On Grouping Individual Wire Segments into Equivalent Wires or Chains, and Introduction of Multiple Domain Basis Functions

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Abstract

The paper introduces a method to cover several wire segments with a single basis function, describes related practical algorithms, and gives some results. The process involves three steps: identifying chains of wire segments, splitting the chains into shorter sub-chains, and assigning basis functions to the resulting geometrical structures. The method may improve efficiency of computations when modeling complex curved and quasi-static structures. In addition, the approach extends the boundaries of applicability for a thin wire kernel. An example is considered comparing three different splitting algorithms.

Introduction

The method reported in this paper has been realized in the framework of the method of moments (MoM) [1] under the thin wire approximation [2]. The method is intended as an extension to the already existing implementations of MoM. It is assumed that an existing MoM implementation uses piecewise linear geometrical elements and sub-domain or large domain basis functions [1].

In a traditional formulation, usage of sub-domain or even large domain basis functions links the number of unknowns to the number of wire segments. This hard link leads to inefficiencies when modeling structures where the complexity of the current distribution is much lower than that of the geometry, e.g. curved structures, such as helical antennas. Additional complications may arise due to usage of overly short wire segments, especially in conjunction with a delta gap source [1]. A short wire segment, whose length is not much larger than its radius, dissatisfies the thin wire approximation. A delta gap source applied at a junction of wires of finite cross-section is prone to producing incorrect input impedance. This worsens when the length of the adjacent wire segments is not long. The proposed technique combines/groups wire segments that are connected in series, thereby uniting them into *chains*, where a chain defines a new, composite domain. A new basis function may then be defined on the composite domain. This basis function is thus referred to as *multiple domain basis function* (MDBF).

The advantage of the technique is in a higher efficiency of modeling curved structures, and structures with small features. The reduction in the memory usage is proportional to the square of the reduction in the number of unknowns. It may also be noted that the highest degree of efficiency is achievable with higher order basis functions. This topic was however left out of this paper and some discussions may be found in [3].

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The papers [4], [5] have considered application of a similar method using rooftop basis and pulse testing functions. The work reported in this paper is based on a more robust Galerkin procedure using rooftop basis functions for both expansion and testing. Several practical, fully automatic, algorithms for splitting the chains into sub-chains have been developed and tested, and are reported on in this paper. In addition, this paper reports on the finding that an arbitrary suboptimal splitting of a chain may lead to an increased condition number of the impedance matrix, and also to less accurate results.

Background – Basic Steps of MoM

The method of moments (MoM) approximates a set of integral equations with a system of linear algebraic equations. One of the most popular forms of the MoM, described, for instance, in [1], [2], includes the following steps. In the first step, the geometrical structure is meshed into elementary geometrical elements. In this paper, thin wire segments are used. The maximum and minimum length of a mesh segment is determined by the ability to model the unknown using given approximating functions and constraints of the thin wire approximation. In the next step, the current on the geometrical elements, the unknown, is described as a series of known functions (referred to as basis functions) with yet unknown constant coefficients I_j. A vector of these unknown coefficients, $\mathbf{I}$, is the unknown in the system of linear algebraic equations. This paper uses one of the most popular basis functions – piecewise linear (PWL) functions, often referred to as the rooftop basis functions.

The traditional MoM considers electromagnetic coupling between individual geometrical elements (here, wire segments) through the basis functions assigned to these elements. The couplings constitute the matrix elements Z_{ij} of the so-called *impedance matrix*, $\mathbf{Z}$.
The product of the impedance matrix and the unknown coefficients vector, $\mathbf{Z}\cdot\mathbf{I}$, equals the excitation vector, $\mathbf{V}$: $\mathbf{Z}\cdot\mathbf{I} = \mathbf{V}$.

Core of the Method – Combining / Grouping Domains of Basis Functions

It is assumed that it is possible to express the relationship between a longer vector of original (old) unknowns $\mathbf{I}$ and the shorter vector with new unknowns $\tilde{\mathbf{I}}$ in a matrix form, as $\mathbf{I} = \mathbf{M}\tilde{\mathbf{I}}$. Herein, $\mathbf{M}$ denotes a matrix grouping/aggregating basis functions. Each row of this matrix contains weights defining which new basis functions are involved in the formation of the old basis function, and with what weight.
The expression relating the old unknowns to the new ones may be substituted into the original system of linear equations $\mathbf{ZI} = \mathbf{V}$. The resultant system $\mathbf{ZM}\tilde{\mathbf{I}} = \mathbf{V}$ is then left-multiplied by the transposed transformation matrix $\mathbf{M}$ to obtain the new system of linear equations: $\underbrace{\mathbf{M}^T\mathbf{ZM}}_{\tilde{\mathbf{Z}}}\tilde{\mathbf{I}} = \underbrace{\mathbf{M}^T\mathbf{V}}_{\tilde{\mathbf{V}}}$. This system may be rewritten in a short form as $\tilde{\mathbf{Z}}\tilde{\mathbf{I}} = \tilde{\mathbf{V}}$.
Once this new system is solved and the new unknowns $\tilde{\mathbf{I}}$ obtained, the original unknowns may be computed from $\mathbf{I} = \mathbf{M}\tilde{\mathbf{I}}$.

Practical Aspect - On Splitting the Chains into Shorter Sub-Chains

When several electrically short wire segments are considered as a single equivalent wire (chain), the total length of this single equivalent wire may be prohibitively large for the new basis function to be able to describe the current variation accurately. In addition,

some integration subroutines may have limitations on the maximum supported electrical length of the integration interval, subject to the required accuracy. Under such circumstances, it becomes crucial to split the chain into shorter sub-chains of supported length. This function is equivalent to applying a finer mesh onto an existing geometrical element.

It is possible to split the single equivalent wire into arbitrary sub-segments. Such flexibility may be desired in order to achieve the best possible accuracy and/or efficiency. However, the inter-relation between the original and new impedance matrices and excitation vectors becomes complex, and complicates practical implementations.

On the other hand, it is usually possible to keep the elements of a chain intact and still be able to split the chain into sub-chains. For a chain of N wire segments, it is possible to find a split that is *optimal* in some sense, within 2^{N-1} iterations. The word optimal may refer to, for example, attempting to keep the distribution of lengths of the sub-chains uniform, or maximizing the minimal length of a sub-chain. However, the total number of required iterations for a complex structure with many chains may quickly become prohibitively large, making the method impractical. It was necessary to find an algorithm which could find a split close to the optimal split within less time, e.g. within N iterations.

Three versions of the chain-splitting algorithm have been developed. The source codes of the Matlab implementations have been excluded due to space constraints but may be found in [3]. The first, initial version, denoted with the letter *A*, was designed to be simple, yet to minimize the number of sub-chains by maximizing the length of each sub-chain *individually*. The maximum permitted electrical length of a sub-chain was a parameter. It was found that this algorithm may sometimes result in an undesirable end sub-chain which contains only one short wire segment. The presence of a very short segment was found to have a negative effect on the condition number of the impedance matrix and on the accuracy of the solution. This effect is due to an excessively high ratio of the longest to the shortest chain, and is in line with theoretical considerations from elsewhere [6].

The next version of the chain-splitting algorithm, denoted with the letter *B*, was designed to consider the wire segments in pairs rather than individually. It was found that this reduced the probability of having an excessive condition number and also softened results for the worst-case scenarios.

The version *C* of the algorithm attempted to produce sub-chains of maximally equal length. This algorithm has demonstrated the best results (lower condition number and better accuracy), as compared to the two previous algorithms. This will be shown in the next section.

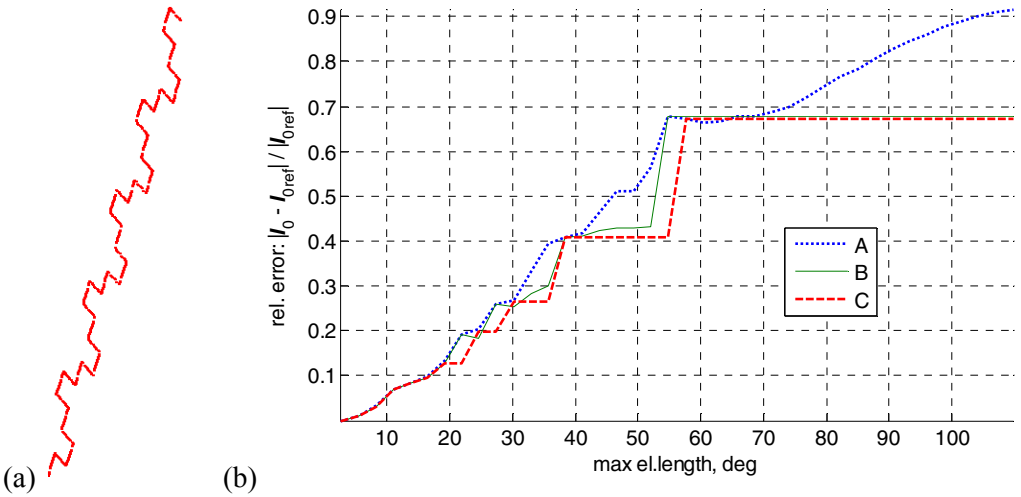
It was also observed that the splits produced by the algorithms *B* and *C* matched some of the optimal results obtained by iterating through all possible combinations of splitting the chain into sub-chains.

Example – Meander Monopole

The meander monopole shown in Fig. 1a is composed of 1 m long, straight wire segments, of radius 1 mm, and with total height 20 m. The ratio of wire segment length to radius was set sufficiently large to satisfy the thin wire approximation conditions well.

This aims to ensure validity of the numerically obtained results, even under the finest meshing of the chain into sub-chains.

The plot shown in Fig. 1b shows a relative error in current at the driving point versus the maximum permitted electrical length of a sub-chain, calculated at the first resonance frequency, 2.28485 MHz. The plot illustrates two aspects: (i) the expected monotonic growth in error with an increase in roughness of the equivalent mesh (expressed via the maximum permitted electrical length of a sub-chain), and (ii) the advantages of the chain splitting algorithm *C* compared to the algorithms *A* and *B*.



(a) (b)

Fig. 1. Geometry of meander monopole (a), and calculated results (b).

Concluding Remarks

A method for grouping sub- and large domain basis functions into multiple domain basis functions was developed, implemented and has demonstrated results applicable in practice. Although the current realization is limited to chains of wire segments, the method may be readily extended onto quadrilaterals. Also, the method may be readily extended on arbitrary shapes of basis functions, whereby the chosen profile is linearly interpolated between the nodes of the respective chain.

References

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