extraction of Three-Dimensional Motion and Geometric Invariants from Range Dependent Signals", Multidimensional Systems and Signal Processing, 14, 161–181, Kluwer Academic Publishers. Manufactured in The Netherlands, 2003.

[11] M. Y. Abdul Gaffar and W. A. J. Nel, "Investigating the effect of a target's time-varying Doppler generating axis of rotation on ISAR image distortion," in IET Radar Conference 2007, Oct. session 5b, No 4, 2007.

[12] P. V. O'Neill. "Advanced Engineering Mathematics", p. 738, Wadsworth Publishing Company, Third Edition, 1991.

[13] V. C. Chen and W. J. Miceli, "Effect of Roll, Pitch and Yaw Motions on ISAR Imaging," in SPIE Proc. Radar Processing, Technology and Applications VI, vol. 3810, Denver, CO, July 1999, pp. 149-15

[14] Z. She, D. A. Gray, and R. E. Bogner, "Autofocus for inverse synthetic aperture radar (ISAR) imaging," Signal Processing, vol. 81, pp. 275-291, 2001.