Using Piecewise Sinusoidal Basis Functions as Multiple Domain Basis Functions

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Abstract

This paper discusses application of the piecewise sinusoidal (PWS) basis function (BF) over a chain of several wire segments, i.e. as a multiple domain basis functions. The usage of PWS BF is compared to results based on the piecewise linear (PWL) basis functions. An example of meander monopole demonstrates the advantages associated with the PWS BF.

Introduction

The paper reports on an approach where piecewise sinusoidal (PWS) basis functions are continuously applied over several wire segments connected in series. The novelty of this approach, as compared to classical works like [1], is in the ability to define the number of PWS BFs independently from a fixed number of the wire segments. In addition, compared to the work [2], more sophisticated PWS BFs are used instead of piecewise linear (PWL) BFs. This enhanced approach leads to a substantial gain in the efficiency and accuracy of modeling. Both the complexity of implementation and computational complexity for PWS BF measured against PWL or any other shape BFs, remain exactly the same.

The approach is based on the concept of multiple domain basis functions (MDBF) reported in [2], where an arbitrary-shaped basis function is permitted to be piecewise-linearly interpolated over a chain. The term *chain* refers to an arbitrary number of wire segments connected in series, so that no more than two wires are connected at any given point. In principle, the method does not have any restrictions in defining a basis function over any arbitrary situated set of wire segments or quadrilaterals.

As discussed in [2], the MDBFs permit decoupling the number of unknowns from the number of wire segments. As such, they offer a higher efficiency for modeling curved structures, and structures with small features than a traditional method of moments (MoM). The memory usage is decreased in proportion to the square of the reduction in the number of unknowns. It may also be noted that the highest degree of efficiency is achievable with higher order basis functions. This topic is outside of the scope of this paper and is discussed in [3].

The approach discussed in this paper involves the MDBFs within the Galerkin procedure of the method of moments (MoM) [4]. This enhances robustness compared to the procedure derived in [5], where rooftop and pulse functions were used for both expansion and testing. Like in [2], this paper takes advantage of the thin wire approximation [1].

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It is assumed that the MoM procedure results in the linear algebraic equations $\mathbf{Z}\cdot\mathbf{I}=\mathbf{V}$, where $\mathbf{Z}$ is the impedance matrix [1], [4], $\mathbf{I}$ is the column vector of unknowns, and $\mathbf{V}$ is the column vector describing excitations.

In applying the MDBFs, it is assumed that the relationship between a longer vector of original (old) unknowns $\mathbf{I}$ and the shorter vector with new unknowns $\tilde{\mathbf{I}}$ exists, and may be written in a matrix form as $\mathbf{I}=\mathbf{M}\tilde{\mathbf{I}}$. Herein, $\mathbf{M}$ denotes a matrix grouping/aggregating basis functions. Each row of this matrix contains weights defining which new basis functions are involved in the formation of the respective old basis functions, and with what weights.

The expression relating the old unknowns to the new ones may be substituted into the original system of linear equations $\mathbf{Z}\mathbf{I}=\mathbf{V}$. The resultant system $\mathbf{Z}\mathbf{M}\tilde{\mathbf{I}}=\mathbf{V}$ is then left-multiplied by the transposed transformation matrix $\mathbf{M}$ to obtain the new system of linear equations: $\underbrace{\mathbf{M}^T\mathbf{Z}\mathbf{M}}_{\tilde{\mathbf{Z}}}\tilde{\mathbf{I}}=\underbrace{\mathbf{M}^T\mathbf{V}}_{\tilde{\mathbf{V}}}$. This system may be rewritten in a short form as $\tilde{\mathbf{Z}}\tilde{\mathbf{I}}=\tilde{\mathbf{V}}$. Once this new system is solved and the new unknowns $\tilde{\mathbf{I}}$ obtained, the original unknowns may be computed from $\mathbf{I}=\mathbf{M}\tilde{\mathbf{I}}$.

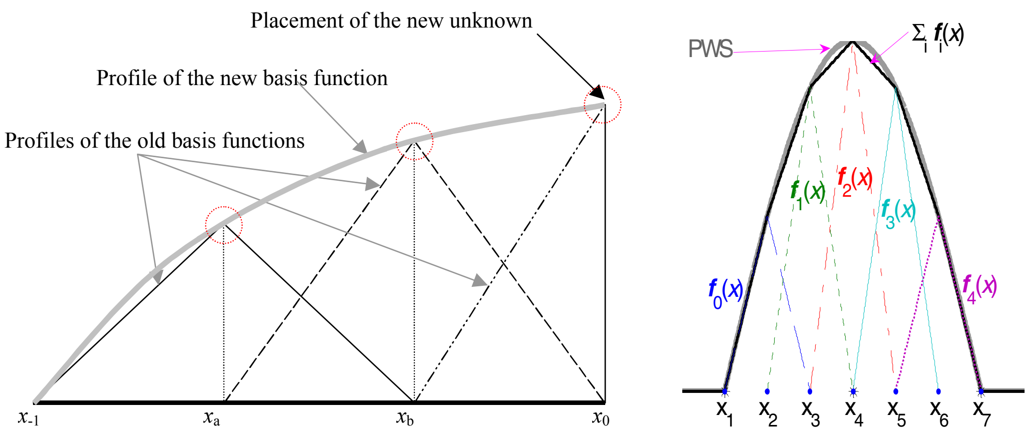
It may be mentioned that the results of a MDBF based approach may be made equal to the results of a traditional MoM with the same original expansion functions, if the matrix $\mathbf{M}$ is an identity matrix.

Application of PWS Basis Functions instead of PWL Basis Functions

The general idea behind expressing the original basis functions via new MDBFs may be illustrated with Fig. 1a. The weights corresponding to the old basis functions to form a piecewise linear approximation may be easily computed by translating the centers of the original triangular basis functions into the co-ordinate system of the new basis function, and computing the PWS function at these points. This is essentially the same procedure as required by PWL functions applied to chains [2]. The only difference between the two is the shape (profile) of the interpolating function. The rest of the details (e.g. calculation for the positions of the old unknowns) is exactly the same.

It is clear that the only modification required in a program to support this extension of the basis functions defined on chains is calculation of the weights for matrix **M**. There is no need for any modifications to the subroutines for integration and calculation of the impedance matrix elements, which are usually the most complex and difficult to change parts of a program realizing MoM.

The only clear limitation of this approach is the requirement of a small electrical length for the wire segments composing a chain, so that the piecewise linear approximation of the applied basis function would be sufficiently accurate. This approximation will produce best results when wire segments are small, i.e. the density of the initial mesh is high. Then, as illustrated in Fig. 1b, the difference between the piecewise linear approximation due to the PWL functions used in the computing matrix elements, and the sinusoidal profile of PWS function will not be large.



(a) (b)

Fig. 1. The relationship between the new piecewise sinusoidal (PWS) basis function and the old/original piecewise linear (PWL) basis functions. (a) The original unknown amplitudes may be readily expressed using a product of the new unknown and the weights at the positions of the original unknowns (x_a, x_b, x_0), denoted with circles. (b) An example of this approximation.

Example – Meander Monopole

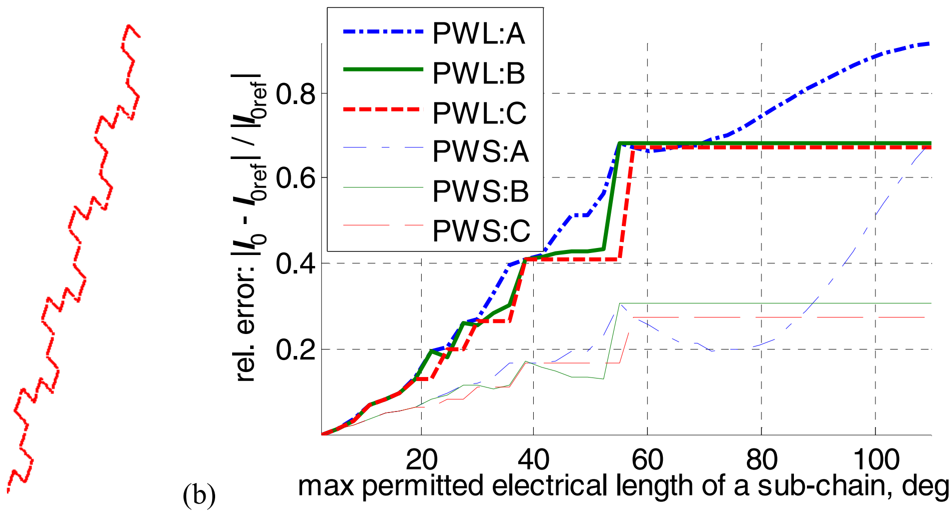
This example compares the results of using PWL and PWS profiles for the MDBFs applied to a meander monopole on a perfectly conducting ground plane. The meander monopole shown in Fig. 2a is the same antenna as was used in [2]. It is composed of 1 m long straight wire segments of radius 1 mm, with total length of wire 40 m, and height above ground plane 20 m. The ratio of wire segment length to radius, 1000:1, was set to be sufficiently large to satisfy the thin wire approximation conditions. This aims to ensure validity of the numerically obtained results, even under the finest meshing of the chain into sub-chains.

The plot shown in Fig. 2b shows a relative error in the electrical current at the driving point versus the maximum permitted electrical length of a sub-chain, calculated at the first resonance frequency, 2.28485 MHz. The plot illustrates the following: PWS approximation gives much smaller error in approximating current distribution. The conclusions already stated in [2] still hold, namely (i) error grows with an increase in the roughness of the equivalent mesh (expressed via the maximum permitted electrical length of a sub-chain), and (ii) the equal-length based chain-splitting algorithm *C* [2] gives better results compared to the algorithms *A* and *B* [2] (although at a possible expense of few extra unknowns).

It is also noticeable that method *A* is particularly prone to errors whether PWL or PWS BF is used. Fig. 3 gives additional detail to this conclusion by highlighting the spread of the calculated current distributions away from the reference profile. Also, it is clear that the PWS is far better in approximating the current.

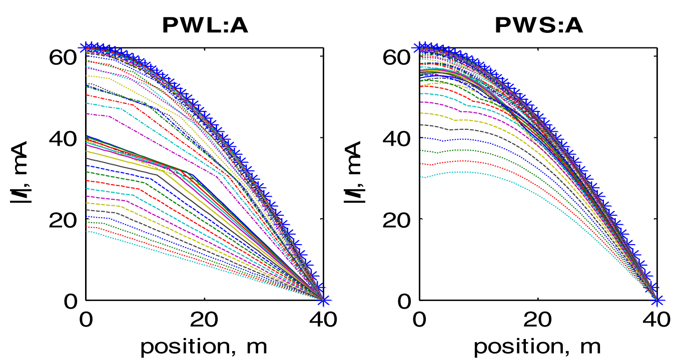
Concluding Remarks

A method for grouping sub- and large domain basis functions into multiple domain basis functions was expanded to support piecewise sinusoidal basis functions. The results have demonstrated improvements in current distribution estimation. Although the current realization is limited to chains of wire segments, the method may be readily extended onto quadrilaterals. Also, the method may be effortlessly extended on arbitrary shapes of basis functions, whereby the chosen profile is linearly interpolated between the nodes of the respective chain.



(a) (b)

Fig. 2. Geometry of meander monopole (a); calculated error of MDBF approximation (b).



(a) (b)

Fig. 3. Current distribution along the wire segments using splitting method *A*, and either (a) piecewise linear (PWL) or (b) piecewise sinusoidal (PWS) MDBF. Stars (*) denote the most accurate (reference) solution using a direct method of moments solution.

References

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