Time domain simulation of piezoelectric excitation of guided waves in rails using waveguide finite elements

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ABSTRACT

Piezoelectric transducers are commonly used to excite waves in elastic waveguides such as pipes, rock bolts and rails. While it is possible to simulate the operation of these transducers attached to the waveguide, in the time domain, using conventional finite element methods available in commercial software, these models tend to be very large. An alternative method is to use specially formulated waveguide finite elements (sometimes called Semi-Analytical Finite Elements). Models using these elements require only a two-dimensional finite element mesh of the cross-section of the waveguide. The waveguide finite element model was combined with a conventional 3-D finite element model of the piezoelectric transducer to compute the frequency response of the waveguide. However, it is difficult to experimentally verify such a frequency domain model. Experiments are usually conducted by exciting a transducer, attached to the waveguide, with a short time signal such as a tone-burst and measuring the response at a position along the waveguide before reflections from the ends of the waveguide are encountered. The measured signals are a combination of all the modes that are excited in the waveguide and separating the individual modes of wave propagation is difficult if there are numerous modes present. Instead of converting the measured signals to the frequency domain we transform the modeled frequency responses to time domain signals in order to verify the models against experiment. The frequency response was computed at many frequency points and multiplied by the frequency spectrum of the excitation signal, before an inverse Fourier transform was used to transform from the frequency domain to the time domain. The time response of a rail, excited by a rectangular piezoelectric ceramic patch, was computed and found to compare favorably with measurements performed using a laser vibrometer. By using this approach it is possible to determine which modes of propagation dominate the response and to predict the signals that would be obtained at large distances, which cannot be measured in the lab, and would be computationally infeasible using conventional finite element modeling.

Keywords: elastic waveguide, finite element method, piezoelectric excitation, time domain simulation

1. INTRODUCTION

Piezoelectric transducers are commonly used to excite waves in elastic waveguides such as pipes, rock bolts and rails. The operation of these transducers can be simulated in the time domain using conventional finite element methods available in commercial software. These models have to extend a few wavelengths along the waveguide and can result in very large numerical problems. This is especially true if the dispersion of the waves over large distances is to be investigated. An alternative method of analyzing waves that propagate in waveguides, which are infinite in one dimension, is to use specially formulated waveguide finite elements. These elements are not available in commercial software packages but have been implemented by a few research groups [1-6]. These models require only a two-dimensional finite element mesh of the cross-section of the waveguide. The propagating and evanescent waves supported by the waveguide can be computed from these models and the response to harmonic point forces can be computed [7]. This harmonic forced response was used to combine waveguide element models with conventional 3-D finite element models of a piezoelectric transducer [8]. These models provide the frequency response of the waveguide as a superposition of the frequency response of each wave of propagation and are useful for designing a transducer or transducer array that will effectively excite a particular wave at a specified frequency and wavenumber. This is particularly important if resonant transducers are to be used for long range guided wave inspection or monitoring. However, it is difficult to experimentally verify such a frequency domain model. Experiments are usually conducted by

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exciting a transducer, attached to the waveguide, with a short time signal such as a tone-burst and measuring the response at a position along the waveguide before reflections from the ends of the waveguide are encountered. The measured responses contain multiple waves if they are excited. The problem of separating the waves can be approached by taking many time domain measurements along a section of the waveguide and then performing a two-dimensional FFT to separate the modes with different wavenumbers. This has been demonstrated for a low frequency excitation that excites only two propagating waves [9] but becomes more difficult at higher frequencies where multiple waves with similar wavenumbers can exist. Instead of converting the measured signals to the frequency domain it is possible to transform the modeled frequency responses to time domain signals to verify the models against experiment. This can be achieved by computing the frequency response at many frequency points, multiplying this by the frequency spectrum of the excitation signal, and performing an inverse Fourier transform. This method is used in this paper to obtain time signals that can be directly compared to measured time signals. Waves traveling in a rail, excited by a rectangular piezoelectric ceramic patch are predicted numerically and compared to responses measured using a laser vibrometer.

The objective of this work is to develop the method for computing time responses and to use this method to verify the numerical model of a piezoelectric transducer attached to an infinite waveguide by direct comparison to experimental measurements.

2. NUMERICAL MODEL DEVELOPMENT

A general numerical modeling method for analyzing the excitation of complex waves in waveguides of complex (but constant) cross-section by arbitrary piezoelectric transducers has been developed [8]. The method makes use of specially developed waveguide finite elements, which require only a two-dimensional mesh of the cross-section of the waveguide. The response of the waveguide to harmonic excitation is used to determine a complex boundary condition representing the waveguide in a finite element model of the piezoelectric transducer. This model allows computation of the frequency response of the transducer when attached to the waveguide. The forces at the interface between the transducer and waveguide are computed and used to determine the response of the waveguide. The frequency response of each mode of the waveguide is computed and the contribution from each mode can be summed to provide the total frequency response. The method has advantages over conventional time domain simulation of a length of waveguide as it requires only a two dimensional model of the waveguide, provides frequency response information directly (no Fourier transforms required) and provides the response of the individual modes, which can be difficult to extract from time domain simulations. However, for comparison with measured time domain signals it is necessary to use the frequency response to predict the time response. This is achieved by multiplying the frequency spectrum of the excitation with the frequency response, at the measurement point, and performing and inverse Fourier transform. This section provides an overview of the mathematical details of the method.

2.1. Formulation of the Waveguide Finite Elements

The finite elements used to describe the waveguide make use of complex exponential function to describe the variation of the displacement field along the waveguide and interpolation functions over the area of the element. This means that only a two dimensional mesh of the cross-section of the waveguide is required. The displacement fields (u, v, w) in an elastic waveguide, extending in the z direction (as shown in figure 1), can be written as;

$$u(x,y,z,t) = u(x,y)\cdot e^{-j(\kappa z-\omega t)}$$
$$v(x,y,z,t) = v(x,y)\cdot e^{-j(\kappa z-\omega t)}$$
$$w(x,y,z,t) = j\cdot w(x,y)\cdot e^{-j(\kappa z-\omega t)} \tag{1}$$

where, z is the coordinate in the direction along the waveguide, κ the wavenumber and ω the frequency.

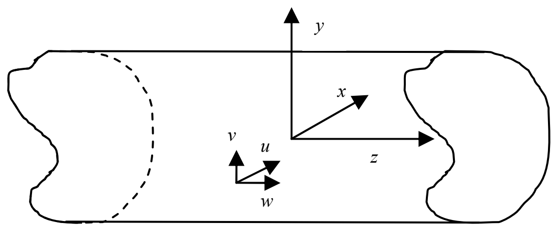


Figure 1: Waveguide coordinates and displacements.

The formulation used here follows that presented by Gavrić [2]. Formulations of Hayashi [3] and Damljanović and Weaver [4] are similar to this formulation. Hayashi's formulation results in complex equations of motion while Damljanović and Weaver also derived complex equations of motion and then applied a transformation to obtain equations of motion comparable to those of Gavrić.

The strain and strain energy of the waveguide can be separated into terms that are independent, linearly dependent or quadratically dependent on the wavenumber. Applying conventional finite element discretization to these terms yields three elemental stiffness matrices with these dependencies. After assembling the elemental matrices into global matrices the system of equations of motion, for the waveguide, is as follows:

$$M\ddot{u} + \left[\kappa^2 \cdot K_2 + \kappa \cdot K_1 + K_0\right]u = f \tag{2}$$

The waves supported by the waveguide may be determined from the free vibration problem ($f = 0$) by performing an eigensolution. If a constant, real wavenumber (κ) is selected a real eigenproblem must be solved for the frequencies (ω) and mode shapes (ψ) of the propagating waves that correspond to this wavenumber. If the frequency is specified a complex eigenproblem must be solved. The wavenumbers that are obtained, by solving this problem, can be real, imaginary or complex and occur in pairs with opposite sign corresponding to waves traveling in opposite directions. If the number of nodes in the model is denoted N, the eigensolution results in $6N$ eigenvalue-eigenvector pairs κ_r and ψ_r.

2.2. Combining waveguide and conventional finite element models

The approach adopted was to use the waveguide finite element model to calculate the receptance of the waveguide to point forces. The receptance is used as a boundary condition, representing the waveguide in a conventional finite element model of the piezoelectric patch actuator. The forces applied to the waveguide are computed and then applied to the waveguide model to compute the response of the waveguide [8].

The forced response of the waveguide finite element model was developed by Damljanović and Weaver [7]. Their finite element formulation is slightly different to that used here but the method of solving the forced response still applies. The equations of motion may be solved by a method similar to that used for solving multi-degree-of-freedom damped oscillator systems. Equation 2 is complemented with an identity as follows,

$$\begin{bmatrix} K_0 - \omega^2 M & 0 \\ 0 & -K_2 \end{bmatrix}\begin{Bmatrix} u \\ \kappa u \end{Bmatrix} + \kappa \begin{bmatrix} K_1 & K_2 \\ K_2 & 0 \end{bmatrix}\begin{Bmatrix} u \\ \kappa u \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{3}$$

so that it may be written in the form

$$A\bar{u} - \kappa B\bar{u} = \bar{f}\,. \tag{4}$$

The solution to the forced vibration problem is found by applying a Fourier transform to equation 4 to obtain an equation in the wavenumber domain, solution in the wavenumber domain and inverse Fourier transform to obtain the solution in the spatial domain [7],

$$\bar{u}(z) = -j\sum_{r=1}^{3N}\psi_r \frac{\psi_r^{\,T}\bar{f}(z)}{\psi_r^{\,T}B\psi_r}e^{-j\kappa_r z} \tag{5}$$

where the summation is performed only over the positive real poles, negative imaginary poles and complex poles with negative imaginary parts.

The response of the waveguide to forces at each of the degrees of freedom (dof) in contact with the piezoelectric transducer may be computed by equation 5 and the receptance r_{ij} is defined as the response at dof *i* due to a unit force applied at dof *j*, i.e. $u_i = r_{ij} f_j$. The displacements at the interface dof (u_{in}) due to loads at the interface dof can then be related by the receptance matrix,

$$u_{in} = R f_{in} . \tag{6}$$

Recalling that the response of the waveguide was computed for a particular frequency of harmonic excitation, the inverse of this matrix (D_w) is the dynamic stiffness matrix of the interface dof's, at this frequency, i.e.

$$D_w u_{in} = f_{in} . \tag{7}$$

The dynamic stiffness matrix of the waveguide is symmetric but fully populated. It is also complex representing the mass/stiffness loading and the damping due to energy being radiated along the waveguide.

A dynamic stiffness for the piezoelectric patch can be computed at the excitation frequency and this matrix (D_p) can be partitioned into degrees of freedom in contact with the patch (u_{in}) and degrees of freedom not in contact (u_n). The two dynamic stiffness matrices can then be combined to represent the piezoelectric patch attached to the waveguide.

$$\begin{bmatrix} D_{p_{nn}} & D_{p_{nin}} \\ D_{p_{inn}} & D_{p_{inin}} + D_w \end{bmatrix} \begin{Bmatrix} u_n \\ u_{in} \end{Bmatrix} = \begin{Bmatrix} f_n \\ f_{in} \end{Bmatrix} \tag{8}$$

The forces in this equation include electrically induced piezoelectric forces. This equation allows the computation of the forced harmonic response of the piezoelectric patch (attached to the waveguide). The forces applied to the waveguide can be computed by substituting the interface displacements into equation 7. The response of the waveguide can then be computed by substituting these interface forces into equation 5. Our interest will often be in the amplitude of response of a particular mode of wave propagation rather than the amplitude at a specific point on the waveguide. This response is written as,

$$p_r(z) = -j \frac{\psi_r^{\,T} \bar{f}}{\psi_r^{\,T} B \psi_r} e^{-j\kappa_r z} . \tag{9}$$

2.3. Time Domain Simulations

The frequency content of a time domain excitation signal (*v(t)*), such as a tone-burst, is given by the Fourier transform of the signal.

$$v(\omega) = \int_{-\infty}^{\infty} v(t) \cdot e^{-j\omega t} dt \tag{10}$$

This can be computed using the Fast Fourier Transform algorithm.

The response to this excitation at distance *z* is then simply
$r(z,\omega) = v(\omega) \cdot u(z,\omega)$, which is expanded to

$$r(z,\omega) = v(\omega) \cdot -j \sum_r^{3N} \psi_r(\omega) \frac{\psi_r^{\,T}(\omega) \cdot \bar{f}(z)}{\psi_r^{\,T}(\omega) \cdot B \cdot \psi_r(\omega)} e^{-j\kappa(\omega) z} \tag{11}$$

The time-domain response can be computed using the inverse Fourier transform,

$$r(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} r(z,\omega) \cdot e^{j\omega t} d\omega \tag{12}$$

Once again, this can be computed using the inverse FFT algorithm.

2.4. Mode Separation Technique

It is clear from equation 11 that the total response is a linear combination of the response of each mode in the waveguide. In some cases it may be useful to evaluate the response of only selected modes, especially the propagating modes. To do this we require the mode shape and wavenumber of that particular mode as a function of frequency. As the wavenumber versus frequency curves generally cross each other a technique is required to separate the modes. A technique was developed which utilized the orthogonality property of the mode shapes.

$$\begin{aligned} \psi_r^T(\omega) \cdot B \cdot \psi_s(\omega) &= 0 \\ \psi_r^T(\omega) \cdot B \cdot \psi_r(\omega) &\neq 0 \end{aligned} \qquad (13)$$

The B-orthogonality of the real mode shapes at frequency step k to those at frequency step $k+1$ was computed.

$$\Theta = \psi^T(\omega_k) \cdot B \cdot \psi(\omega_{k+1}) \qquad (14)$$

If the wavenumber versus frequency curves have not crossed in this frequency interval then the diagonal terms in the matrix Θ will be the largest terms. The presence of an off-diagonal term that is larger than the diagonal term indicates that the curves have crossed. This is then taken into account in the numbering of the waves.

3. PIEZOELECTRIC PATCH ON A RAIL

The method described above was applied to a rail excited by a piezoelectric patch. The mesh used is shown in figure 2. The PZT4 piezoelectric patch was 30x10x2mm in dimension and was modeled using three-dimensional elements, which extended 10mm along the axis of the rail.

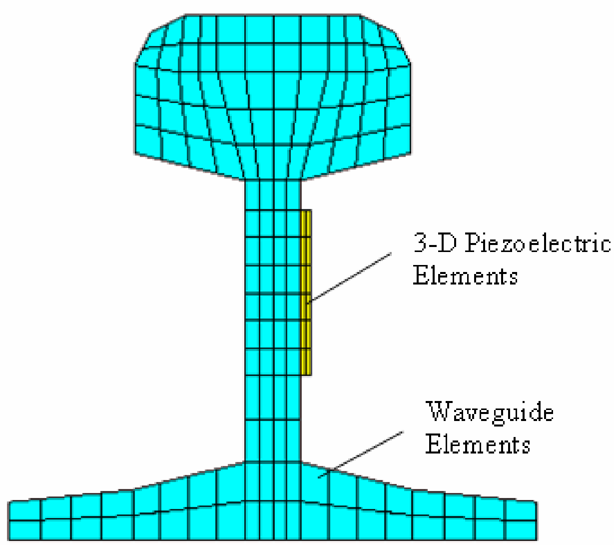


Figure 2: Finite element mesh rail and piezoelectric patch.

Equation 3, with $f=0$, was used to compute the dispersion characteristics of the rail by setting the frequency and computing the wavenumbers (complex) at equally spaced frequency points. The first 20 real wavenumbers (propagating modes) are plotted in figure 3. The mode separation technique outlined in section 2.4 was used to automatically identify the modes when the curves intersect. The corresponding group velocities are plotted in figure 4. The group velocities were computed using an analytical expression based on the eigensolution without resorting to a finite difference approximation [5]. In the procedure points with negative group velocity were excluded and the wavenumber was only plotted once the group velocity became positive. In this way the problem of having two wavenumbers for a mode at a particular frequency was avoided. The wavenumber and group velocity are properties of the rail only and do not involve the piezoelectric patch. Figure 5 shows the amplitude of each of the first 20 propagating waves excited by the patch when driven with a 1 V_{peak} harmonic excitation.

In order to verify the proposed numerical method, time responses were computed and compared to measured responses. The measurements were performed using a National Instruments data acquisition card to generate the signal and capture the response. The excitation used was a Hanning windowed 7.5 cycle 15 kHz tone-burst with duration of 0.5ms. The signal was amplified to 200V_{peak} using an A-303 piezo driver amplifier from AA-Lab Systems and the displacement response was measured using a Polytec laser vibrometer system. The laser vibrometer system comprised the OFV-505 sensor head and the OFV-5000 controller equipped with the VD-06 digital velocity decoder and the DD-500 displacement decoder. The signals from the laser vibrometer were captured by the data acquisition card and the average of 100 measurements was recorded. The measured signal was converted to a displacement in meters and divided by 200 so that it could be compared to the computed results where a 1 V_{peak} excitation was used.

The horizontal response of the web of the rail at a distance of 1.25m from the piezoelectric patch was computed using the method of section 2.3. The computed response is shown in figure 6a, while the equivalent measured response is shown in figure 6b. The computed and measured responses shown in figure 6 appear to be similar in shape and magnitude. The computed result was based on only the first eight propagating waves. If the tenth propagating wave was included the predicted response was far larger. It is believed that his occurred because this wave cuts-on at a little over 15 kHz and at this frequency it has resonant behavior as shown by the modal amplitude which is effectively infinite at the cut-on frequency (see figure 5). This occurs because no material damping has been included in the modeling. Wave propagation in damped rails was analyzed by Bartoli et al. [6] who compute the attenuation of the waves, which appears to be infinite at the cut-on frequency. Damping has not been included in the modeling in this paper but it is suspected that including damping would solve the problem of the large response at the cut-on frequency.

The computed and measured responses of the top of the rail in the vertical direction, again at a distance of 1.25m, are shown in figure 7. Again it appears that the responses are qualitatively similar. In this case, however, the experimental result is three to four times smaller than the computed result. The cause for this difference has not been established.

The frequency of 15 kHz was chosen for investigation because in addition to the two cut-on frequencies there is also a mode with large dispersion at this frequency. This is evident in the wavenumber versus frequency plot where this curve changes slope drastically or in the group velocity plot where the group velocity decreases rapidly to a minimum close to 15 kHz and then increases again. The response, in the vertical direction at 1.25 m from the piezoelectric patch, of only this mode of propagation is shown in figure 8 along with the group velocity of only this mode. Because of the large dispersion the time response appears to contain two modes of propagation but this is due to the differences in group velocity around this frequency.

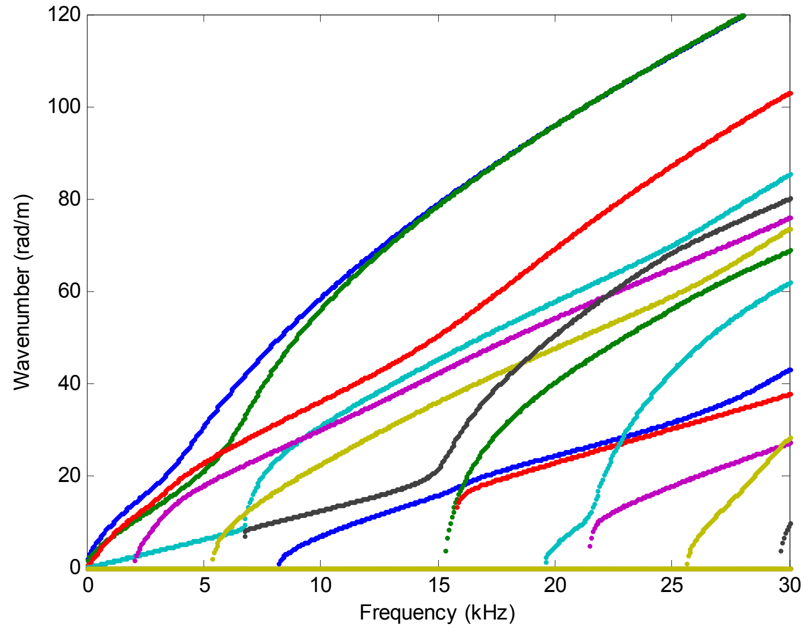


Figure 3: Wavenumber – frequency plot of the 1st 20 propagating waves.

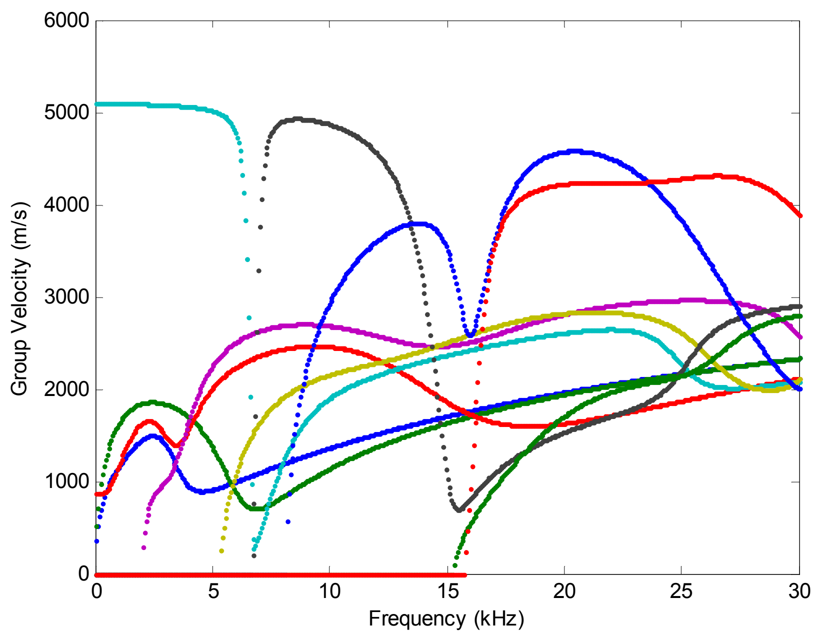


Figure 4: Group velocity – frequency plot of the 1st 20 propagating waves.

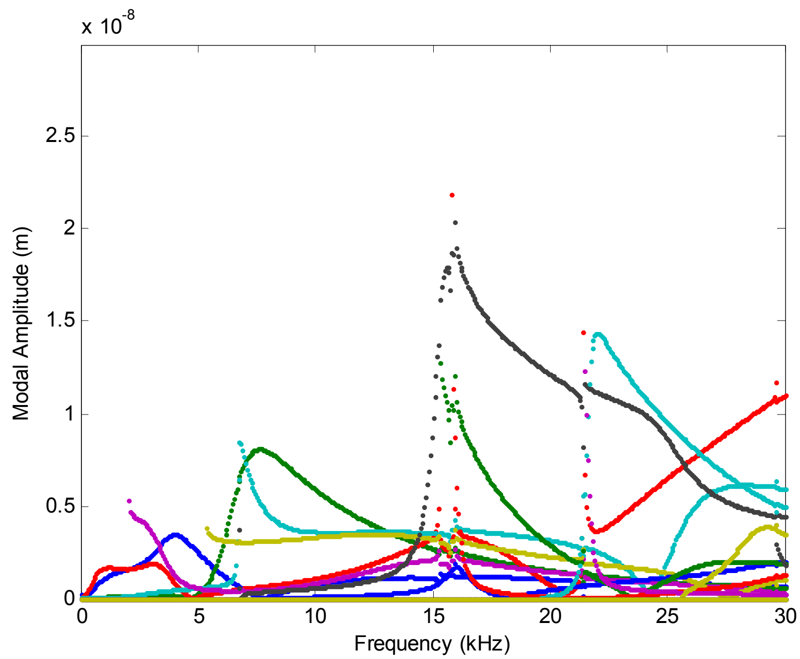
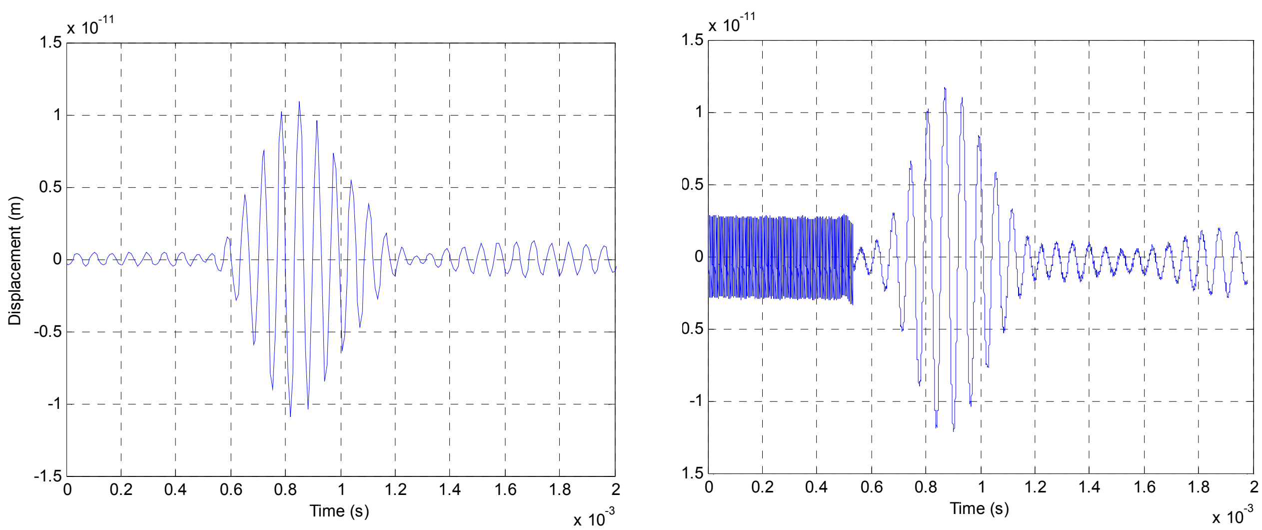


Figure 5: Amplitude – frequency plot of the 1st 20 propagating waves.



(a) (b)

Figure 6: Computed and Measured Horizontal Displacements.

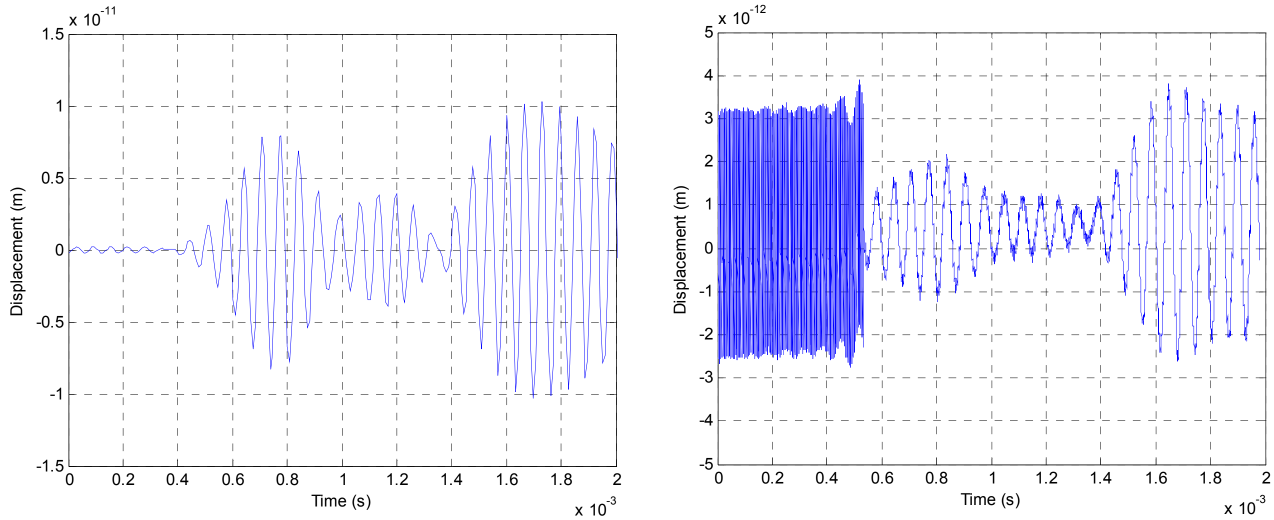


Figure 7: Computed and measured vertical displacements.

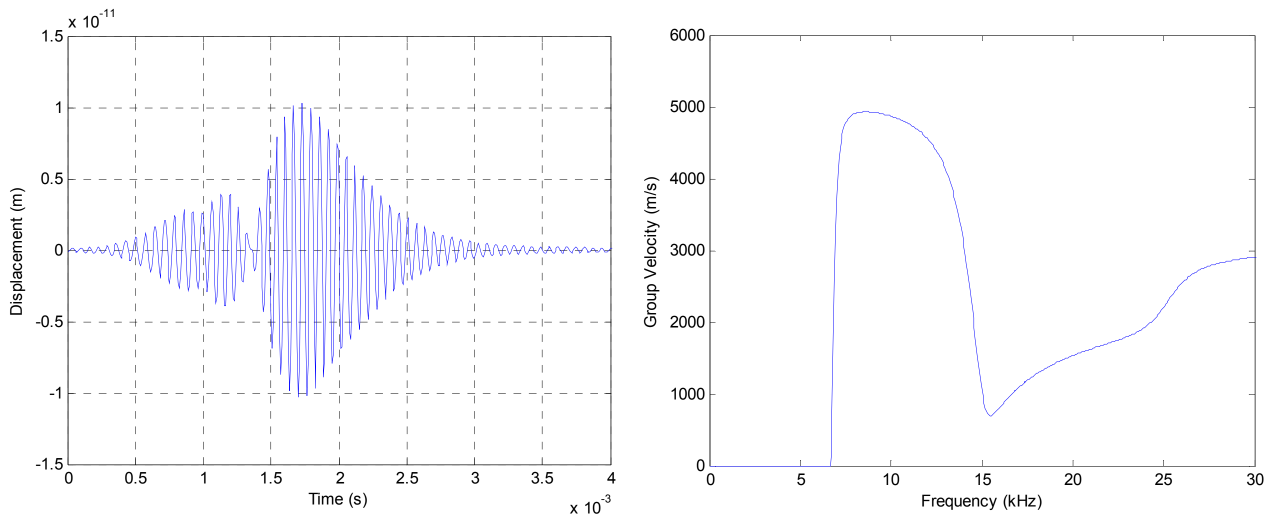


Figure 8: Computed response and group velocity of a highly dispersive mode.

4. CONCLUSIONS

A method for computing the time domain displacement responses of one-dimensional waveguides, due to excitation by piezoelectric transducers, has been developed. The method was applied to compute the waves propagating in a rail when excited by a rectangular piezoelectric patch transducer. The computed responses compared favorably to experimental measurements performed with a laser vibrometer.

The method can compute the response of individual waves of propagation thus providing insight into measured responses, which are a superposition of all the waves. It is possible to determine which modes of propagation dominate the response and to predict the signals that would be obtained at large distances, which cannot be measured in the lab nor be computed using conventional finite element models.

The waveguide model did not include damping and excessive responses were computed at the cut-on frequency of a mode of propagation. It is recommended that damping be added in future to determine if damping will eliminate this problem.

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