Influence of the least-squares phase on optical vortices in strongly scintillated beams

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The optical vortices that exist in strongly scintillated beams make it difficult for conventional adaptive optics systems to remove the phase distortions. When the least-squares reconstructed phase is removed, the vortices still remain. However, we found that the removal of the least-squares phase induces a portion of the vortices to be annihilated during subsequent propagation, causing a reduction in the total number of vortices. This can be understood in terms of the restoration of equilibrium between explicit vortices, which are visible in the phase function, and vortex bound states which are somehow encoded in the continuous phase fluctuations. Numerical simulations are provided to show that the total number of optical vortices in a strongly scintillated beam can be reduced significantly after a few steps of least-squares phase corrections.

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I. INTRODUCTION

Adaptive optics (AO) systems [1] are widely used to correct scintillated optical beams. In weakly turbulent atmospheric conditions, the wavefront in the receiver aperture of the AO system is only weakly perturbed and can be accurately reconstructed by so-called wavefront reconstructors [2–4] that employ least-squares methods. In such a case the reconstructed wavefront is continuous, as it is formed by a deformable mirror.

When the turbulence in the optical path becomes stronger, such as when a laser beam propagates horizontally near the ground for several kilometers, the wavefront in the receiver aperture of the AO system will be severely distorted [5, 6] with the present of numerous optical vortices [7–9]. Around these vortex cores, the phase increases or decreases by a value of 2π and the amplitude vanishes at these vortex cores. The direction of phase increment indicates the sign of the vortex, a positive or negative one. The removal of the least-squares phase can not get rid of optical vortices. It can only remove the continuous phase fluctuations. However, we find that the behavior of vortex dipoles (pairs of oppositely charged optical vortices) change after the least-squares phase has been removed. In this paper, we show that some of the vortex dipoles in a least-squares corrected beam will subsequently annihilate after a distance of free-space propagation. We also show that several consecutive least-squares corrections can remove a significant number of the optical vortices from the beam.

We begin by reviewing the basic theory of random vortex fields and discuss the concept of an optical vortex plasma in Section II. Statistical results from numerical simulations showing the effect of least-squares phase corrections on the total number of optical vortices in strongly scintillated beams are provided in Section III. Finally, we summarize our conclusions in Section IV.

II. OPTICAL VORTEX PLASMA MODEL

Random vortex fields have been studied extensively [10–17]. It was found, among other things, that neighboring vortices in a random vortex field tend to have opposite topological charges [13, 18], and that the total number of vortices is inversely proportional to the coherence area of the random wave field [11]. In a random wave field, saddles, phase singularities and extrema can be created or converted from one to another with the topological index of the wave field being conserved [14, 19]. The total number of vortices can be variable due to the creation and annihilation of vortices while the net topological charges and total angular momentum are conserved during free-space propagation [20].

Recently the idea was put forward [21] that one can consider the optical vortices in a random wave field as a plasma consisting of three species of particles: positive vortices, negative vortices, and neutral bound states. While the positive and negative vortices are visible in the phase function the neutral bound states are not visible and are in some way encoded in the phase and/or amplitude of the wave. The reason behind the postulated existence of the neutral component lies in the apparent tendency for optical beams to maintain the average number of optical vortices. The rate of dipole annihilations in a random wave field is balanced by the rate of dipole creations. This is reminiscent of a plasma in equilibrium where the rate of ionization is balanced by the rate of recombination. For this reason it is reasonable to model a random vortex field as a plasma, which contains, in addition to the (topologically) charged particles, also the

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neutral bound states.

A scintillated wave field can be viewed as a Gaussian random field. During free-space propagation, the total number of optical vortices fluctuates around an average number of vortices, due to a balance between the annihilation and the creation of vortex dipoles. A scintillated beam, therefore, presents a scenario in which one can apply and test this plasma model for optical vortices. A state of equilibrium implies that the three components exist in specific fixed ratios. In other word, for a given optical vortex density there must be a certain density of neutral bound states.

According to the model the neutral component is represented by bound states of vortices, which are somehow encoded in the continuous phase, the amplitude and the distribution of the vortices. By removing the continuous phase one would remove part of the neutral component and thereby perturb the system away from equilibrium. To restore the equilibrium more of the optical vortices would need to recombine to supplement the depleted neutral component. Under the assumption that the total (explicit plus encoded) vortex number is conserved, the resulting beam will have a lower optical vortex density after equilibrium has been reached.

In what follows we find that this is indeed what happens. By removing the least-squares phase of the scintillated beam we find that the vortex density drops. This serves as partial confirmation of the optical vortex plasma model.

III. NUMERICAL SIMULATIONS AND RESULTS

Numerical simulations are used to evaluate the influence of the removal of the least-squares phase on optical vortex dipoles in strongly scintillated beams. The simulations are conceptually represented by the diagram in Fig. 1.

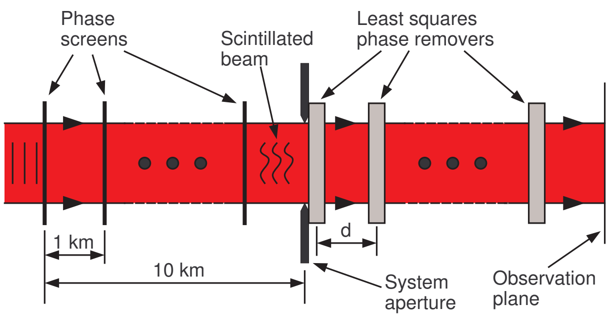

FIG. 1: (Color online)Geometry for the simulations of the scintillation of an optical beam with multiple subsequent least-squares phase corrections.

To generate a scintillated beam, an optical beam with wavelength $\lambda = 500$ nm is propagated over a distance of 10 km, through a turbulent atmosphere. Ten equally spaced random phase screens are used along the optical path to simulate the turbulent atmosphere. Each phase screen represents a 1 km thick turbulent atmospheric layer [22–24].

All the phase screens and propagation are conducted on an $N \times N$ array. To avoid the aliasing problems caused by using a fast Fourier transform (FFT) in the numerical simulations, the relationship between the array size N along one direction of the FFT array, the sampling space Δ and the propagation distance z should satisfy $N \geq 2\lambda z/\Delta^2$ [24]. Due to a limited amount of computer memory, N can not be very large. In our simulations, N is set to be 512 and Δ is set to be 2 mm. Therefore, the size of the beam waist becomes very large — on the order of 0.5 m. In the real world, the size of the beam waist may only be a couple of centimeters or even smaller. To avoid any confusion, the propagation distance in the following discussion will be expressed in terms of the size of the beam waist.

The strength of the turbulence is parameterized with the structure constant of the index of refractive fluctuations C_n^2. For each simulation, C_n^2 is set to a constant value along the entire optical path. This method provides a reasonable agreement between real world data and the simulation data. Figure 2 shows the amplitude and the phase function in the system aperture for a simulated example of a scintillated beam with $C_n^2 = 4 \times 10^{-15}$ m$^{-2/3}$. The Rytov variance is,

$$\sigma_{\chi,R}^2 = 0.307 \left(\frac{2\pi}{\lambda}\right)^{7/6} L^{11/6} C_n^2 \approx 5.07, \qquad (1)$$

for a plane wave propagating along such an optical path.

In this case, the turbulence is strong enough to create a significant number of optical vortices. For this example, there are 381 positive and 381 negative optical vortices in the system aperture. The net topological charge is zero and remains zero for the duration of the beam propagation through free space.

The scintillated beam, which contains all the vortices, now enters the system aperture, shown in Fig. 1. A least-squares phase remover is put directly behind the system aperture. This least-squares phase remover is just a conventional adaptive optics system. It measures the incident wavefront with a wavefront sensor such as a Shack-Hartmann wavefront sensor and then reconstruct a continuous phase with a least-squares method. With the aid of a deformable mirror the least-squares phase can then be removed from the original distorted wavefront. In Fig. 3 the corrected phase function is shown, for the example shown in Fig. 2. The removal of the least-squares phase can not directly remove any of the optical vortices. However, one can see that the background phase around the vortex cores becomes smooth. As a result, the neutral vortex bound states that are encoded in the continuous phase fluctuations are removed together with the least-squares phase. This influences the behavior of vortex dipoles. As the beam propagates beyond the least-squares phase remover some optical vortices will be

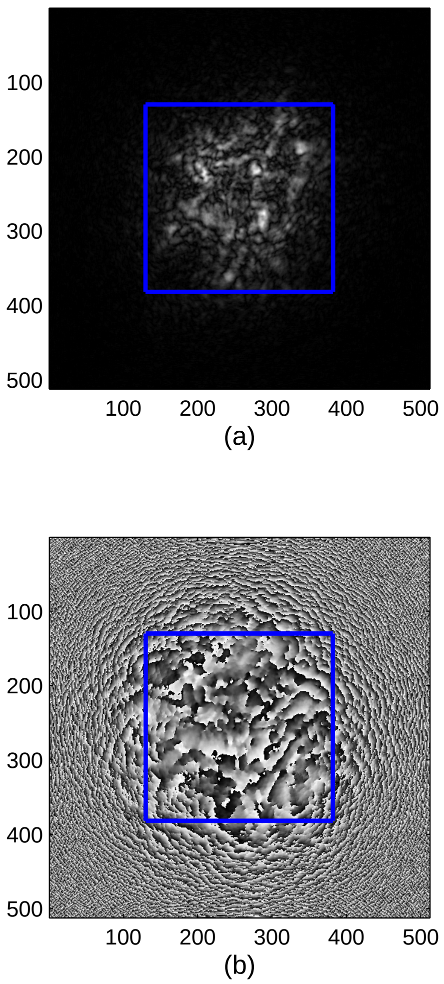

FIG. 2: (Color online)Amplitude (a) and phase (b) of a scintillated beam, with $C_n^2 = 4 \times 10^{-15}$ m$^{-2/3}$. In total there are 381 positive and 381 negative optical vortices in the system aperture, which is represented by the blue rectangles.

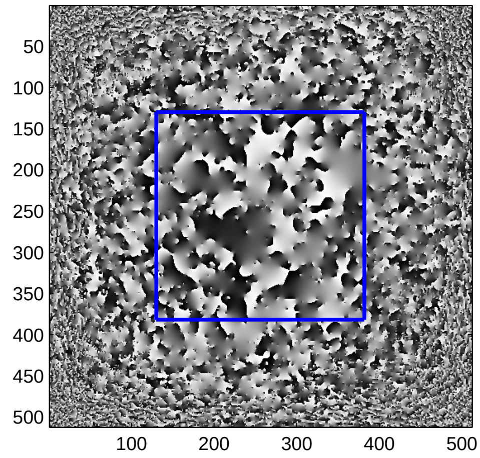

FIG. 3: (Color online)Phase function with the least-squares phase removed from the original phase function as shown in Fig. 2 (b).

turned into neutral vortex bound states by annihilating each other in oppositely charged pairs (vortex dipoles).

To see the influence of the least-squares correction, we record the total number of optical vortices at regular intervals during propagation of the corrected beam in free space. Each interval is a unit of propagation distance (A.U.) which is about 20 times the size of the beam waist. For comparison, we do the same for the uncorrected beam. This process is repeated for 200 different corrected beams and 200 different uncorrected beams to obtain statistical averages of the total number of vortices. The curves for the statistical averages of the total number of optical vortices, together with their standard deviations, are shown in Fig. 4 as a function of the distances of free-space propagation, for the case where $C_n^2 = 4 \times 10^{-15}$ m$^{-2/3}$. To show the relationship clearly, we divide the curve for each simulation by the initial total number of vortices that exist in the system aperture. As a result, the initial average total number of vortices of both curves equals unity with a vanishing standard deviation.

One can see that the average total number of vortices in the uncorrected beam has a very slight decline during beam propagation, less than the standard deviation. The reason for this decline is that some of the optical vortices located near the edges of the aperture tend to move out of the system aperture during propagation. If one ignores the vortices near the edges, one can see that the total number of vortices in a uncorrected beam neither increase, nor decrease as a general trend. The vortex field in a uncorrected beam is in equilibrium, which implies that the rate of annihilation of vortex dipoles is balanced by the creation of vortex dipoles. In this way the average number of optical vortices in an uncorrected beam is conserved [25, 26].

In Fig. 4, the curve with the blue stars shows the average total number of optical vortices after the least-squares phase has been removed from the original scintillated beam. It drops down exponentially, converging to a value of almost half the initial number of vortices. The reason can be explained as follows. After all the neutral vortex bound states that were encoded in the continuous phase function have been removed, together with the least-squares phase, more of the explicit vortices were converted into neutral vortex bound states through vortex dipole annihilation to restore the equilibrium for which the rate of dipole annihilation is again balanced by the rate of dipole creation. The result is that, for larger distances of propagation this curve asymptotically tends toward a lower fixed value, as shown in Fig. 4. This new equilibrium state now contains fewer explicit optical vortices and presumably also fewer neutral vortex bound

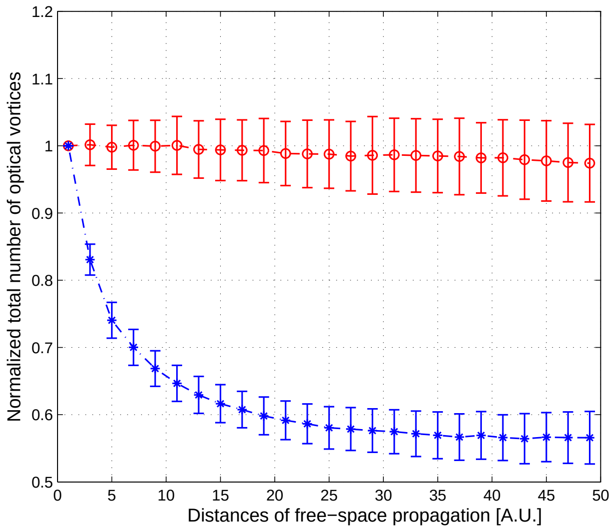

FIG. 4: (Color online)Normalized total number of vortices as a function of the distance of free-space propagation, for the case where $C_n^2 = 4\times10^{-15}$ m$^{-2/3}$. The circles (red) show the average total number of optical vortices without any correction. The stars (blue) show the average total number of optical vortices with least-squares phase correction. Each value of the total number of vortices is divided by the initial total number of optical vortices that exist in the system aperture for each simulation. Error bars indicate the standard deviations.

states.

Once the equilibrium is restored, the newly formed neural bound states are again partially encoded in the fluctuations of the continuous phase. As a result the continuous phase again becomes distorted after some distance of propagation. For this reason one can repeat the removal of the least-squares phase several times, as shown in Fig. 1. In each such step the newly created bound states are removed by removing the least-squares phase. Then the new corrected beam is allowed to propagate further to restore the equilibrium again and thereby further reduce the average total number of explicit vortices.

Figure 5 shows the curve of the average total number of optical vortices, which is normalized in the same way as in Fig. 4, with four such least-squares correction steps. In this procedure, the least-squares phase is measured and removed every time once the equilibrium for the previous step has been restored, in other words, when vortex dipole annihilation is balanced by vortex dipole creation. These points are indicated by the vertical red dashed lines in Fig. 5. One can see that after each step of correction, the average total number of vortices is reduced further. However, the reduction becomes smaller for each successive step. This indicates that the ability of getting rid of optical vortices by removing the least-squares phase becomes progressively less effective. We do not currently understand this loss in effectiveness.

The results above suggest that one can remove more optical vortices from a scintillated beam by implementing

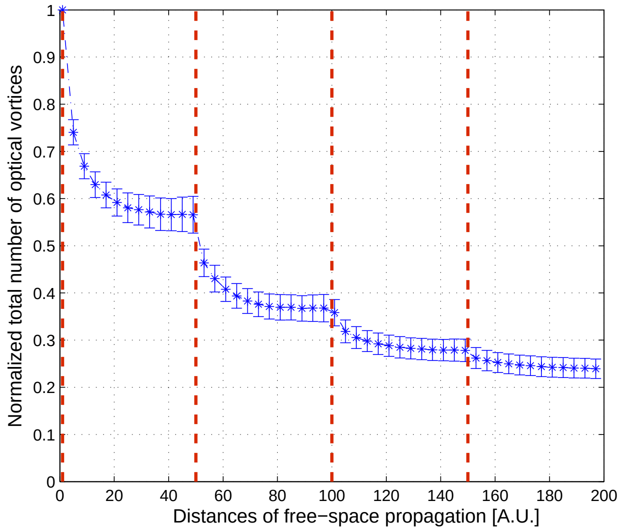

FIG. 5: (Color online)Normalized total number of vortices as a function of the distance of free-space propagation. The stars (blue) show the average total number of vortices. Each total number of vortices is normalized by the initial total number of vortices in the system aperture for each simulation. Error bars indicate the standard deviations. The red dashed lines show the points where the least-squares phase is removed.

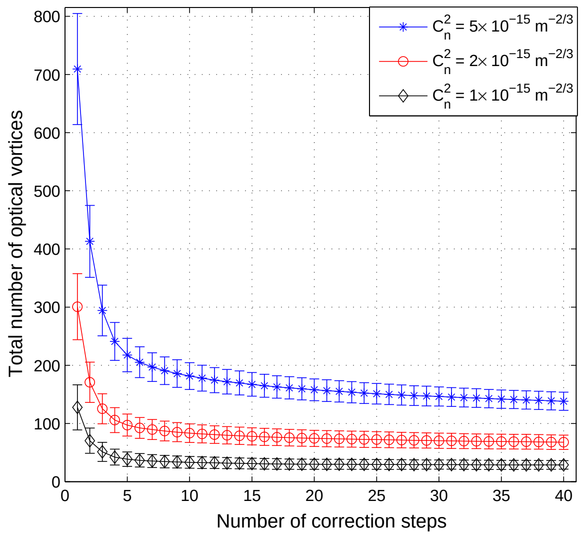

FIG. 6: (Color online)Total number of vortices as a function of the number of correction steps. The stars (blue), circles (red) and diamonds (black) show the average total numbers of vortices when the turbulent strength are respectively, $C_n^2 = 5\times10^{-15}$ m$^{-2/3}$, $C_n^2 = 2\times10^{-15}$ m$^{-2/3}$ and $C_n^2 = 1\times10^{-15}$ m$^{-2/3}$. Error bars indicate the standard deviations.

several consecutive least-squares corrections, as shown in Fig. 1. We simulated such a multi-step least-squares phase removal procedure consisting of $n = 40$ correction steps for three different values of C_n^2 (5×10^{-15} m$^{-2/3}$, 2×10^{-15} m$^{-2/3}$ and 1×10^{-15} m$^{-2/3}$). After each correction step, the corrected beam is allowed to propagate in free

TABLE I: The total number of optical vortices for different turbulence strengths. N_I and N_F are the initial and final number of optical vortices respectively. R is the ratio of N_F to N_I.

C_n^2 ($m^{-2/3}$)	N_I	N_F	R(%)
1×10^{-15}	130±40	30 ± 7	23±5.8
2×10^{-15}	300±60	70 ±12	23±4.1
5×10^{-15}	700±90	140±16	20±2.2

space over a distance of d (about 500 times the size of the beam waist) to allow the beam to reach an equilibrium before the next correction step. For each value of C_n^2 the simulation is repeated 200 times to compute average numbers of vortices together with standard deviations. In these simulations we only record the initial number of vortices and the number of vortices before each correction step.

The statistical curves are shown in Fig. 6. One can see that the average total number of optical vortices drops down exponentially as a function of the number of correction steps, asymptotically approaching some finite number of vortices. Although this asymptotic value is significantly less than the initial number of vortices, the final number of vortices cannot be reduced below this asymptotic value regardless of how many correction steps are used. One can also see that the asymptotic value for the number of vortices depends on the turbulence strength (C_n^2) that was used to produce the initial number of vortices.

If one computes the ratio of the final (asymptotic) number of optical vortices to the initial number of vortices, one finds that, as shown in Table I, about 80% of the initial vortices are removed so that only about 20% of the initial vortices remain. Note that this is the case for other values of C_n^2. In other words, this ratio does not change significantly due to different turbulence strength. The fact that the final number of optical vortices does not approach zero indicates a limitation in the ability of a multi-step least-squares phase correction system to remove optical vortices.

IV. CONCLUSIONS

We have investigated the behavior of vortices in strongly scintillated beams after the least-squares phase has been removed. It is found that some vortex dipoles with short separation distances will annihilate during the subsequent free-space propagation. Using numerical simulations and statistics, we found that about 80% of the initial optical vortices can be removed by having several steps of cascaded least-squares phase corrections and free-space beam propagations. The remaining 20% of optical vortices can not be removed by adding more least-squares phase corrections. Therefore, although the removal of least-square phase in a strongly scintillated beam can help to get rid of optical vortices, this ability is limited.

Acknowledgments

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[1] R. K. Tyson, *Principles of adaptive optics* (Academic Press, San Diego, 1991), ISBN 0-12-705900-8.
[2] R. H. Hudgin, J. Opt. Soc. Am. A **67**, 375 (1977).
[3] D. L. Fried, J. Opt. Soc. Am. A **67**, 370 (1977).
[4] W. H. Southwell, J. Opt. Soc. Am. A **70**, 998 (1980).
[5] M. C. Roggemann and B. Welsh, *Imaging through turbulence* (CRC Press, Boca Raton, 1996), ISBN 0-8493-3787-9.
[6] L. C. Andrews and R. L. Phillips, *Laser beam propagation through random media* (SPIE Optical Engineering Press, 1998), ISBN 0-8194-2787-X.
[7] P. Coullet, L. Gil, and F. Rocca, Opt. Commun. **73**, 403 (1989).
[8] D. L. Fried and J. L. Vaughn, Appl. Opt. **31**, 2865 (1992).
[9] M. Chen, F. S. Roux, and J. C. Olivier, J. Opt. Soc. Am. A **24**, 1994 (2007).
[10] N. Baranova, A. Mamaev, N. Pilipetsky, V. Shkunov, and B. Zel'dovich, J. Opt. Soc. Am. **73**, 525 (1983).
[11] I. Freund, N. Shvartsman, and V. Freilikher, Opt. Commun. **101**, 247 (1993).
[12] I. Freund, J. Opt. Soc. Am. A **11**, 1644 (1994).
[13] N. Shvartsman and I. Freund, Phys. Rev. Lett. **72**, 1008 (1994).
[14] I. Freund, Physical Review E **52**, 2348 (1995).
[15] M. V. Berry and M. R. Dennis, Proc. R. Soc. Lond. A **456**, 2059 (2000).
[16] M. R. Dennis, Ph.D. thesis, H. H. Wills Physics Laboratory, University of Bristol (2001).
[17] M. R. Dennis, J. Phys. A: Math. Gen. **34**, 297 (2001).
[18] K. Staliunas, A. Berzanskis, and V. Jarutis, Opt. Commun. **120**, 23 (1995).
[19] I. Freund and D. A. Kessler, Opt. Commun. **187**, 71 (2001).
[20] V. N. G. M. S. Soskin and M. V. Vasnetsov, Phys. Rev. A **56**, 4064 (1997).
[21] F. S. Roux, Phys. Rev. A **79**, 013810 (2009).
[22] J. M. Martin and S. M. Flatté, Appl. Opt. **27**, 2111 (1988).
[23] J. M. Martin and S. M. Flatté, J. Opt. Soc. Am. A **7**, 838 (1990).
[24] M. C. Roggemann and A. C. Koivunen, J. Opt. Soc. Am. A **17**, 911 (2000).

[25] F. S. Roux, Opt. Commun. **266**, 433 (2006).
[26] G. Gbur and R. K. Tyson, J. Opt. Soc. Am. A **25**, 225 (2008).