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ANALYSIS OF A NEW UNIDIMENSIONAL MODEL AND LATERAL VIBRATIONS OF 1-3 PIEZOCOMPOSITE SIDE SCAN SONAR ARRAY

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Abstract

A novel unidimensional model of 1-3 piezocomposite ultrasonic transducers is presented. A new electromechanical model is the generalization of the Smith-Auld unidimentional model, which is formulated in terms of a variational approach. It takes into account the lateral motions of the piezoelectric pillars and polymer, which are supposed to be proportional to the strain in the axial direction. A new set of equivalent elastic, electrical and electromechanical constants of 1-3 piezocomposite is derived. Lamb modes of the 1-3 piezocomposites are investigated in term of the Certon-Patat membrane model by means of direct variational method application. A new design of a 1-3 piezocomposite side scan SONAR array is considered. An implementation of the array is proposed with generation of acoustic beam patterns at three different frequencies: 80 kHz, 300 kHz and 510 kHz.

Introduction

Multi-frequency side scan transmit arrays broadly used in underwater acoustic measurements. A three frequency side scan array is designed in the Centre for Integrated Sensing Systems of CSIR based on the technology of 1-3 piezocomposites. Thickness modes are designed using a new unidimensional model which is a generalization of the known Smith-Auld model[1]. Proper prediction of lateral resonances is important for increasing bandwidth and improving performance of transducers. Prediction of lateral resonances is based on Certon-Patat "membrane" model[2], [3]. A modification of this theory based on a variational approach. The implementation of the array is given with generation of acoustic beam patterns at frequencies: 80 kHz, 300 kHz and 510 kHz and electrical impedance for three apertures is measured.

Unidimensional Model of 1-3 Piezocomposite

Typical 1-3 piezocomposites are shown in Figure 1. The proposed unidimensional model is a generalization of the Smith-Auld model[1] and is based on the Rayleigh theory of longitudinal oscillations of bars. Displacements and strains are supposed to be

$$u_i = -\nu_i x w_z{}'; \qquad v_i = -\nu_i y w_z{}'; \qquad (i = 1, 2)$$

$$S_1 = u_x{}'; \quad S_2 = v_y{}'; \quad S_3 = w_z{}'; \quad S_4 = S_5 = S_6 = 0 \tag{1}$$

where the equivalent Poisson's ratios and modules of elasticity (1 – piezoelectric material, 2 – isotropic polymer) are

$$\nu_1 = \frac{c_{13}^D h_{33} - c_{33}^D h_{31}}{\left(c_{11}^D + c_{12}^D\right) h_{33} - 2c_{13}^D h_{31}}; \qquad \nu_2 = \frac{E_2}{2G_2} - 1 = \frac{\lambda_2}{2\left(\lambda_2 + \mu_2\right)};$$

$$E_1 = c_{33}^D + 2\nu_1{}^2\left(c_{11}^D + c_{12}^D\right) - 4\nu_1 c_{13}^D; \qquad E_2 = \frac{\mu_2\left(3\lambda_2 + 2\mu_2\right)}{\lambda_2 + \mu_2} \tag{2}$$

Lagrangian of the system is

$$L = L\left[\dot w, \dot w', w', w\left(H/2\right), w\left(-H/2\right), D_3, \lambda\right] =$$

$$\frac{1}{2}\int_{-H/2}^{H/2}\left(F_1\dot w^2 + F_2\dot w'^2 - F_3 w'^2\right)dz + \frac{1}{2}F_4 D_3{}^2 + \lambda\left[F_5 D_3 - F_6 w\left(H/2\right) + F_6 w\left(-H/2\right) - V\right] \tag{3}$$

where H – height of the pillar, V – applied voltage, D_3 - electric displacement, λ - Lagrange multiplier

$$F = \rho_1 A_1 + \rho_2 A_2; \quad F_2 = \rho_1 \nu_1{}^2 I_1 + \rho_2 \nu_2{}^2 I_2; \quad F_3 = \left(E_1 A_1 + E_2 A_2\right) + 8E_2\left[\frac{a}{\Delta}\left(\nu_1 a + \nu_2 \Delta\right)^2 + \left[\nu_2\left(a + \Delta\right)\right]^2\right];$$

$$F_4 = \beta_{33}^S A_1 H; \qquad F_5 = \beta_{33}^S H; \qquad F_6 = h_{33} - 2h_{31}\nu_1 \tag{4}$$

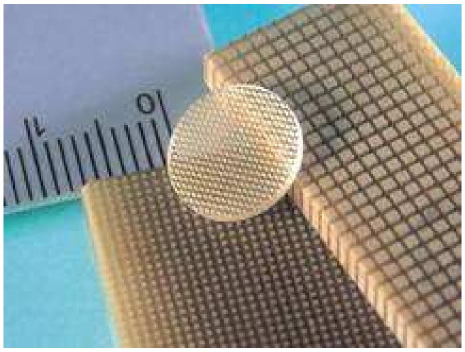

Figure 1. Different types of 1-3 piezocomposites

Geometry of the cell is shown in Figure 2.

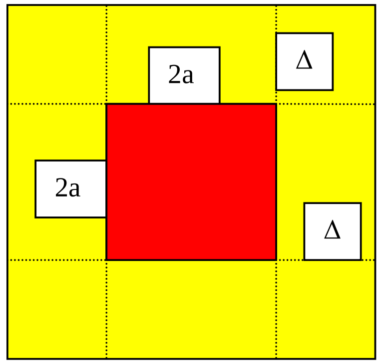

Figure 2. In-plane of elementary cell for development of unidimentional model

Equations of motion are obtained from the Lagrangian (24) as follows:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{w}}\right)-\frac{\partial^2}{\partial t\,\partial z}\left(\frac{\partial L}{\partial \dot{w}'}\right)+\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial w'}\right)=F_1\ddot{w}-F_2\ddot{w}''-F_3 w''=0;\quad \frac{\partial L}{\partial D_3}=F_4 D_3+\lambda F_5=0 \quad (5)$$

Boundary conditions are:

- mechanical boundary conditions:

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{w}'}\right)-\frac{\partial L}{\partial w'}\right]_{z=\pm H/2}=\left[F_2\ddot{w}'+F_3 w'\right]_{z=\pm H/2}=0 \quad (6)$$

- electrical boundary conditions:

$$\frac{\partial L}{\partial \lambda}=F_5 D_3-F_6 w\left(H/2\right)+F_6 w\left(-H/2\right)-V=0 \quad (7)$$

Suppose that

$$w=\widehat{W}(z)\cdot e^{i\omega t};\quad D_3=\widehat{D}_3\cdot e^{i\omega t};\quad V=\widehat{V}\cdot e^{i\omega t};\quad \lambda=\widehat{\Lambda}\cdot e^{i\omega t} \quad (8)$$

In this case the first equation (26) and boundary conditions (27) – (28) could be rewritten as:

$$\widehat{W}''+\eta^2\widehat{W}=0\quad\left(\eta=\eta(\omega)=\omega\cdot\sqrt{\frac{F_1}{F_3-\omega^2F_2}},\quad 0<\omega<\sqrt{\frac{F_3}{F_2}}\right);\qquad \widehat{\Lambda}=-\frac{F_4}{F_5}\widehat{D}_3; \quad (9)$$

$$\left(F_3-\omega^2F_2\right)\left[\widehat{W}'\right]_{z=\pm H/2}=0;\qquad F_6\widehat{W}\left(-H/2\right)-F_6\widehat{W}\left(H/2\right)+F_5\widehat{D}_3=V \quad (10)$$

First equation (30) has the following solution:

$$\widehat{W}=\widehat{W}(z)=\widehat{C}\cdot\cos(\eta z)+\widehat{S}\cdot\sin(\eta z) \quad (11)$$

Unknown constants $\hat{C}$ and $\hat{S}$ as well as $\hat{D}_3$ could be found from the system of boundary conditions (10). Physical meaning of the Lagrange coefficient $\lambda = -A_1 D_3 \cdot e^{i\omega t}$ is a charge on both faces of the piezoelectric pillar, which maintain the electro-mechanical motion of the1-3 piezocomposite.

Lateral Vibrations of 1-3 Piezocomposite

The elementary cell in Certon and Patat’s approach[2], [3] is formed by a right-angled triangle and so the only symmetric solutions of lateral vibrations are considered (Figure 3). If periods of the 1-3 piezocomposite are different in x- and y-directions asymmetric solutions are slightly piezoelectrically coupled. In the present paper the elementary cell is formed by a rectangle due to periodicity in x- and y-directions and hence, the symmetric and asymmetric solutions are considered (Figure 4).

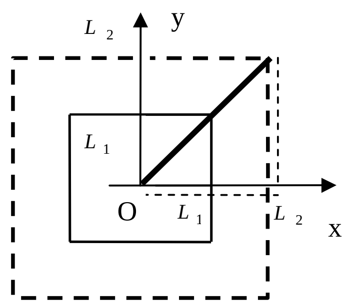

Figure 3. Certon-Patat elementary cell

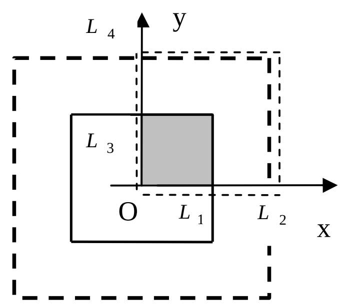

Figure 4. New elementary cell

Lateral modes, dependent on x and y, are considered separately. Our aim is to define lateral motions of 1-3 piezocomposites. It could be done by means of the following simplifications. Let us assume that:

- Lateral displacements $u^{(j)} = v^{(j)} = 0$.
- Thickness motion is neglected, i.e. the thickness strain is zero $S_z^{(j)} = 0 \,:\, w_z'^{(j)} = 0$.

In the frames of these assumptions the “membrane” model of lateral vibrations of 1-3 piezocomposite could be described. In this case:

$$S_1^{(j)} = S_2^{(j)} = S_3^{(j)} = S_6^{(j)} = 0; \quad S_4^{(j)} = w_y'^{(j)}; \quad S_5^{(j)} = w_x'^{(j)} \tag{10}$$

To simplify the model we consider steady-state vibrations:

$$w^{(j)}(x,y,t) = W^{(j)}(x,y) \cdot e^{i\omega t} \tag{11}$$

and the model could be described by a simplified Lagrangian:

$$L = \frac{1}{2}\sum_{j=1}^{2} \int_{(A_j)} \left\{ \rho^{(j)} \omega^2 W^{(j)2} - c_{44}^{(j)} \left[W_x'^{(j)2} + W_y'^{(j)2} \right] \right\} dA_j \tag{12}$$

The Euler-Lagrange equations for this Lagrangian are:

$$\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial W_x'^{(j)}}\right)+\frac{\partial}{\partial y}\left(\frac{\partial L}{\partial W_y'^{(j)}}\right)-\frac{\partial L}{\partial W^{(j)}}=0, \quad (j=1,2) \tag{13}$$

or

$$\rho^{(1)}\omega^2 W^{(1)}=c_{44}^{(1)D}\left[W_{xx}''^{(1)}+W_{yy}''^{(1)}\right]; \qquad \rho^{(2)}\omega^2 W^{(2)}=c_{44}^{(2)}\left[W_{xx}''^{(1)}+W_{yy}''^{(1)}\right] \tag{14}$$

Boundary conditions, which could be obtained from the Lagrangian (12) are:

$$\begin{array}{llll}
x=L_1, \quad 0\le y<L_3: & W^{(1)}-W^{(2)}=0; \quad c_{44}^{(1)D}W_x'^{(1)}-c_{44}^{(2)}W_x'^{(2)}=0 & & \\
y=L_3, \quad 0\le x<L_1: & W^{(1)}-W^{(2)}=0; \quad c_{44}^{(1)D}W_y'^{(1)}-c_{44}^{(2)}W_y'^{(2)}=0 & & \\
x=0, \quad 0\le y<L_3: & W_x'^{(1)}=0 & x=0, \quad L_3\le y\le L_4: & W_x'^{(2)}=0 \\
y=0, \quad 0\le x<L_1: & W_y'^{(1)}=0 & y=0, \quad L_1\le x\le L_2: & W_y'^{(2)}=0 \\
x=L_2, \quad 0\le y\le L_4: & W_x'^{(2)}=0 & y=L_4, \quad 0\le x\le L_2: & W_y'^{(2)}=0
\end{array} \tag{15}$$

Using Rayleigh method, an approximate solution which automatically satisfied the last six equations of the system (15) is

$$W=W(x,y)\approx\sum_{m=0}^{M}\sum_{n=0}^{N}C_{m,n}\cos\left(\frac{m\pi}{L_2}x\right)\cdot\cos\left(\frac{n\pi}{L_4}y\right) \tag{16}$$

There are *(M+1)(N+1)* unknown constants $C_{m,n}$ in this expressions.

In contrast to the approach, used by Certon and Patat[2] who considered only symmetric vibrations, the elementary cell is formed by not a right-angled triangle but a rectangle due to periodicity in x- and y-directions (Fig. 4) and hence, the symmetric and asymmetric solutions are considered (Figures 5, 6). For 56% volume fraction the corresponding frequencies are 1.75 MHz and 2.39 MHz (symmetric modes) and 2.18 MHz and 2.91 MHz (asymmetric modes).

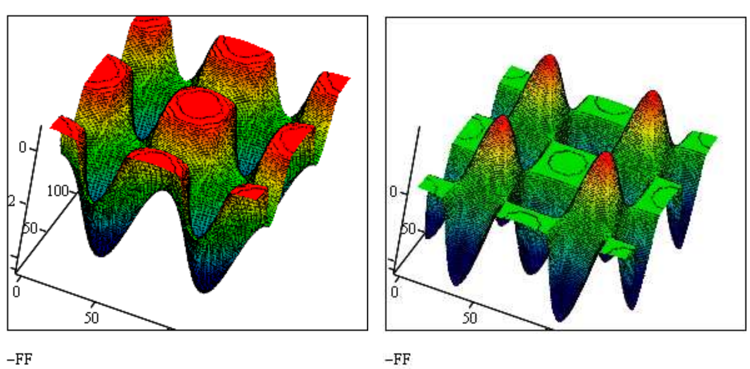

Figure 5. Symmetric eigenfunctions

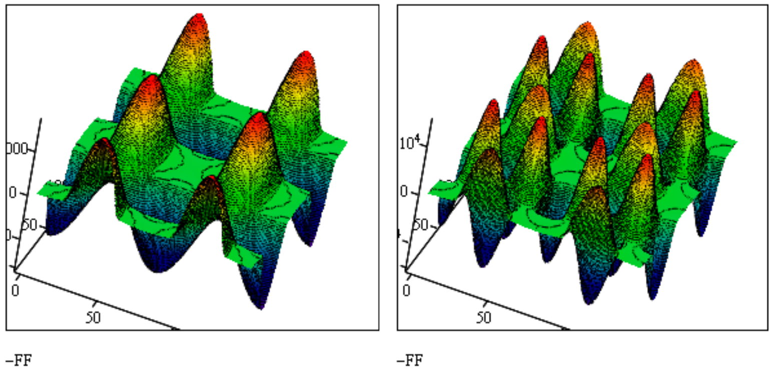

Figure 6. Asymmetric eigenfunctions

Symmetric eigenfunctions are shown in Figure 5. They are well piezoelectrically coupled and hence, could be simply detected by means of electric measurements.

1-3 Piezocomposite Array

The Transmit Sonar Array below was developed to operate as a single array in three frequency modes 80 kHz, 300 kHz and 510 kHz while maintaining a constant vertical and horizontal beam width. This was achieved by using a 1-3 piezocomposite material and electrode patterning. Electrical impedances for three apertures are depicted in Figure 7.

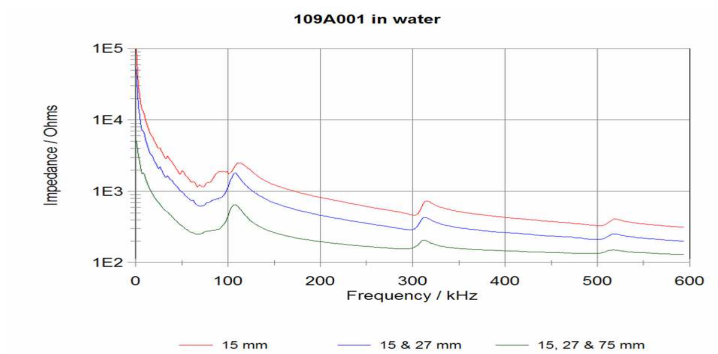

Figure 7. Electrical impedances for three apertures

Implementation of 1-3 piezocomposite transmit sonar array is shown in Figure 8.

Figure 8. *Front view of the multi-frequency array*

Conclusions

New unidimensional model of 1-3 piezocomposites was formulated for design of three frequency piezocomposite transmit side scan sonar array. This model is based on Rayleigh theory of vibrating bars and enables to properly design 1-3 piezocomposite transducers with low aspect ratio of piezoelectric pillars. For estimation of lateral frequencies the "membrane model" was used. Symmetric and asymmetric lateral eigenfunctions were investigated. Both unidimensional model and model of lateral vibrations of 1-3 piezocomposites were formulated using the variational approach. Results of implementation of 1-3 piezocomposite transmit sonar array were demonstrated.

References

1. W.Smith, B.Auld. Modeling 1-3 composite piezoelectrics: thickness mode oscillations, IEEE Trans. Ultrason., Ferroelelct., Freq. Contr., Vol.38, No.1, Jan. 1991, pp. 40-47.
2. D.Certon, F.Patat, F.Levassort, G.Feuillard, B.Karlsson. Lateral Resonances in 1-3 Piezoelectric Periodic Composite: Modeling and Experimental Results, JASA, 101 (4), April 1997, pp. 2043-2051.
3. D.Certon, O.Casula, F.Patat, D.Royer. Theoretical and Experimental Investigations of Lateral Modes in 1-3 Piezocomposites, IEEE Trans. Ultrason., Ferroelelct., Freq. Contr., Vol.44, No.3, May 1997, pp. 643-651.