

Analysis of Route Availability in Inter-working Multi-hop Wireless Networks.

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Abstract

In order to link a source-destination node pair in inter-working multi-hop wireless networks, links or routes must first be available. It is only after establishing the availability of links and routes between nodes that factors which affect connectivity e.g. interference can be considered. Connectivity in multi-hop wireless networks has been studied. However, the studies focused on network connectivity in ad-hoc networks. Since the next generation of wireless networks will be inter-working, an understanding of connectivity as it applies to such networks is needed. Specifically, this paper emphasizes that an analysis of route connectivity rather than network connectivity is needed for inter-working multi-hop wireless networks. With a focus on route connectivity, a route availability model for inter-working multi-hop wireless networks is presented.

1. Introduction

Connectivity is a fundamental property of any network. Normally, in all networks, links are the basic elements that ensure connectivity. In wired networks, links are provided by communication cables and these links are stable and predictable to a large extent. On the other hand, in wireless networks, links are provided by the air interface (wireless channel).

Generally, in wireless networks, nodes have to be within an appreciable distance of each other before a communication link can be established between them. Any node that is not within the recommended range is said to be out of the network. In single hop wireless networks, it is sufficient for each node to be within the transmission range of at least one of the centralized base stations in order to communicate with another node. For multi-hop wireless networks, if source-destination pairs are not within each other's transmission range, packets reach their destination nodes after some hops on nodes in between the source and destination. One of the advantages of multi-hop communications is that it ensures efficient spatial re-

use. In multi-hop wireless networks, the choice of the next hop depends on whether the node on this hop is able to link up to another node in the communication path en-route to the destination node. Most importantly, an available link must also be reliable for a good quality communication to be established between node pairs. One major characteristic of the wireless channel that affects the quality of communication is the variation in its strength over time and frequency. As a result of the variation, communication links in wireless networks tend to be unpredictable. Moreover, this variation affects the connectivity between two communicating nodes.

Another factor that affects connectivity between two communicating nodes is mobility. Since mobility may cause connected radio links to be disconnected, a critical issue is for nodes in the network to be able to communicate on links that can be sustained throughout the packet transmission duration. Therefore, the links between nodes have to be able to ensure connectivity.

The developments of the theory of connectivity for wireless networks have been done in research works such as [1-7]. However, most of the theoretical and analytical investigations have been developed around with ad-hoc sensor networks. This research studies the theory of connectivity in inter-working multi-hop wireless networks. Since the existence of a link or route between node pairs is essential for connectivity, the contribution of this paper is to provide an analysis of route availability between source-destination nodes in inter-working multi-hop wireless networks.

Availability is the measure of the amount of nodes that are reachable by a node. It is also the probability that a link or route exist between any two nodes. Ultimately, it is determined by the probability that at least a node exists within a certain distance range from a particular node. In a wireless network, the availability of a link or route between node pairs depends on the distance between them, their transmission range and the network's node density. Link and route availability are probabilistic factors since the wireless network is stochastic in nature. The probability that a route is

available between source-destination pair depends on the probability that intermediate nodes en-route to the destination have a link to another node closer to the destination node. Link availability is the probability that two nodes are within at most the maximum transmission range that is sufficient for a communication link to be established between them. Route availability is the probability that the adequate number of links that will form the communication path between a source-destination pair exists. In particular, this paper presents an analysis of the inter-dependency that exists between link and route availability in inter-working wireless multi-hop networks. Such analysis is needed to determine the availability of a route for packet transmission. To avoid ambiguity, a link refers to the connection between any node pair in the network, while a route refers to the last mile connection path between a source and destination pair.

For this analysis, the fundamental models that are needed to represent the inter-working multi-hop wireless networks are: 1) A model for the spatial distribution of nodes: *Nodes are independently located and the average density of the nodes is uniform throughout the network.* 2) A model for the link distance between nodes: *The model gives the probability that a node has a link to other nodes in the network. If the maximum transmission range of any node is R , then an independent communication link is available for any two nodes separated by a distance less than or equal to R . If β is the distance between two nodes, a link is available between them as long as $\beta \leq R$. Note that β refers to the distance between specific node pairs. The distance may be a single hop distance between and it may be the multi-hop distance between any source-destination pair.*

The outline of this paper is as follows. In section 2, the network model, the node distribution model and the node degree model are described. An analysis of the link models is given in Section 3. Section 4 presents the route availability model and section 5 concludes the paper.

2. Network Model

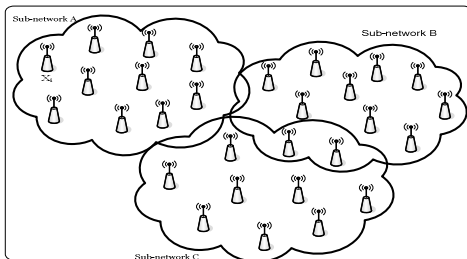


Figure 1. Network Ω

The network in fig 1 represents a set of inter-working wireless networks. Network Ω contain three subset networks (sub-networks) A, B, C. The total number of nodes in Ω is denoted N_{Ω} , while the number of nodes in each of these sub-networks A, B, C are N_a , N_b and N_c respectively, where $N_a + N_b + N_c = N_{\Omega}$ and $N_a = N_b = N_c = N$ (i.e. the sub-networks contain the same number of nodes). All nodes have the same transmission capability and packets are transmitted from the source node towards the destination node via a multi-hop path.

2.1. Node Distribution Model

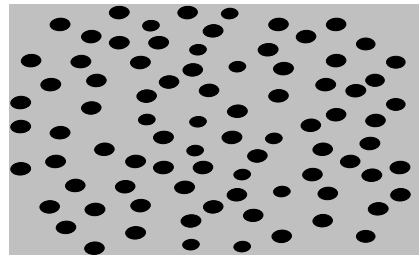


Figure 2. Spatial Point Pattern.

Consider each sub-network in fig.1 as a collection of random nodes whose realization is called a spatial point pattern as shown in fig.2. These nodes are contained in a Euclidean space of 2-dimensions (R^2), and their positions in the network are independent of each other. The lack of dependence between these nodes is called *complete spatial randomness (csr)* [8]. From theory, these nodes can be said to form a realization of a Planar Homogeneous Poisson Point Process. With regards to the analysis of spatial point pattern, the distribution theory of the nodes under complete spatial randomness is well known under the theory of the Nearest Neighbor Distance (NND). These theories are used to analyze point patterns in biological sciences and are also applicable to wireless networks [4]. Note that the distance between a node and any of the nodes nearest to it is the nearest neighbor distance.

In any of the sub-networks in fig. 1, nodes are independently and randomly placed on this 2-dimensional space with area A . For each sub-network, the node density $\rho = N/A$ (number of nodes per unit area). These nodes are distributed uniformly within the area A of each sub-network. The maximum transmission range of each node is R and the Euclidean distance between any two nodes, X_i and X_j ($\forall i, j \in \mathbb{Z}$ and $i \neq j$) in the network is represented by $d(X_i, X_j)$. In a multi-hop wireless network, two nodes are able to communicate with each other if $d(X_i, X_j) \leq R$ [2].

2.1.1. Node Degree.

The degree of a node in wireless multi-hop networks is defined as the number of neighbor nodes that it has [9]. A node is said to be a neighbor node to another node if the distance between the two nodes is less than or equal to their transmission range, which means that both nodes have a direct link to each other. Therefore, a node's degree is the number of nodes within its transmission range.

The node degree of a node X_i is denoted by $D(X_i)$. In an instance where for a node, $D(.)=0$, the node is termed a "lone node". The existence of a "lone node" in a multi-hop wireless network is an undesirable condition. Although a lone node maybe useless in terms of connectivity in a static multi-hop wireless network, yet in a mobile scenario, it becomes useful as it moves into the transmission range of another node or when another node moves into the node's transmission range. The desirable condition for connectivity in a multi-hop wireless network is for all nodes to have $D(.) > 0$. The probability that $D(.) > 0$ for any node pair is the same as the probability that a link is available for the node and it is given by equation 1. R is the transmission range of the node and $f(x)$ is the probability density function of the distance between any two nodes.

$$P(D(.) > 0) = P(\text{link availability}) = \int_0^R f(x) dx \quad (1)$$

3. Link Models

3.1. Link Distance Distribution Model

In multi-hop wireless networks, the probability that a multi-hop communication path is available is related to the availability of the individual links that make up the path. Therefore, it is important to analyze the distribution of the link distances between nodes in multi-hop wireless networks [10]. Let β denote the NND of a chosen node. For any two nodes, X_i and X_j , $\beta = d(X_i, X_j) \forall i, j \in \mathbb{Z}^+, i \neq j$. With theorem 1 stated below, the probability that $\beta > R$ can be evaluated.

Theorem 1: For a Homogeneous Poisson Point Process in \mathcal{R}^2 , the probability that there are no points within a distance y of an arbitrary point (p) is $e^{-\lambda\pi y^2}$, where the parameter λ is the expected number of points per unit area [8, pg 636].

The above theorem applies to any of the sub-networks in fig. 1 in the following ways: 1) For an arbitrary node, the probability that there are no nodes within a distance $\beta \leq R$, (probability that a node has no neighbor) is:

$$P(D(.) = 0) = P(\beta > R) = e^{-\rho\pi R^2} \text{ for } R > 0 \quad (2)$$

2) Also, for an arbitrary node, the probability that the distance between a randomly chosen node and its nearest neighbor node is less than the node's transmission range R (the probability that a node has at least one neighbor) is:

$$P(D(.) > 0) = 1 - e^{-\rho\pi R^2} \quad \forall R > 0 \quad (3)$$

$$P(\beta \leq R) = \begin{cases} 1 - e^{-\rho\pi R^2} & \forall R > 0, \\ zow & \end{cases} \quad (4)$$

Equations 3 and 4 only hold as long as $\beta \leq R$. Equation 4 represents the cumulative distribution function (CDF); ($F_\beta(R)$) of the distance between any two randomly positioned nodes in any of the sub-networks in fig. 1. It also represents the probability that a link exists. Assuming that links become non-existent independently, this quantity can be taken as the probability that a link exists in a binomial trial. If the trial is repeated N times, then an estimate of the number of existing links for any node is given by $N \times F_\beta(R)$ [12].

3.2. Link Availability model

As long as $\beta \leq R$, a link is available (exists) between any two arbitrary nodes [11] [12]. Therefore, the CDF ($F_\beta(R)$) of the link distance β can be taken as the probability that at least a link is available for transmission. Thus, the availability of a link in a network is a function of R , β and the number of nodes in the network. Let P_{link} represent the availability of a 1-hop link for any node as expressed in equation 5. A node becomes a lone node (no link is available) once $\beta > R$. It is assumed that links in the network are identical.

$$\therefore P_{\text{link}} = \begin{cases} 1 - e^{-\rho\pi R^2} & \text{for } 0 < \beta \leq R \\ 0 & \text{for } \beta > R \end{cases} \quad (5)$$

Fig. 3 gives a plot of the availability of a link as the value R takes on increases. A network scenario in which $N=20$ nodes in a 10 square unit area has been considered. At $R=0.2$, only 22.2% of the total nodes are available for a 1-hop link to any node and 99.8% of nodes are available if $R=1$. All (100%) of the links are available once $R>1$, which means that every node has a link to all other nodes in the network. This phenomenon indicates that the network is fully connected. For a network with N nodes in area A , as R increases, P_{link} increases. From Poisson distribution, equation 3 is analogous to equation 6, which is the probability of > 0 nodes in the radio area πR^2 , for any value of R . The probability that the degree of a node is equal to n is expressed in 6.

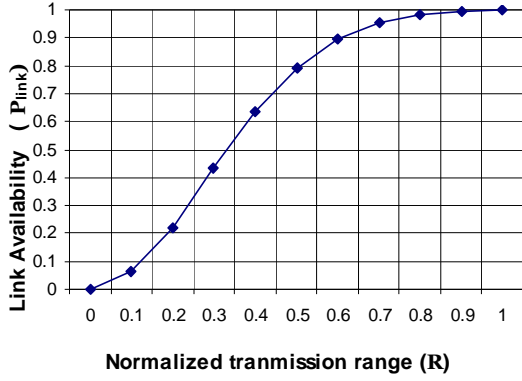


Figure 3. Link Availability vs Normalized transmission range

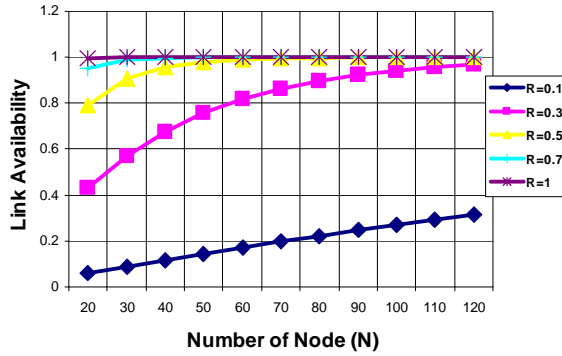


Figure 4. Plink vs Number of Nodes (N) for different values of R

$$P(D(\cdot) = n) = \frac{(\rho\pi R^2)^n}{n!} e^{-\rho\pi R^2} \text{ for } R > 0 \quad (6)$$

$$P(D(\cdot) > 0) = \sum_{n=1}^{N-1} \frac{(\rho\pi R^2)^n}{n!} e^{-\rho\pi R^2} \text{ for } R > 0 \quad (7)$$

Therefore, the number of available 1-hop link for any node, given its transmission radius can be expressed as $P_{link}(N-1)$ for N nodes in the network. Note that a maximum of $N-1$ links are potentially available to all node in a network of N nodes. In figure 3, it can be observed that the CDF of β , ($F_{\beta}(R)$), is a monotonically increasing function. Consequently, in a network with area A and N nodes, P_{link} increases as R increases.

Fig. 4 gives a plot of P_{link} at fixed transmission range (R) as the number of nodes in the network increases. Also, $A=10$ square units, and N was increased from 20 nodes to 120 nodes at fixed node transmission values of 0.1, 0.3, 0.5, 0.7 and 1. From figure 4, generally for all the cases considered, P_{link} increases as N increases; indicating that in multi-hop wireless networks, the probability of having an available link is higher in a

dense network. For high values of R , the P_{link} is at very high values for large N in the network. If R is the same for all nodes, then the upper bound for P_{link} is:

$$P_{link-upper} = 1 - e^{-\rho\pi R^2} \text{ for } R > 0 \quad (8)$$

The next section deals with the analysis of route availability.

4. Route Availability Model

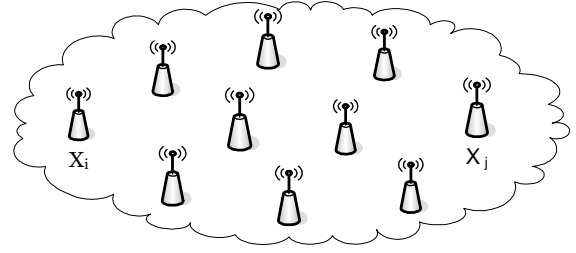


Figure 5. A Subnetwork

If the distance (β) between a source and destination is greater than R , then $P_{link}=0$, therefore a multi-link (multi-hop) route has to be utilized for packet transmission. In this case, multiple hop routes in the direction of the destination node are used. [13] explains the different methods, which can be used to achieve this. To ensure end to end route availability, each intermediate node on the route must have at least two neighbour nodes. These two neighbours are for the purpose of packet reception from the preceding node and packet transmission to the subsequent node.

Let l represent the link (or hops) between any two nodes in the network, where $l \in L$ and L is the set of all links that exits in the network. If a transmitted packet from a node have to hop on a total of l links to arrive at the destination node, then, $l-1$ intermediate nodes will be required on this route. The number of hops depends on β , and the transmission range (R) of the source node and the intermediate nodes. Note that in this paper, the transmission ranges of all nodes in the network are equal. The minimum number of links (hops) that can connect any two nodes together is:

$$l = \left\lceil \frac{\beta}{R} \right\rceil \quad (9)$$

$\lceil x \rceil$ represents the greatest integer that is greater than x .

However, a bound for l exists in every network. The bound occurs when β happens to be equal to the maximum distance (β_{max}) that can be between any two nodes in the network. In this case, the maximum number of hops $l_{max} = \beta_{max}/R$ cannot be exceeded.

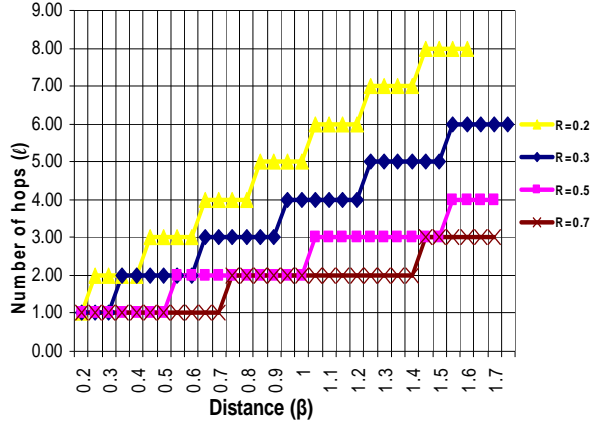


Figure 6. Number of hops (l) vs distance between nodes.

Fig. 6 shows a plot of the number of hops versus the distance between a source-destination node pair. As in section 3.2, a 20 node network with an area of 10 square units has been considered. The value of β was increased from 0.2 units to 1.8 units, while the number of hops was observed for constant transmission range ($R=0.2, 0.3, 0.6$ and 0.7). Figure 6 confirms that the longer the distance between node pairs, relative to their transmission range, the more the number of hops (links) that will be utilized to transmit a packet from source to destination. Equation 10 summarizes the minimum number of hops (l_{\min}) for different values of β , and a general expression for evaluating l_{\min} is obtained.

$$l_{\min} = \begin{cases} 1, \forall 0 < \beta \leq R \\ 2, \forall R < \beta \leq 2R \\ 3, \forall 2R < \beta \leq 3R \\ \dots\dots\dots \\ l, \forall (l-1)R < \beta \leq lR \end{cases} \quad (10)$$

Consider the sub-network in fig. 5 with N nodes. If a route is to be established between X_i and X_j , where X_j is the destination, there will be $N-2$ intermediate nodes between X_i and X_j . Depending on β and R , the maximum number of hops that can be used to transmit packets from X_i to X_j is $N-1$ and the minimum number of hop is 1. To establish a route with a definite number of hops (e.g. l hops), R has to be at a certain maximum value as illustrated in fig.6. If R is lower than the maximum value, more hops will be utilized to set up such a route. In order to evaluate the probability that any source-destination node pair is linked by a certain number of hops, the following must be fulfilled:

1) The distance between the source-destination pair must fulfil the general expression for l_{\min} in equation 8.

2) There should be at least a node between the distance $(l-1)R$ and lR and;

3) Every node along the route should have at least a neighbour node that is within the transmission range of another node 2hops away from it.

The condition in (2) is such that nodes must exist between distance $(l-1)R$ and lR and the probability of this happening is expressed in equation 11:

$$\sum_{n=1}^{N-1} \frac{(\rho\pi(lR)^2)^n}{n!} e^{-\rho\pi(lR)^2} - \sum_{n=1}^{N-1} \frac{(\rho\pi((l-1)R)^2)^n}{n!} e^{-\rho\pi((l-1)R)^2} \quad (11)$$

For condition (3), let A_{int} be the area of intersection of any two nodes along the route, which are 2-hops away from each other. From [14],

$$A_{\text{int}} = R^2 \left[2 \cos^{-1} \left(\frac{\beta}{2R} \right) - \sin \left(2 \cos^{-1} \left(\frac{\beta}{2R} \right) \right) \right] \quad (12)$$

The probability of at least 1 node in area A_{int} , where n_{int} is number of nodes in area (A_{int}) is:

$$P(n_{\text{int}} > 0) = 1 - e^{-\rho A_{\text{int}}} \quad \forall A_{\text{int}} > 0 \quad (13)$$

For an l -hop route, equation 11 is expressed as:

$$P(n_{\text{int}} > 0) = (1 - e^{-\rho A_{\text{int}}})^{l-1} \quad \forall A_{\text{int}} > 0 \quad (14)$$

Finally, the probability of an l -hop route, ($P_{l\text{-hop}}$) between X_i and X_j is given by the multiplication of equation 11 and 14. However, as the network's node density increases, for constant R , equation 14 tends towards 1. From equation 3, 7 and 11, an asymptotic probability for an l -hop route can be evaluated with equation 13.

$$P_{l\text{-hop}} = \left(1 - e^{-\rho\pi(lR)^2} \right) - \left(1 - e^{-\rho\pi((l-1)R)^2} \right) \quad (15) \\ = e^{-\rho\pi((l-1)R)^2} - e^{-\rho\pi(lR)^2}$$

Using the network scenario in section 3b, $P_{l\text{-hop}}$ versus l is as shown in figure 7. The sum of $P_{l\text{-hop}}=1$. From the data in figure 7, the probability that $l > l_{\max}$ tends to zero. In case of node or link failures, an alternative detour needs to be available at any point in order to ensure end-to-end packet transmission. This alternative route may require more than the minimum number of hops or the same number of hops as l . So now, what is the probability that a source-destination pair will be connected irrespective of the number of hops from source to destination? Let P_r denote the probability that a route is available. Equation 16 gives the route availability for $\beta/R \leq l \leq l_{\max}$. P_r depends on the probability of establishing an l -hop route between any pair of source-destination node in the inter-working multi-hop wireless network. It also depends on β (distance between the source-destination nodes) and R (transmission range of nodes).

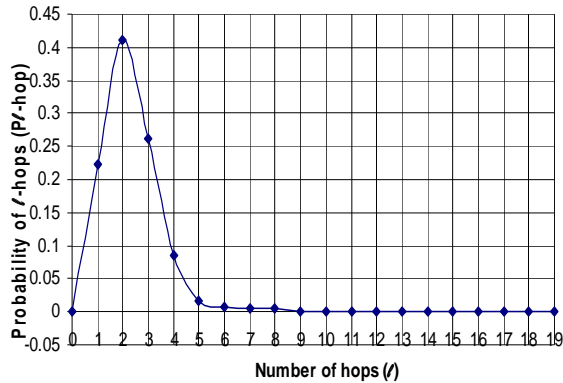


Figure 7. Probability of l-hops vs number of hops in a 20 node network

$$P_r = \sum_{l=\frac{\beta}{R}}^{l_{\max}} P_{l\text{-hop}} \quad (16)$$

5. Conclusion and Future Work

In this paper an analysis of the link and route availability in wireless multi-hop networks has been presented. The research work focuses on route connectivity in inter-working multi-hop wireless networks. In multi-hop wireless networks such as mobile ad-hoc networks, a network connectivity analysis is needed. However, in multi-hop inter-working wireless networks, an analysis of the route connectivity is more desirable.

For there to be connectivity between a source-destination node pair in an inter-working multi-hop wireless network, a route has to be available. Route availability is dependent on the availability of a link between the node pairs. A distance-dependent model of link availability has been assumed. However, this model does not accurately represent the stochastic nature of the wireless channel. For optimal resource dimensioning and quality of service in inter-working multi-hop wireless networks, the randomness in the wireless environment needs to be considered. The second part of this research work includes the physical layer factors that affect the availability of a link in the evaluation of P_{link} . Although, in mobile multi-hop network, β would be stochastic parameter, yet the channel model does not include the effect of attenuation, interference and fading on the wireless channel. In the second part of this research work, the parameters that induce randomness into the wireless channel are considered in the development of a link reliability model for inter-working multi-hop wireless networks.

6. References

- [1] Farhadi, G and Beaulieu, N.C, "On the Connectivity and Average Delay of Mobile Ad Hoc Networks," in Proc. IEEE International Conference on Communications, 2006. ICC, June 2006, pg 3868 – 3872.
- [2] C. Bettstetter "On the minimum node degree and connectivity of a wireless multi-hop network," in Proc. ACM MobiHoc, 2002, pp. 89-91.
- [3] P. Santi and D. M. Blough, "An evaluation of connectivity in mobile wireless ad hoc networks," in Proc. IEEE DSN, June 2002.
- [4] C. H. Foh and B. S. Lee, "A closed form network connectivity formula for one-dimensional MANETs," in Proc. IEEE ICC, June 2004.
- [5] C. H. Foh, G. Liu, B. S. Lee, B-C Seet, K-J Wong, and C.P. Fu, "Network Connectivity of One-Dimensional MANETs with Random Waypoint Movement," IEEE Communications Letters, Vol. 9, No. 1, Jan. 2005, pg. 31-33
- [6] J. Li, L.L.H. Andrew, C.H Foh, M. Zukerman and M.F.Neuts, "Meeting connectivity requirements in a wireless multihop network," IEEE Communications Letters, Vol. 10, Issue 1, Jan 2006, pg. 19 – 21.
- [7] C. Bettstetter and C. Hartmann, "Connectivity of wireless multihop networks in a shadow fading environment," Wireless Networks, Vol. 11, Springer, 2005, pg. 571-579.
- [8] N. A. C. Cressie, "Statistics for Spatial Data," John Wiley and Sons, 1991.
- [9] Q. Ling and Z. Tian, "Minimum Node Degree and K-connectivity of a Wireless Multi-hop Network in Bounded Area," in Proc. IEEE Globecom 2007, pg. 1296-1301.
- [10] Leonard E. Miller. "Distribution of link distances in a wireless network." Journal of Research of the National Institute of Standards and Technology, 2001, 106(2), pg 401-412,
- [11] L. E. Miller, "Joint Distribution of Link Distances," in Proc. Conference on Information Science and Systems, The John Hopkins University, March 12-14, 2003.
- [12] Z. Kong and E.M.Yeh, "Connectivity and latency in Large-Scale Wireless Networks with Unreliable Links." in Proc. IEEE INFOCOM 2008, April 13th -18th, Phoenix, Arizona, pg. 394-402.
- [13] A. Boukerche, "Handbook of algorithms for wireless networking and mobile computing." CRC Press, 2006, ISBN 1584884657, 9781584884651.
- [14] L. E. Miller, "Probability of a Two Hop Connection in a Random Mobile Network," in Proc. Conference on Information Science and Systems, The John Hopkins University, March 21-23, 2001.