Intracavity generation of longitudinal dependant Bessel like beams

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ABSTRACT

In this paper we report on two resonator systems for producing LDBLBs with arbitrary initial parameters. First resonator system is based on doublet of divergence and convergence lenses with spherical aberration. Second resonator system is based on doublet of axicon and axicon-lens. For both resonator systems the LDBLB is output of fundamental mode, but the spectral properties of obtaining LDBLBs are different.

Keywords: intra–cavity beam shaping, Bessel beams, non-diffraction beams**.**

1. INTRODUTION

The zero-order Bessel beam J_0 as a mathematical construction was firstly introduced by Durnin [1]. Such beams have been produced by many different techniques, including illuminating an annular ring and transforming the field through a lens [2], a hologram [3], an axicon [4, 5], and recently intra–cavity techniques [6-9] and the use of anisotropic crystals [10, 11]. The use of a refractive axicon, or conical lens, provides the most efficient method for producing Bessel beams, due mainly to the higher transmittance compared to an annular slit, and because an axicon produces no higher-order diffracted beams as in the case of holographic elements.

Much of the interest in Bessel beams is connected with the non-diffracting nature of these beams as well as with the effect of self-reconstruction of the transverse profile after shadowing (see, for example, [12]). While inside the non-diffracting region the Bessel beam does not change its profile, at the boundary of this region the beam abruptly transforms into a conical field with the characteristic ring-shaped intensity distribution ("double-face" effect). The significant difference between the near-field and the far-field intensity pattern can be considered a disadvantage of such beams, in contrast to Gaussian beams which preserve their profile while propagating in free space.

The "double-face" effect can be partially weakened by generating a Bessel beam with a very small cone angle, γ, as the non-diffraction beam length is inversely proportional to γ. There is an elegant possibility to eliminate the "double-face" effect for Bessel beams via the generation of Bessel beams with decreasing cone angle γ during beam propagation. In so doing, if at $z\to\infty$ the limiting value of the angle $\gamma(z)$ is zero, then such beams will have the advantages of both Bessel and Gaussian beams. In what follows such beams will be referred to as longitudinal dependent Bessel-like beams (BLBs). Such BLBs have been generated by a combination of axicons and lens systems with spherical aberration and anastigmatic lens axicons with the reflecting spherical surfaces [12-16]. Such schemes have been investigated before, mainly with the aim of obtaining a uniform on-axis profile, a constant diameter central spot size, as well as to minimize astigmatism, an aberration that is typically large for conical optics [17]. In the papers [18-19] the possibility is shown of achieving of a high transverse resolution at large distances in a scheme of a defocused Galilean-type telescope with negative spherical aberration. However until now there has not been a detailed investigation of the transverse structure in the near and far field of longitudinal dependent BLBs [LDBLB], nor the possibility of managing the axial intensity of such BLBs through parameter choice of the implementation method. In particular, of great practical interest is the study of how to produce BLBs with an on-axis intensity that is higher than that of the input Gaussian beam.

The methods of producing such of LDBLBs can be divided into two classes, namely extra– and intra–cavity beam shaping. Extra–cavity (external) beam shaping can be achieved by manipulating the output beam from a laser with suitably chosen amplitude and/or phase elements, and has been extensively reviewed to date [18, 23]. The second method of producing such beam intensity profiles, intra–cavity beam shaping, is based on generating a LDBLB directly as the cavity output mode. Unfortunately such laser beams are not solutions to the eigenmode equations of laser

Laser Beam Shaping X, edited by Andrew Forbes, Todd E. Lizotte, Proc. of SPIE Vol. 7430,
743010 · © 2009 SPIE · CCC code: 0277-786X/09/$18 · doi: 10.1117/12.827971

resonators with spherical curvature mirrors, and thus cannot be achieved (at least not as a single mode) from conventional resonator designs.

The key problem is how to calculate the required non–spherical curvature mirrors of the resonator in order to obtain a desired output field. One method to do this is to combination of so called *reverse propagation technique* proposed by Belanger and Pare [20-22] and well known extra-cavity designs of obtaining LDBLBs [18, 23].

In this paper we report on two resonator systems for producing LDBLBs with arbitrary initial parameters. First resonator system is based on doublet of divergence and convergence lenses with spherical aberration [18]. Second resonator system is based on doublet of axicon and axicon-lens [23]. For both resonator systems the LDBLB is output of fundamental mode, but the spectral properties of obtaining LDBLBs are different. We have tried to give possible explanation and differences in experimental nondifraction properties of outputs of given resonator systems.

2. RESONATOR BASED ON TRANSMITTIN GALILEAN TELESCOPE SCHEME

One of the possible optical schemes which allows obtaining the Bessel beam with z - dependent cone angle $\gamma = \gamma(z)$ is shown in Fig. 1(a). This scheme consists of doublet of divergence and convergence lenses with spherical aberration and firstly was discovered by Tadashi Aruga [18] during telescope systems investigation.

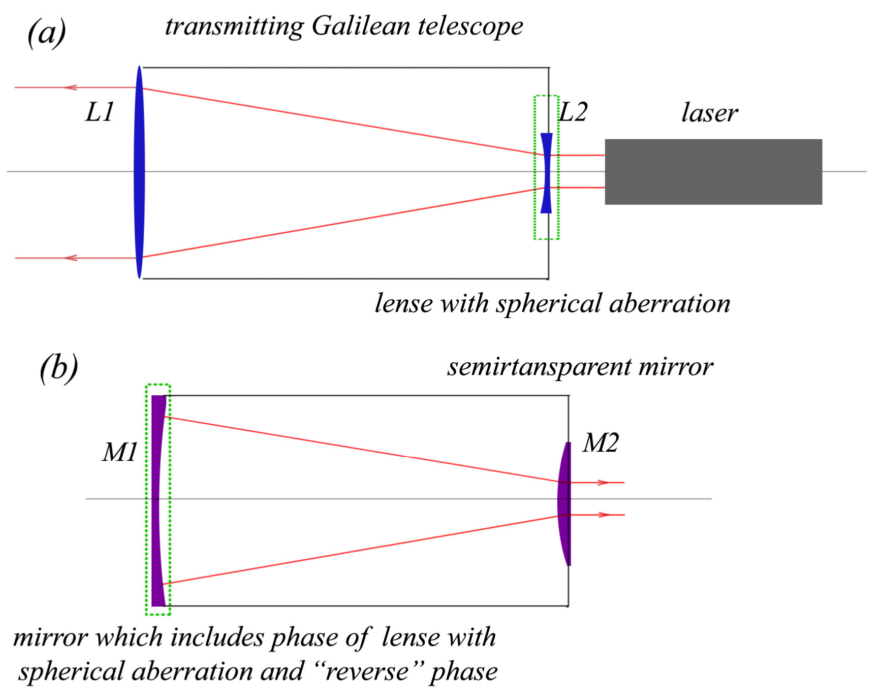

Fig.1. The transmitting Galilean telescope system for producing LDBLB (a). The resonator system based on Galilean telescope for producing LDBLB (b)

On fig. 1 we can see how the transmitting Galilean telescope firstly proposed by T. Aruga can be transformed into resonator system with similar output beam properties. For this we use so called “reverse propagation technique” [20-22] namely two mirrors: M2 which include phase of lens L2 and M1 which include the phase of divergence lens L1 and conjugate resulting phase after the same lens (“reverse” phase) for determination of fundamental mode similar as beam propagating in transmitting Galilean telescope (see fig. 1(a)) situated on similar with telescope distance. For producing the LDBLB Aruga has used the telescope with large aperture diameter around 10 cm. The large aperture allows raising spherical aberration coefficient easily because of this order of aberration depends strongly on the radius of lens or mirror. Obviously that the resonator system with similar aperture is difficult to design, and is one of the key problems with such resonator systems – how to increase the spherical aberration of the output beam. To solve this problem we propose using a convergence mirror with spherical aberration instead of a diverging lens with spherical aberration and, consequently, divergence mirror with spherical aberration in resonator scheme. How we can see from fig.1 and fig. 3 the beam radius on convergence mirror is almost twice larger then on divergence one, therefore for divergence mirror we can use almost 16 times weaker spherical aberration. But for this case, we have to change the output mirror into divergence mirror as well. From numerical and analytical analyses we can conclude that Aruga scheme and resonator scheme fig. 1 are produced completely similar output beams for next cases. The using large Fresnel number, to avoid the influence the mirror boundaries on fundamental mode properties and, if impact of mirror M2, as transmitted mirror, into output of fundamental mode phase we can determine as constant shift. It is practicable by choosing right shape of backward side of mirror or by using mirrors with constant thin thickness.

The main problem of this resonator system is to find the shape of mirror M1 because of the shape of mirror M2 and distance between mirrors can be taken directly from transmitting Galilean telescope.

The shape of mirror M1 has to be determined by reverse propagation technique, namely the phase shift of the field after reflecting off mirror M1 must be equal to the conjugate phase of the field after propagation through telescope system from eyepiece to objective taking into account the spherical aberration on lens L1 instead of L2.

Lets assume that laser beam on eyepiece is Laguerre-Gaussian beam of zero order with flat phase and width - w_0. The eyepiece is non-aberrated thin lens with focal length f_1. The objective is convergence lens with f – focal length, spherical aberration constant – β and z_1 – distance between lenses.

To find the curvature of wave front at output plate we have to calculate next diffraction Fresnel integral:

$$a_1(\rho, z_1) = -\frac{ik_0}{2z_1}\exp[\frac{ik_0\rho^2}{2z_1}]\int_0^R \exp\left(-\frac{\rho_1^2}{w_0^2}+\frac{ik\rho_1^2}{2f_1}+\frac{ik\rho_1^2}{2z_1}\right)J_0\left(\frac{k_0\rho\rho_0}{z_1}\right)\rho_1 d\rho_1, \tag{1}$$

where $k_0=2\pi/\lambda$, λ – wave length and R – we can define as resonator mirror radius.

To find the solution of integral (1) we have used stationary phase approximation:

$$a_1(\rho, z_1) = \frac{w_0}{w(z_1, f_1)}\exp\left(-\frac{\rho^2}{w(z_1)^2}+\frac{ik_0\rho^2}{2R(z_1)}-i\alpha(z_1, f_1)\right), \tag{2}$$

where $w(z_1)=w_0\sqrt{1+(z_1/f_1)^2+(z_1/z_0)^2}$, $R(z_1,f_1)=z_1\dfrac{1+(z_0/z_1)^2(1+z_1/f_1)^2}{1+(z_0/z_1 f_1(1+z_1/f_1))}$, $tg(\alpha(z_1,f_1))=\dfrac{z_1/z_0}{1+z_1/f_1}$, $z_0=k_0 w_0^2/2$.

Consequently to find field on output plate we have to multiply Eq. 2 by transmission function of lens L1 with spherical aberration namely by $\exp\left(-\left(ik_0\rho^2/2f\right)+i\beta r^4\right)$. Now we are able to find the reflected phase shift and consequently shape of mirror M1 using *reverse propagation technique* [20-22]. We can find reflected phase shift of mirror M1 by direct solution of integral (1) as well (see fig 2a).

Throughout this paper a concept resonator with the following parameters is used to illustrate the two approaches to LDBLB generation: wavelength of λ = *632* nm; optical path length between the mirrors of $z_1=0.65$ *m,* initial width of Gaussian beam $w_0=0.5$ *mm*, lenses with focal lengths *f1=-0.4, f2=0.7*, aberration coefficient $\beta=16\ 10^4$, mirror radiuses are equal *R=5 mm*, and corresponding these parameters Fresnel number of resonators $N_f=60$.

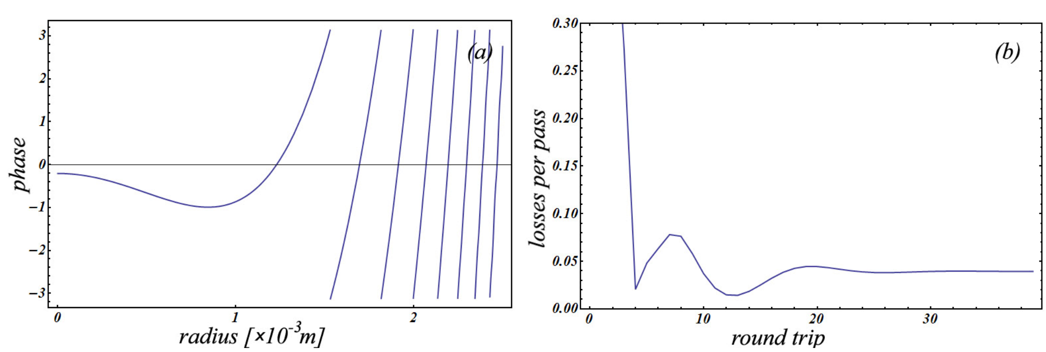

Fig.2. The phase of mirror *M1* obtained by direct solution of integral (1), (a).
The fundamental mode stabilization (Fox-Li analyses) (b).

To simulate fundamental mode stabilization and to proof of operational integrity of designed resonator system we will use well known Fox-Li analysis.

On Fig. 2b we can see fundamental mode stabilization graph of Fox-Li algorithm operation namely losses of fundamental mode stabilization per round trip. We note that even for large Fresnel numbers ($N_f=60$) the stabilization of the fundamental mode (which corresponds to generation of LDBLB) proceeds quickly. We suppose that corresponding behavior is due to resonator stabilization parameter namely given resonator system which based on telescope scheme lie on boundary of stabilized and non stabilized states and implemented spherical aberration induce the resonator to non stabilized state and leads to quick stabilization of fundamental mode due to large losses of initial field.

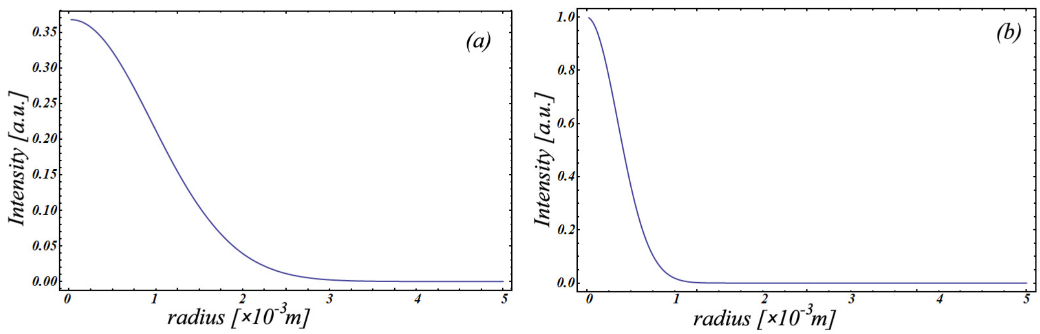

Fig. 3. The intensity profiles on resonator mirror M1 (a) and M2 (b) after stabilization.

On fig. 3 the intensity profiles at resonator mirrors are presented. How we can see inside resonator after stabilization required fields are obtained successfully namely Gaussian field on mirror M2 and field corresponding Eq. 2 on mirror M1.

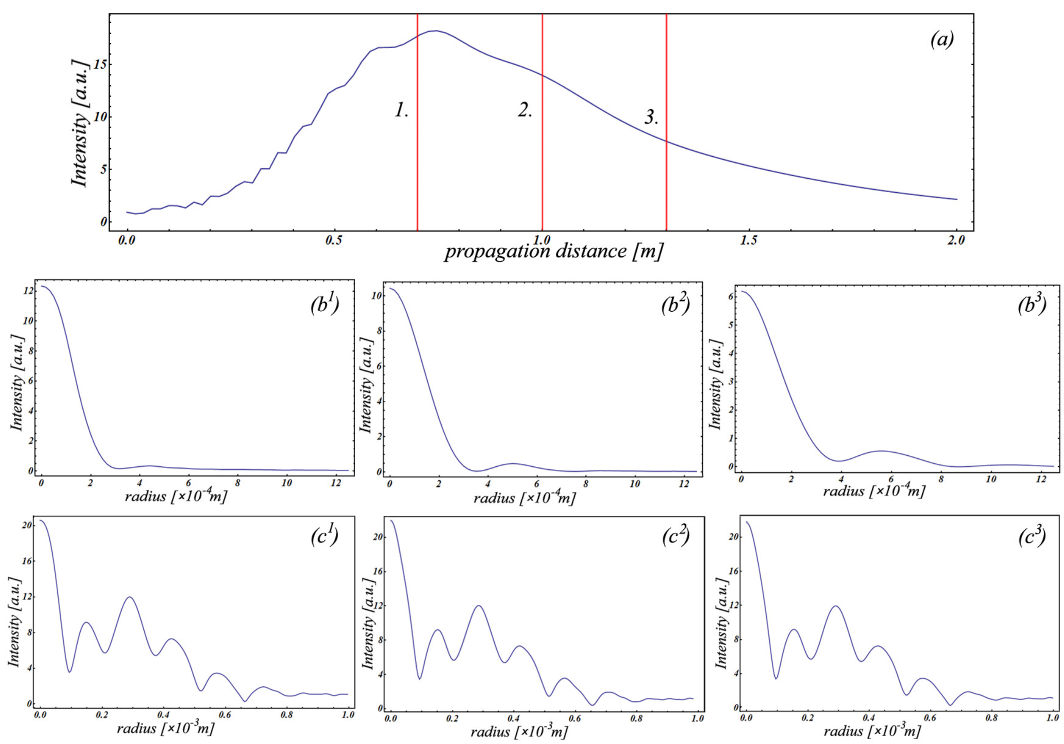

Fig.4. The propagation properties of output beam. (a) – longitudinal dependence of central peak, (b) – cross section of propagated beam on distance 0.7 m (1), 1.0 m (2) and 1.3 m (3) correspondingly. (c) – spectrum view of obtained beam profiles.

Figure 4 (a) illustrates the change in the on-axis intensity with propagation distance z. As is seen, it is a one-peaked curve typical for Bessel beams. When z increases up to several meters or more, there occurs a slow monotonic decrease in the peak on-axis intensity with propagation distance as is typical of Gaussian beams.

How we can see from fig. 4 (b) during the propagation the intensity profile of obtained beam is getting wider due to this nature we call this beam longitudinal dependent as contrasted to Bessel beams which do not change longitudinally intensity profile. To see nondifraction properties of this beam let look at spectrum fig. 4 (c) of obtained beam. How we can see the spectrum consist of two main peaks. First central peak belongs to Gaussian nature of beam namely Gaussian view of spectrum and second off-center peak with small oscillations similar to Bessel-Gauss beam spectrums namely displacement Gaussian view of spectrum. The oscillation on second off centre peak can be responsible for composite nature of Bessel spectrum of obtained LDBLB namely the Bessel part of spectrum we can represent as sum of several Bessel spectrums with close cone angles. Due to dual nature of obtained beam firstly: this beam can propagate long distance as contrasted to Bessel beams and secondly: has non diffraction properties as contrasted to Gaussian beams. [18]

2. RESONATOR BESED ON DOUBLET OF AXICON –LENSE AND AXICON

Second possible optical scheme that allows one to obtain LDBLB with cone angle $\gamma = \gamma(z)$ is shown in Fig. 5(a) [23]. The scheme consists of two axicons, *A1* and *A2*, and a lens *L* with of focal length *F*. The Bessel beams generated by these axicons are characterized by cone angles of γ_1 and γ_2 respectively.

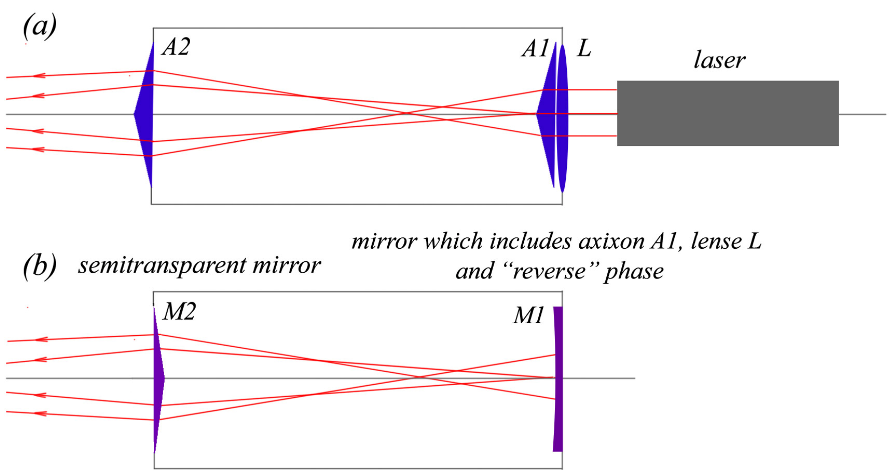

Fig. 5. The scheme based on doublet of axicon-lens and axicon for producing LDBLB (a) and resonator based on this scheme (b).

On fig. 5(a, b) we can see how "the axicons" scheme can be transformed into resonator system with similar output beam properties. The involving of two axicons in this scheme comes to one serious disadvantage of similar resonator system namely apex of axicons is situated on maximum intensity of laser beam. In this case after numerical simulation we are not able to make away with intensity oscillations around centre of obtained beam. To solve this problem we propose using instead of zero order Laguerre-Gaussian beam the first order Laguerre-Gaussian beam with flat phase as initial beam to avoid maximum intensity on apexes of axicons [see fig. 6].

To find the curvature of wave front at output plate of "axicons" scheme we have to calculate next diffraction Fresnel integral:

$$a_1(\rho, z_1) = -\frac{i}{\lambda z_1}\exp\left(\frac{ik_0\rho^2}{2z_1}\right)\int_0^{R_{a1}}\exp\left(-\frac{\rho_1^2}{\rho_0^2} - ik_0\gamma_1\rho_1 - \frac{ik_0\rho\rho_1}{z_1}\cos(\varphi-\varphi_1)\right)\rho_1^2 d\rho_1 d\varphi_1\,, \tag{3}$$

where $\frac{1}{\rho_0^2} = \frac{1}{w_0^2} + \frac{ik_0}{2F} - \frac{ik_0}{2z_1}$, R_{a1} – is the radius of the first axicon, $w_0=1.5mm$ – the half-width of the input Gaussian beam . Applying the stationary phase method yields

$$a_1(\rho, z_1) = -\frac{i\rho F^2}{(z_1 - F)^2}\left(1 - \frac{\gamma_1 z_1}{\rho}\right)^{3/2} \exp\left[\frac{ik_0}{2z_1}\left(\rho^2 + \frac{z_1/F - 1 + iz_1/z_0}{(z_1/F - 1)^2 + (z_1/z_0)^2}(\rho - \gamma_1 z_1)^2\right)\right], \quad (4)$$

where $z_0 = k_0 w_0^2/2$

The last equation represents the field before transmitting of second axicon A2. Consequently to find the field on output plate we have to multiply Eq. 4 by transmission function of axicon A2 namely by $\exp(-ik_0\gamma_2\rho)$. Now we are able to find the reflected phase shift and consequently shape of mirror M2 (Fig.5) using *reverses propagation technique* [20-22]. The same phase we can obtain by direct solution of integral (3) (see. fig. 7 (a)) as well.

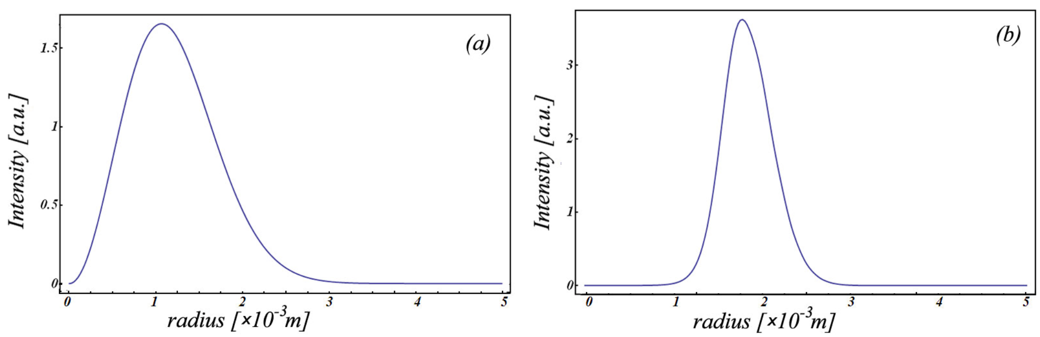

Fig. 6. The intensity profiles on resonator mirror M2 (a) and M1 (b) (see fig. 5) after stabilization.

On Fig. 7 (b) we can see fundamental mode stabilization graph of Fox-Li algorithm operation namely losses of fundamental mode stabilization per round trip. How we can see the stabilization process for this resonator system has different behavior in comparison with scheme based on Galilean telescope. Firstly the stabilization process needs more time and therefore losses of fundamental mode must be less that we can see on figure. We can explain this behavior by completely stable obtained resonator system. Secondly, oscillation process of stabilization takes place which can be explained by interference of low order modes during stabilization and fundamental mode separation.

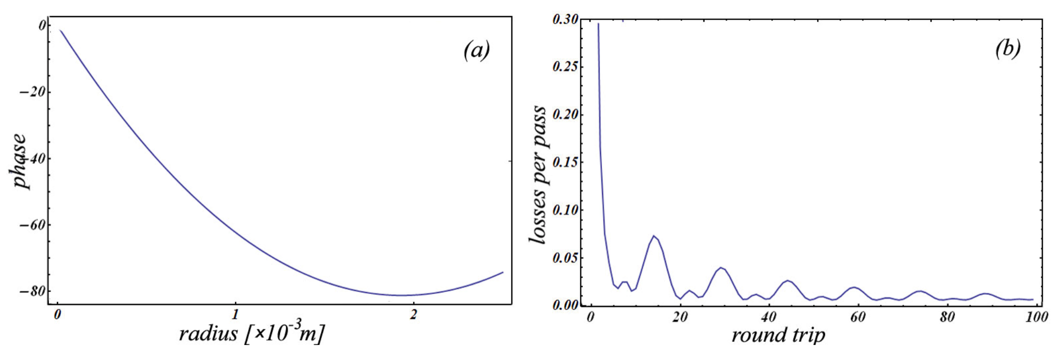

Fig. 7. The phase of mirror *M1* obtained by direct solution of integral (1), (a).
The fundamental mode stabilization (Fox-Li analyses) (b).

The change in the on-axis intensity with propagation distance z we can see on fig. 8. It is a one-peaked curve typical for Bessel beams and with z increases there occurs a slow monotonic decrease in the peak on-axis intensity with propagation distance as is typical of Gaussian beams. The obtained behavior is similar to telescope scheme.

We see from fig. 8 (b) that during propagation the intensity profile of the beam is getting wider and the transverse intensity profile becomes close to a Bessel beam profile with longitudinal changing of cone angle. The intensity profile of spectrum of obtained beam is close to intensity profile of obtaining LDBLB and close to Bessel beam profile as well. From the view of Fourier-spectra profile we can suppose that obtained beam we can describe as sum of infinite number of Bessel- Gauss beams with different and discrete difference in cone angles. This representation of obtained beam can be used for easy explanation of nondifraction properties of this class of longitudinal dependant Bessel like beams.

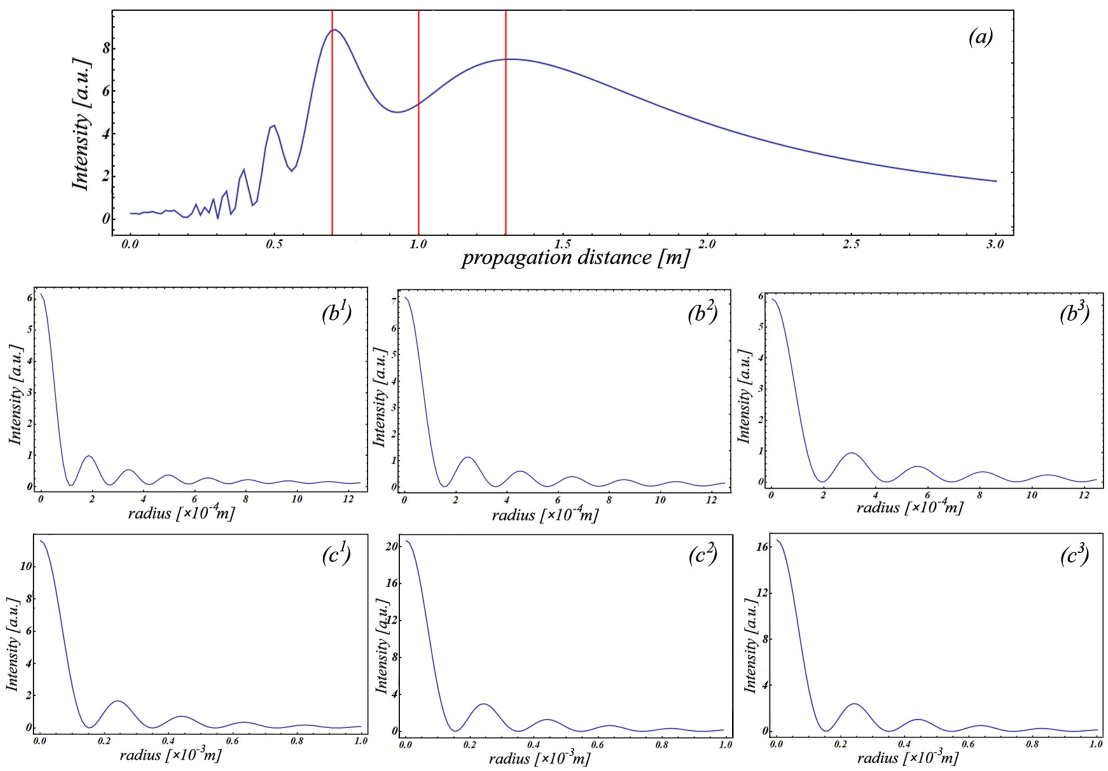

Fig. 8. The propagation properties of output beam of scheme based on doublet of axicon-lens and axicon. (a) – longitudinal dependence of central peak, (b) – cross section of propagated beam on distance 0.7 m (1), 1.0 m (2) and 1.3 m (3) correspondingly. (c) – spectrum view of obtained beam profiles.

3. CONCLUSION

In this paper we report on two resonator systems for producing LDBLBs. The first resonator system is based on a doublet of divergence and convergence lenses with spherical aberration [18]. Second resonator system is based on doublet of axicon and axicon-lens [23]. The difference in the spectrums and intensity profiles of obtained beams must lead to different both propagation properties and a nondifraction property of resulting beams that we can see on fig. 4 and fig. 8. The different stabilization behavior of these systems was observed and results from difference in stability parameter of given cavities (see fig. 2(b) and fig. 7(b)). We can conclude that both resonator systems can be used for producing LDBLB and obtained beams have close but different nondifraction and propagation properties.

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