Superpositions of higher-order Bessel beams and nondiffracting speckle fields

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ABSTRACT

In this paper we present a mechanism for the generation of the superposition of higher-order Bessel beams, which implements a ring slit aperture and spatial light modulator (SLM). Our experimental technique is also adapted to generate nondiffracting speckle fields. We report on illuminating a ring slit aperture with light which has an azimuthal phase dependence, such that the field produced is a superposition of two higher-order Bessel beams. In the case that the phase dependence of the light illuminating the ring slit aperture is random, a nondiffracting speckle field is produced. The experimentally produced fields are in good agreement with those calculated theoretically.

Keywords: Higher-order Bessel beams, nondiffracting speckle fields, ring slit aperture, computer generated holograms, spatial light modulator (SLM)

1. INTRODUCTION

There is an extensive body of literature on generating either zero order or higher-order Bessel beams. Visually, the zero order Bessel beam possesses a bright central maximum, while the higher-orders have a dark central vortex, which propagates over an extended distance in a diffraction-free manner. Zero order Bessel beams can be generated by illuminating a ring slit aperture, placed in the back focal plane of a lens, with a plane wave[1]. Refractive optical elements, such as axicons[2,3], and diffractive optical elements such as computer generated holograms[4,5], can be used to generate both zero and higher-order Bessel beams. However, in this paper we have developed a mechanism for the generation of the superposition of higher-order Bessel beams, which implements the previously mentioned technologies, namely a ring slit aperture and computer generated holograms.

It is known that higher-order Bessel beams can be generated by the illumination of a ring slit aperture or axicon[3] with a beam which has an azimuthal phase dependence, $\exp(im\varphi)$, such as a Laguerre-Gauss beam. In this work, we have adapted this concept so that the light illuminating the ring slit aperture possesses two separable azimuthal phase components, namely $\exp(im\varphi)$ and $\exp(in\varphi)$, producing a superposition of an m^{th} order and n^{th} order Bessel beam. Adapting this concept even further, nondiffracting speckle fields are generated by illuminating the ring slit aperture with light possessing a random phase dependence.

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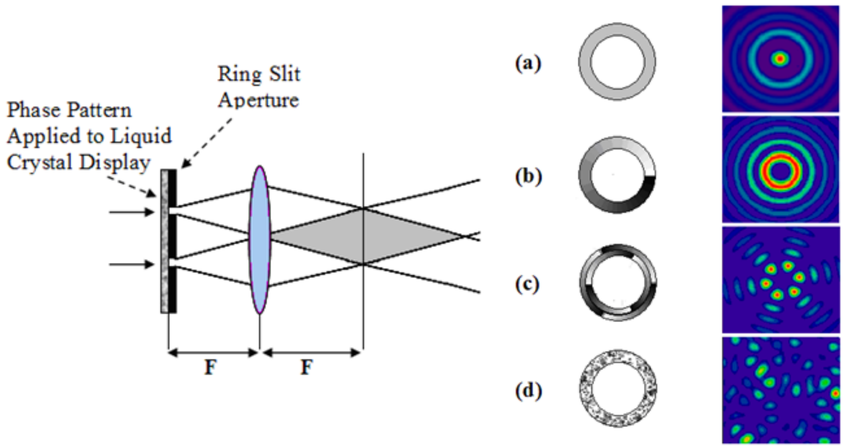

Fig. 1. Extension of Durnin's[1] ring slit experiment. (b) and (c): Illuminating the ring slit aperture with a beam whose angular spectrum carries an azimuthally varying phase generates higher-order and superpositions of higher-order Bessel beams. (d): Illuminating the ring slit aperture with a field which possesses random phases generates a nondiffracting speckle field.

The case depicted in Fig. 1 (a) represents the experiment developed by Durnin[1] which generates a zero order Bessel beam. This experiment involves illuminating a ring slit aperture, placed in the back focal plane of a lens, with a plane wave. Fig. 1 (b) and (c) illustrate the generation of higher-order and superpositions of higher-order Bessel beams by the illumination of a ring slit aperture with a beam whose angular spectrum carries an azimuthally varying phase. Finally, in the case of Fig. 1 (d) the ring slit aperture is illuminated with a field whose angular spectrum carries a randomly varying phase resulting in the generation of a nondiffracting speckle field.

Speckle fields are most frequently observed when light is scattered by a rough surface. The distinction in forming a nondiffracting speckle field consists in using a ring slit aperture to illuminate the random medium. It is known that the Fourier transformation of a ring slit aperture results in the formation of a Bessel field. Introducing a random phase mask results in a superposition of zero and higher-order Bessel beams having random amplitudes and phases being formed[6]. This superposition of zero and higher-order Bessel beams produces a nondiffracting speckle field.

2. THEORETICAL BACKGROUND

In our optical setup an SLM and ring slit aperture are used to generate a superposition of two higher-order Bessel beams, as well as a nondiffracting speckle field. The incident Gaussian beam is transformed to a ring field, with radius R and width 2Δ, by a ring slit aperture.

The ring field is projected onto an SLM and the transmission function of the ring slit aperture can be described by the Fourier-Bessel expansion:

$$\tau(r,\varphi) = \sum_{m=0}^{\infty}\left(C_m(r)\exp(im\varphi) + S_m(r)\exp(-im\varphi)\right). \tag{1}$$

In the case of generating a nondiffracting speckle field, the transmission function $\tau(r,\varphi)$ is dependent on both the radius, r, and the angle, φ. C_m and S_m are described by random functions:

$$C_m(r) = c_m(r)\exp(i\phi_m), \tag{2}$$

$$S_m(r) = s_m(r)\exp(i\xi_m), \tag{3}$$

where ϕ_m and ξ_m are random phases. $c_m(r)$ and $s_m(r)$ are amplitudes dependant on the incident Gaussian beam and reflection coefficient of the SLM.

By making use of the following diffraction integral:

$$A(r,\varphi,z)=\frac{-i}{\lambda z}\int_0^{2\pi}\int_{R-\Delta}^{R+\Delta}\tau(r,\varphi)\exp\left[i\frac{k_0}{2f}\left(1-\frac{z}{f}\right)r_1^2\right]\exp\left[-i\frac{k_0 r r_1}{f}\cos(\varphi_1-\varphi)\right]r_1 dr_1 d\varphi_1, \quad (4)$$

where $k_0 = 2\pi/\lambda$ and f is the focal length of the Fourier transforming lens, the nondiffracting speckle field can be described as follows:

$$A(r,\varphi,z)=\frac{-i}{\lambda z}\sum_{m=0}^{\infty}\int_{R-\Delta}^{R+\Delta}\left(\left(c_m(r_1)i^m J_m\left(\frac{k_0 r r_1}{f}\right)\right)\exp(im\varphi+i\phi_m)+\left(s_m(r_1)i^{-m}J_{-m}\left(\frac{k_0 r r_1}{f}\right)\right)\exp(-im\varphi+i\xi_m)\right)\exp\left[\frac{ik_0 r_1^2}{2f}\left(1-\frac{z}{f}\right)\right]r_1 dr_1, \quad (5)$$

where J_m is the m[th] order Bessel function.

The above equation illustrates that the creation of nondiffracting speckle fields lies in forming a superposition of zero and higher-order Bessel beams having random phases.

For the case of generating superpositions of two higher-order Bessel beams, the random functions C_m and S_m are neglected. We simplified matters by dividing the width of the ring slit aperture into two parts, such that the transmission function can be written as follows:

$$\tau(r,\varphi)=\begin{cases}\exp(im\varphi) & R\geq r\geq (R-\Delta)\\ \exp(in\varphi) & R\leq r\leq (R+\Delta)\end{cases}. \quad (6)$$

Similarly, the resulting superposition is calculated numerically by the use of the Kirchoff-Huygens diffraction integral:

$$A_{m,n}(r,\varphi,z)=A_m(r,\varphi,z)+A_n(r,\varphi,z), \quad (7)$$

where

$$A_m(r,\varphi,z)=\frac{-ik_0}{f}\int_{R-\Delta}^{R}\left(i^m\exp(im\varphi)J_m\left(\frac{k_0 r r_1}{f}\right)\right)\exp\left[-\frac{r_1^2}{w^2}+\frac{ik_0 r_1^2}{2f}\left(1-\frac{z}{f}\right)\right]r_1 dr_1, \quad (8)$$

$$A_n(r,\varphi,z)=\frac{-ik_0}{f}\int_{R}^{R+\Delta}\left(i^n\exp(in\varphi)J_n\left(\frac{k_0 r r_1}{f}\right)\right)\exp\left[-\frac{r_1^2}{w^2}+\frac{ik_0 r_1^2}{2f}\left(1-\frac{z}{f}\right)\right]r_1 dr_1, \quad (9)$$

illustrating that the field produced is a superposition of an m[th] and n[th] order Bessel field.

3. EXPERIMENTAL SETUP AND RESULTS

Fig. 2 depicts the general experimental setup for the generation of the superposition of two higher-order Bessel beams, as well as nondiffracting speckle fields. A HeNe laser operating at 633nm was expanded through a telescope before illuminating a ring slit aperture. The ring field was imaged onto the liquid crystal display of the SLM. A HEO1080P SLM containing 1920x1080 pixels was used in our optical setup. The Fourier transform of the ring field reflected off of the liquid crystal display was obtained by the use of lens 5. A 10X objective and CCD camera were used to investigate the resulting field from the start to the end of its propagation. An interferometer, denoted in red in Fig. 2, was introduced in the experimental setup so as to investigate the vortex structure of these fields.

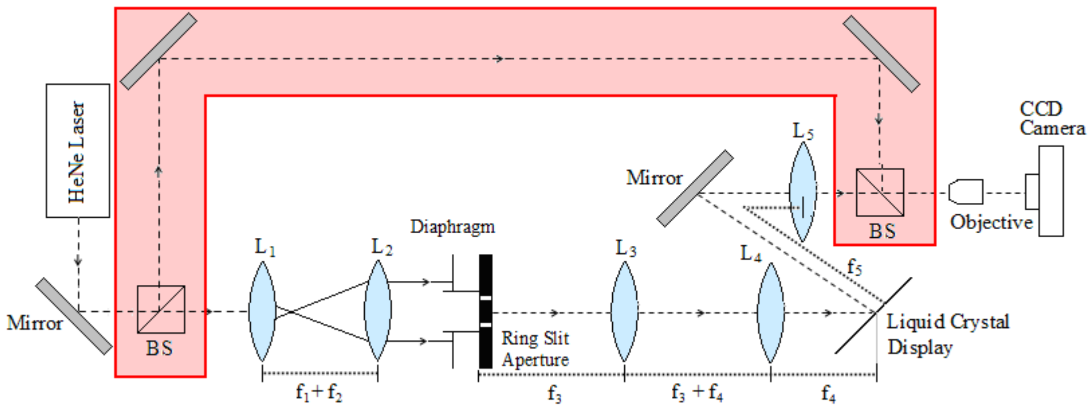

Fig. 2. The experimental design for generating a superposition of two higher-order Bessel beams, as well as producing nondiffracting speckle fields. The interferometer used to interfere the field produced at the Fourier plane with a plane wave is denoted in red.

In this work the ring field produced by the ring slit aperture illuminates a computer generated hologram which consist of two separate parts (such as those represented in Fig. 3). The computer generated holograms are constructed such that the border between the two parts is situated in the centre of the width of the incident ring field. The field produced is then a superposition of two higher-order Bessel beams. The order of the two Bessel beams is denoted by the number of times the phase varies from zero to 2π in the inner and outer parts of the ring field.

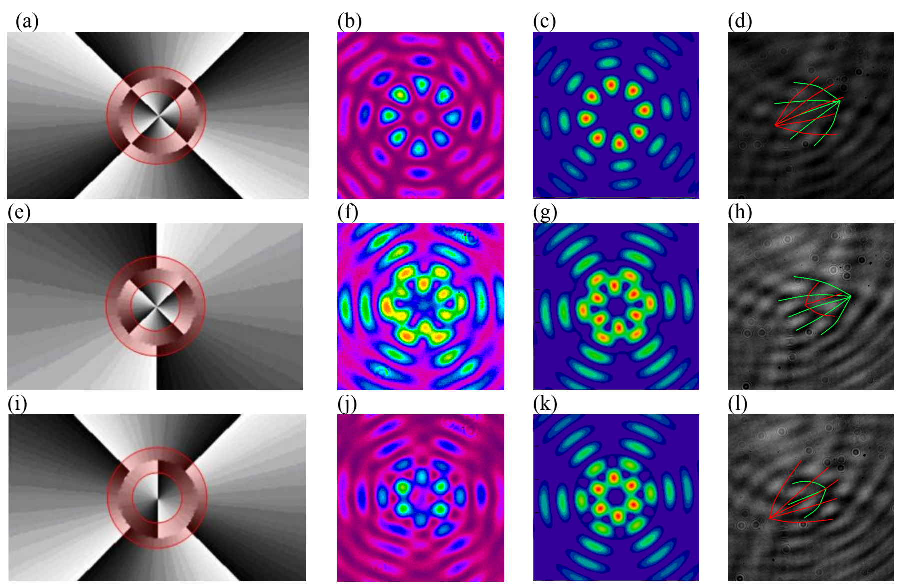

Fig. 3. The columns from left to right represent the computer generated hologram applied to the liquid crystal display of the SLM, the observed intensity distribution of the superposition, the theoretical prediction, and interference pattern of the superposition field and a plane wave, respectively. Data is shown for (a) – (d): $A_{4,-4}$, (e) – (h): $A_{2,-4}$, and (i) – (l): $A_{4,-2}$. The illuminated ring slit is shown as a red overlay on the computer generated hologram.

Due to the azimuthally varying phase which is imparted to the angular spectrum of the beam, the resulting fields possess vortices. The vortex structure of these fields is investigated by interfering these fields with a plane wave. It is known that fork-like patterns can be observed when a wavefront containing a screw dislocation is interfered with a plane wave.

Interfering these fields with a plane wave produces an interference pattern containing two overlapping forks, illustrating the presence of two higher-order Bessel fields. The number of dislocations or spaces between the 'fork-prongs' conveys the order of the Bessel fields involved in the superposition.

Similarly, computer generated holograms can be constructed such that three separate azimuthal phase components fit within the ring slit so as to produce a superposition of three higher-order Bessel beams. This was attempted and the experimental and theoretical results are depicted in Fig. 4. However, superpositions of more than three higher-order Bessel beams were not attempted due to the restricting dimensions of the ring slit aperture and the dimensions of the liquid crystal pixels.

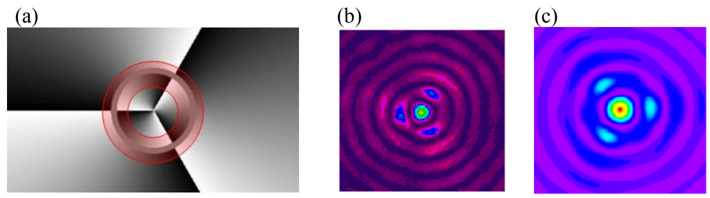

Fig. 4. (a): Computer generated hologram applied to the liquid crystal display. The red ring denotes the section of the hologram which is illuminated by the ring slit. (b): Experimental beam cross-section of the field produced at the Fourier plane. (c): It is in good agreement with the calculated field at the Fourier plane.

Apart from investigating the field produced at the Fourier plane, the propagation of these fields was also investigated. In this work we make use of the correlation function to evaluate the length of these nondiffracting fields. This involves determining the intensity correlation coefficient between pairs of experimentally recorded images as a function of the propagation distance.

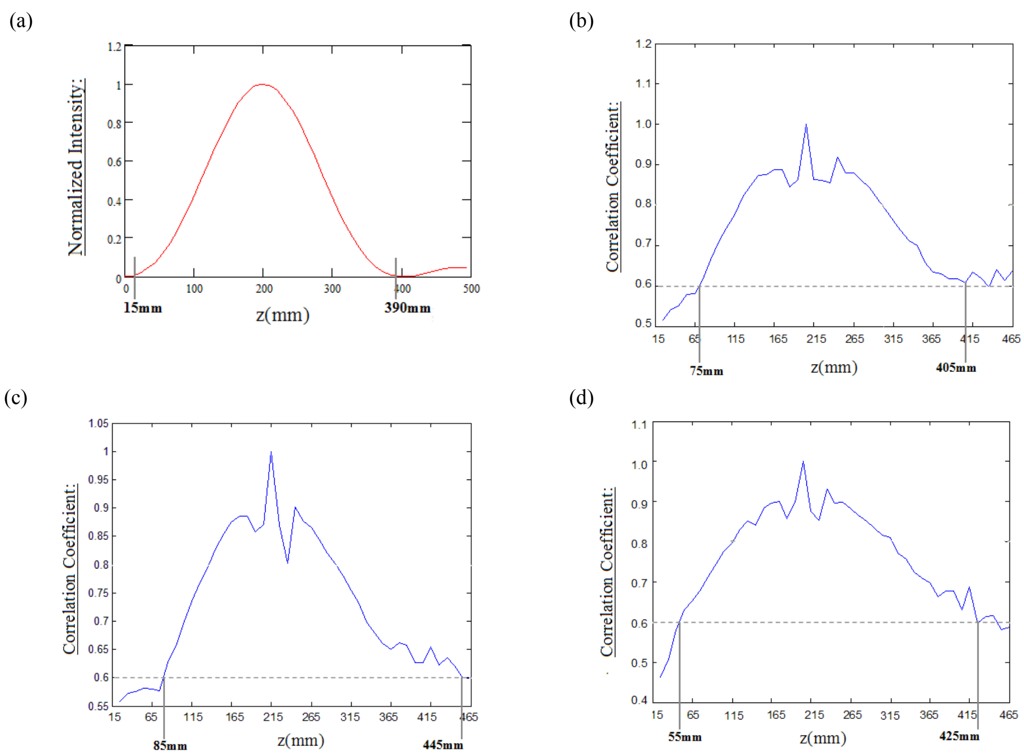

Fig. 5. (a): The theoretically calculated propagation distance. (b) - (d): The experimentally measured propagation distances for $A_{3,-3}$, $A_{4,-4}$ and $A_{2,-4}$ respectively.

Field:	Theoretical Propagation Distance:	Experimental Propagation Distance:
$A_{3,-3}$	375mm	330mm ± 10mm
$A_{4,-4}$	375mm	360mm ± 10mm
$A_{2,-4}$	375mm	370mm ± 10mm

Table. 1. The experimentally measured propagation distances for the fields: $A_{3,-3}$, $A_{4,-4}$ and $A_{2,-4}$ compared with the theoretically calculated propagation distance.

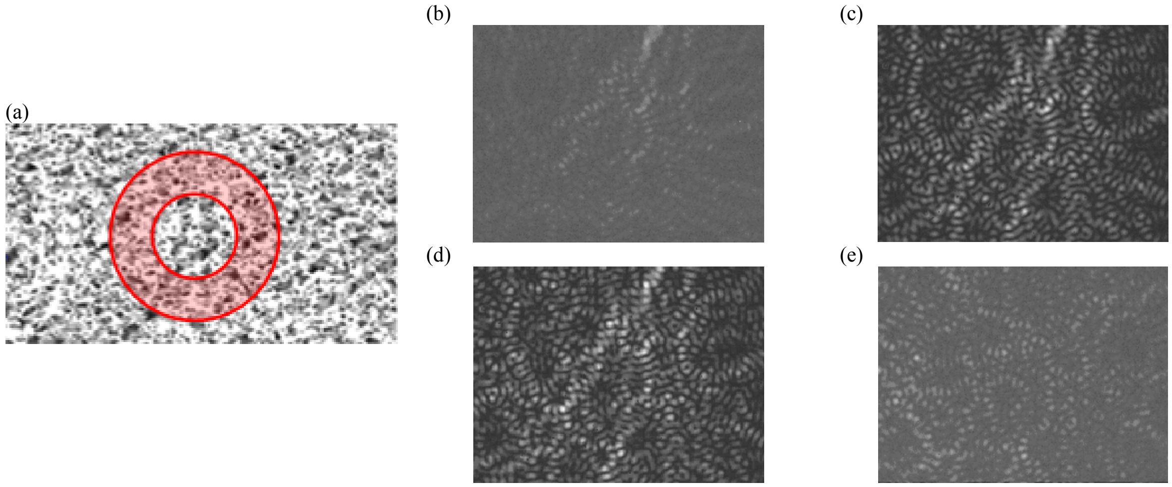

Fig. 6. (a): The random computer generated hologram applied to the liquid crystal display of the SLM. The illuminated ring slit is shown as a red overlay on the hologram. (b) – (e): The intensity profile of the experimentally produced nondiffracting speckle field at intervals along its propagation.

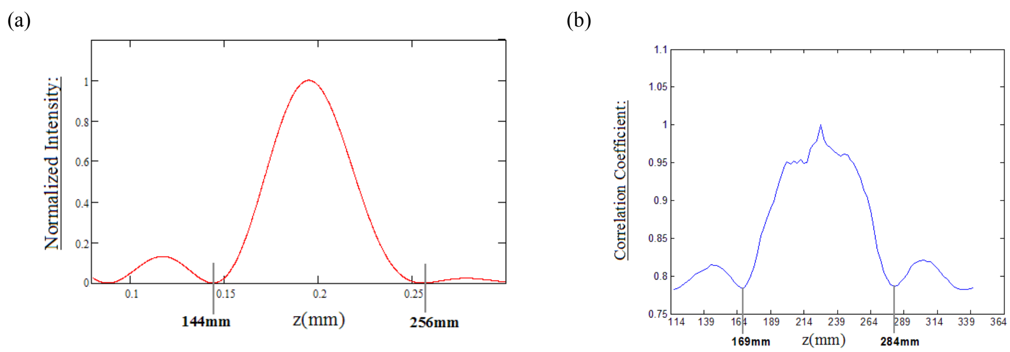

Fig. 7. (a): The theoretically calculated propagation distance. (b): The experimentally measured propagation distance for the nondiffracting speckle field.

Field:	Theoretical Propagation Distance:	Experimental Propagation Distance:
Nondiffracting speckle field	112mm	115mm ± 5mm

Table. 2. The experimentally measured propagation distance for the nondiffracting speckle field compared with the theoretically calculated propagation distance.

4. CONCLUSION

In this paper we have presented a technique to experimentally realize the superposition of two higher-order Bessel beams as well as generate nondiffracting speckle fields. The propagation of these fields was investigated, illustrating that these fields are propagation-invariant in accordance with the predicted result. The wavefront dislocations present in the higher-order Bessel beams were revealed by interferometric means. By interfering these fields with a plane wave, the fork-like patterns observed in the interference pattern reveal the existence of vortexes in these fields. The fields generated by the superposition of higher-order Bessel beams can be used to understand modes present in certain types of resonators. In optical tweezing, beams which carry orbital angular momentum are used to rotate trapped particles. In the case of generating a superposition of two higher-order Bessel beams which possess equal orders but of differing sign, the produced field carries no orbital angular momentum. However, these beams are still able to trap a particle in its intensity distribution and cause it to rotate over a spiral path along the beam's optical axis.

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