Superpositions of Higher-order Bessel Beams and Nondiffracting Speckle Fields

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ABSTRACT

In this report we have developed a mechanism for the generation of the superposition of higher-order Bessel beams[1], which implements a ring slit aperture and spatial light modulator (SLM). Our experimental technique is also adapted to generate nondiffracting speckle fields. We report on illuminating a ring slit aperture with light which has an azimuthal phase dependence, such that the field produced is a superposition of two higher-order Bessel beams. In the case that the phase dependence of the light illuminating the ring slit aperture is random, a nondiffracting speckle field is produced. The experimentally produced fields are in good agreement with those calculated theoretically.

INTRODUCTION

It is known that higher-order Bessel beams can be generated by the illumination of a ring slit aperture or axicon[2] with a beam which has an azimuthal phase dependence, exp(imφ), such as a Laguerre-Gauss beam. In this work, we have adapted this concept so that the light illuminating the ring slit aperture possesses two separable azimuthal phase components, namely exp(imφ) and exp(inφ), producing a superposition of an m^{th} order and n^{th} order Bessel beam. Adapting this concept even further, nondiffracting speckle fields are generated by illuminating the ring slit aperture with light possessing a random phase dependence.

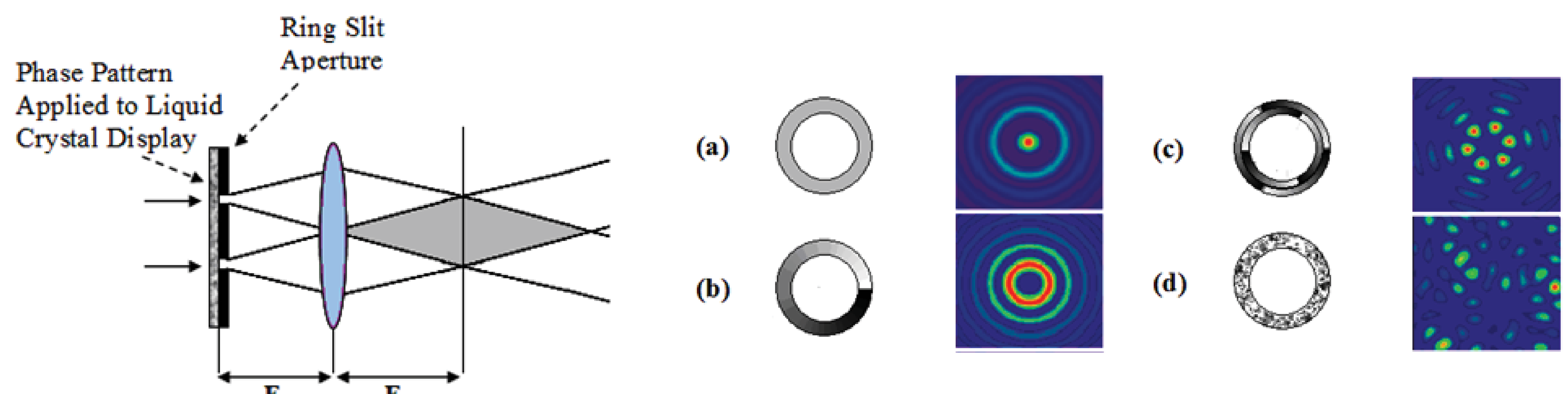

Figure 1: (a): The experiment developed by Durnin[3] to generate a zero order Bessel beam. This experiment involves illuminating a ring slit aperture, placed in the back focal plane of a lens, with a plane wave. (b) and (c): The generation of higher-order and superpositions of higher-order Bessel beams by the illumination of a ring slit aperture with a beam whose angular spectrum carries an azimuthally varying phase. (d): Illuminating the ring slit aperture with a field which possesses random amplitudes and phases generates a nondiffracting speckle field.

THEORETICAL BACKGROUND

In our optical setup an SLM and ring slit aperture are used to generate a superposition of two higher-order Bessel beams, as well as a nondiffracting speckle field. The incident Gaussian beam is transformed to a ring field, with radius *R* and width 2Δ, by a ring slit aperture.

The ring field is projected onto an SLM and the transmission function of the ring slit aperture can be described by the Fourier-Bessel expansion:

$$\tau(r,\varphi) = \sum_{m=0}^{\infty}\left(C_m(r)\exp(im\varphi) + S_m(r)\exp(-im\varphi)\right) \quad \text{... (1)}$$

In the case of generating a nondiffracting speckle field, the transmission function $\tau(r,\varphi)$ is dependent on both the radius, *r*, and the angle, φ. C_m and S_m are described by random functions:

$$C_m(r) = c_m(r)\exp(i\phi_m) \quad \text{... (2)}$$

$$S_m(r) = s_m(r)\exp(i\xi_m) \quad \text{... (3)}$$

By making use of the following diffraction integral:

$$A(r,\varphi,z) = \frac{-i}{\lambda z}\int_0^{2\pi}\int_{R-\Delta}^{R+\Delta}\tau(r,\varphi)\exp\left[-\frac{r_1^2}{w^2} + i\frac{k_0}{2f}\left(1-\frac{z}{f}\right)r_1^2\right]\exp\left[-i\frac{k_0 r r_1}{f}\cos(\varphi_1-\varphi)\right]r_1 dr_1 d\varphi_1 \quad \text{... (4)}$$

The nondiffracting speckle field can be described as follows:

$$A(r,\varphi,z) = \frac{-i}{\lambda z}\sum_{m=0}^{\infty}\int_{R-\Delta}^{R+\Delta}\left(\left(c_m(r_1)i^m J_m\left(\frac{k_0 r r_1}{f}\right)\right)\exp(im\varphi + i\phi_m) + \left(s_m(r_1)i^{-m}J_{-m}\left(\frac{k_0 r r_1}{f}\right)\right)\exp(-im\varphi + i\xi_m)\right)\exp\left[-\frac{r_1^2}{w^2} + \frac{ik_0 r_1^2}{2f}\left(1-\frac{z}{f}\right)\right]r_1 dr_1 \quad \text{... (5)}$$

The above equation illustrates that the creation of nondiffracting speckle fields lies in forming a superposition of zero and higher-order Bessel beams having random amplitudes and phases.

For the case of generating superpositions of two higher-order Bessel beams, the random functions C_m and S_m are neglected. We simplified matters by dividing the width of the ring slit aperture into two parts, such that the transmission function can be written as follows:

$$\tau(r,\varphi) = \begin{cases}\exp(im\varphi) & R \geq r \geq (R-\Delta)\\ \exp(in\varphi) & R \leq r \leq (R+\Delta)\end{cases} \quad \text{... (6)}$$

Similarly, the resulting superposition is calculated numerically by the use of the Kirchoff-Huygens diffraction integral:

$$A_m(r,\varphi,z) = \frac{-ik_0}{f}\int_{R-\Delta}^{R}\left(i^m\exp(im\varphi)J_m\left(\frac{k_0 r r_1}{f}\right)\right)\exp\left[-\frac{r_1^2}{w^2} + \frac{ik_0 r_1^2}{2f}\left(1-\frac{z}{f}\right)\right]r_1 dr_1 \quad \text{... (7)}$$

$$A_n(r,\varphi,z) = \frac{-ik_0}{f}\int_{R}^{R+\Delta}\left(i^n\exp(in\varphi)J_n\left(\frac{k_0 r r_1}{f}\right)\right)\exp\left[-\frac{r_1^2}{w^2} + \frac{ik_0 r_1^2}{2f}\left(1-\frac{z}{f}\right)\right]r_1 dr_1 \quad \text{... (8)}$$

EXPERIMENTAL SETUP AND RESULTS

Figure 2 depicts the general experimental setup for the generation of the superposition of two higher-order Bessel beams, as well as nondiffracting speckle fields. A HeNe laser operating at 633nm was expanded through a telescope before illuminating a ring slit aperture. The ring field was imaged onto the liquid crystal display of the SLM and the Fourier transform of the ring field was obtained by the use of lens 5. A 10X objective and CCD camera were used to investigate the resulting field from the start to the end of its propagation. An interferometer, denoted in red in figure 2, was introduced in the experimental setup so as to investigate the vortex structure of these fields.

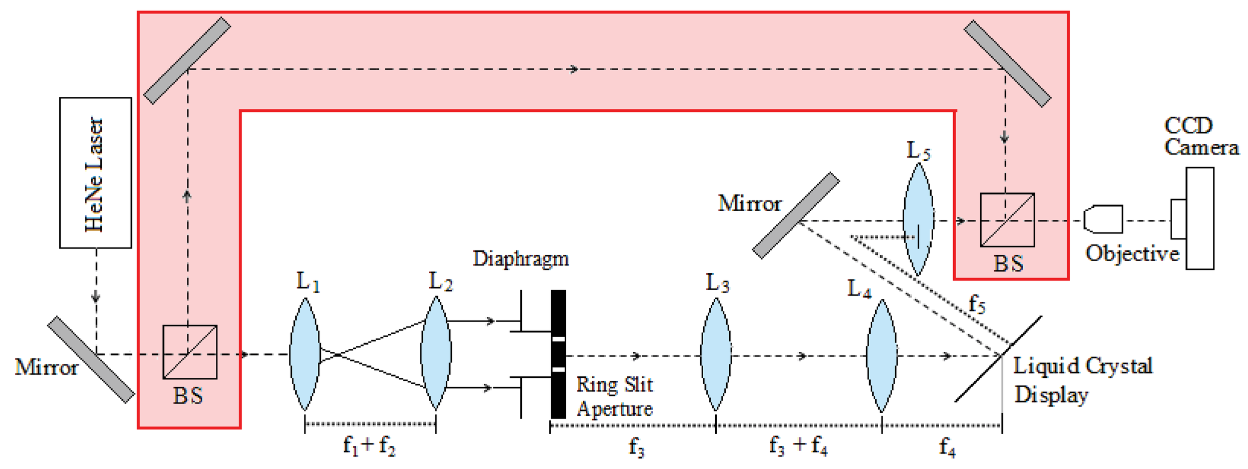

Figure 2: The experimental design for generating a superposition of two higher-order Bessel beams, as well as producing nondiffracting speckle fields. The interferometer used to interfere the field produced at the Fourier plane with a plane wave is denoted in red.

Due to the azimuthally varying phase which is imparted to the angular spectrum of the beam, the resulting fields possess vortices. The vortex structure of these fields is investigated by interfering these fields with a plane wave. It is known that fork-like patterns can be observed when a wavefront containing a screw dislocation is interfered with a plane wave.

Apart from investigating the field produced at the Fourier plane, the propagation of these fields was also investigated. It was noted that the intensity profile of the field rotates during the beams propagation which is in accordance with predicted results[4].

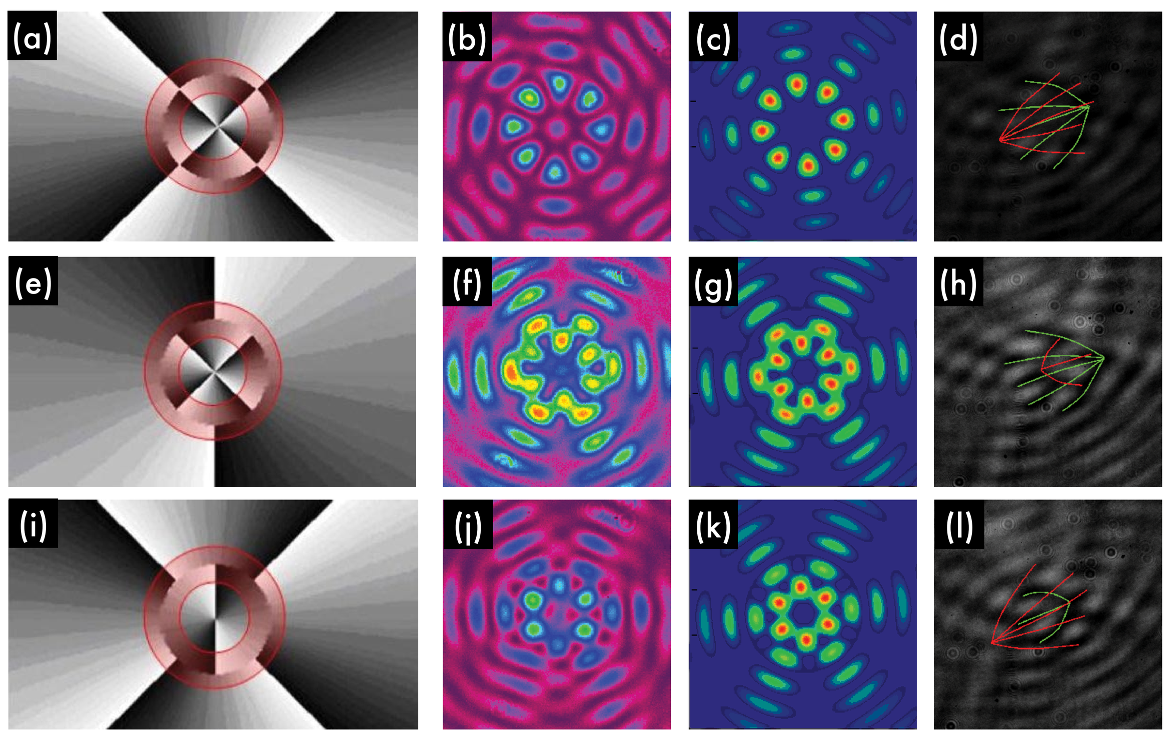

Figure 3: The columns from left to right represent the phase pattern applied to the liquid crystal display of the SLM, the observed intensity distribution of the superposition, the theoretical prediction, and interference pattern of the superposition field and a plane wave, respectively. Data is shown for (a)–(d): $A_{4,-4}$, (e)–(h): $A_{2,-4}$, and (i)–(l): $A_{4,-2}$. The illuminated ring slit is shown as a red overlay on the phase patterns.

Table. 1. The experimentally measured propagation distances for the fields: $A_{3,-3}$, $A_{4,-4}$ and $A_{2,-4}$ compared with the theoretically calculated propagation distance.

Field:	Theoretical Propagation Distance:	Experimental Propagation Distance:
$A_{3,-3}$	375mm	330mm ± 10mm
$A_{4,-4}$	375mm	360mm ± 10mm
$A_{2,-4}$	375mm	370mm ± 10mm

A speckle field is produced when light is scattered by a rough surface. The distinction in forming a nondiffracting speckle field consists in using a ring slit aperture to illuminate the random medium (depicted in figure 1). It is known that the Fourier transformation of a ring slit aperture results in the formation of a Bessel field. Introducing a random phase mask results in a superposition of zero and higher-order Bessel beams having random amplitudes and phases being formed. This superposition of zero and higher-order Bessel beams produces a nondiffracting speckle field.

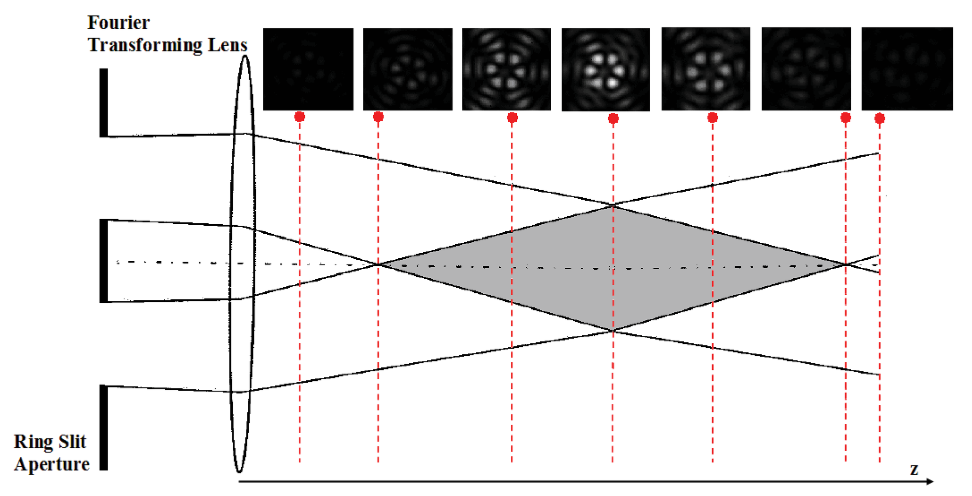

Figure 4: Images of the intensity profile of the experimentally produced field $A_{3,-3}$ at intervals along its propagation.

Table. 2. The experimentally measured propagation distance for the nondiffracting speckle field compared with the theoretically calculated propagation distance.

Field:	Theoretical Propagation Distance:	Experimental Propagation Distance:
Nondiffracting speckle field	112mm	115mm ± 5mm

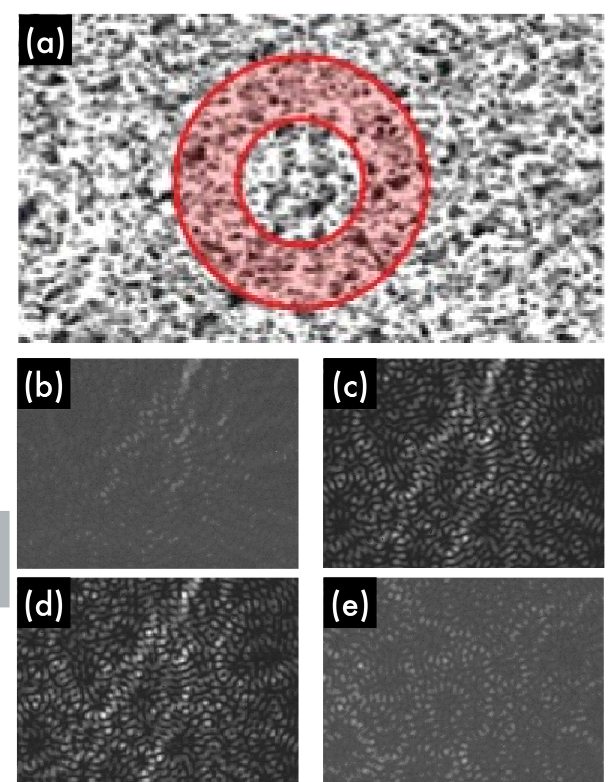

Figure 5: (a): The random phase pattern applied to the liquid crystal display of the SLM. The illuminated ring slit is shown as a red overlay on the phase pattern. (b)–(e): The intensity profile of the experimentally produced nondiffracting speckle field at intervals along its propagation. Near the start and end of the field's propagation the intensity distribution diminishes, but the speckle pattern remains, illustrating that the field is quasi-nondiffracting.

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