INFLUENCE OF CROSS-ANISOTROPY MATERIAL BEHAVIOR ON BACK-CALCULATION ANALYSIS OF MULTI-LAYERED SYSTEMS

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ABSTRACT
Each layer in a pavement structure is compacted during construction. Moreover, since the compaction process is carried along the same direction, it is well known that horizontal and vertical mechanical properties in each layer differ. But when FWD test results are analyzed, it is assumed that layer properties are homogenous and isotropic. The influence of anisotropic material property on pavement performance has never been evaluated. In this research, pavement base layer materials are assumed to be cross-anisotropic and by assuming a variety of horizontal and vertical elastic moduli, surface deflections were computed. These deflections were used to backcalculate equivalent layer moduli assuming isotropic material property. Finally, by using standard performance prediction models, results for the case of cross-anisotropic and isotropic base materials were compared.

KEY WORDS
cross-anisotropy, multi-layered system, backcalculation, flexible pavement

INTRODUCTION

Most software for pavement analysis like CHEVRON, BISAR and GAMES[1] were developed by assuming all materials to be homogeneous and isotropic. But it is well known that base course materials depict cross-anisotropic property[2][3]. Adu-Osei, et al.[4] and Tutumluer et al.[5] obtained this material property from laboratory tests. Further, Masad et al.[6] added cross-anisotropic material property in their stress dependency model and performed Finite Element Analysis (FEA). On the other hand, Wang et al.[7] confirmed existence of cross-anisotropic material property in asphalt concrete material and investigated its effect by using Finite Element Method (FEM). Very important theoretical researches for a semi-infinite medium are the solutions on cross-anisotropic material property presented by Lekhnitskii[8] as well as Gerrard and Harrison[9]. Authors of this paper have also developed software CRANES (CRoss ANisotropic Elastic Systems) for pavement structural analysis considering cross-anisotropic material property.

In this research, CRANES software was used to determine pavement surface deflections by assuming cross-anisotropic material property of the base course only. Using the computed surface deflections, backcalculation analysis for equivalent layer moduli considering isotropic material property was performed using BALM (Back Analysis of Layer Moduli) software. After that, equivalent layer moduli obtained from backcalculation process were used in GAMES software to determine strains at the bottom of asphalt concrete layer (ε_r) and at the top of the subgrade layer (ε_z). Results of the strains obtained by considering cross-anisotropic and isotropic material properties were compared. It was found that cross-anisotropic material property have little effect on the strain results at the bottom of asphalt concrete layer but has significant effect on the strain results at the top of the subgrade layer.

Theory
Cross-Anisotropy

Derivation for the case of axi-symmetric loading is presented hereunder. By neglecting body forces and similar to the isotropic case, the equilibrium equation may be written as follows:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad \textbf{(1-a)}$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0 \qquad \textbf{(1-b)}$$

Deflection in r for axi-symmetric case may be represented as $u = u(r,z)$, whereas deflection in z may be represented as $w = w(r,z)$. Circumferential (θ) deflection is zero. Normal stresses in r, θ and z directions may be represented as σ_r, σ_θ, and σ_z, respectively, while shear stress in $r-z$ section will be τ_{rz}. Strains corresponding the these stresses are ε_r, ε_θ, ε_z and γ_{rz}. Strains-deflection relationship is similar to the case where material property is assumed to be isotropic and may be written shown below:

$$\varepsilon_r = \frac{\partial u}{\partial r}, \quad \varepsilon_\theta = \frac{u}{r}, \quad \varepsilon_z = \frac{\partial w}{\partial z}, \quad \gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \tag{2}$$

The difference between anisotropic and isotropic material properties is in the expressions of strains in terms of stresses. Strain-stress relationship for cross-anisotropic material may be written as follows:

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \\ \gamma_{rz} \end{Bmatrix} = \begin{bmatrix} 1/E_h & -v_h/E_h & -v_v/E_v & 0 \\ -v_h/E_h & 1/E_h & -v_v/E_v & 0 \\ -v_v/E_v & -v_v/E_v & 1/E_v & 0 \\ 0 & 0 & 0 & 1/G \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{rz} \end{Bmatrix} \tag{3}$$

where E_h : elastic modulus in horizontal direction

E_v : elastic modulus in vertical direction

v_h : Poisson's ratio for effect of horizontal stress on horizontal strain

v_v : Poisson's ratio for effect of vertical stress on vertical strain

G : shear modulus

In case of isotropic material, strains and stresses may be expressed using only two parameters, namely elastic modulus and Poisson's ratio, whereas in case of cross-anisotropic material, five parameters are used to express strain-stress relationships, namely, moduli of elasticity in horizontal and vertical directions, Poisson's ratios for effects of stresses on strains in vertical and horizontal directions and shear modulus.

Stresses and deflections may be determined by adapting Hankel transform and based on strain-stress relationship, strains may also be determined. By using the procedure explained above, CRANES software, which is capable of analyzing the effect of cross-anisotropic material, was developed.

WORKED EXAMPLE
Example model
It is well known that compacted granular base materials are cross-anisotropic. Bearing that in mind, an example of a three-layer system consisting of cross-anisotropic base course is shown in **Figure 1**. Materials in the first, second and third layers are asphalt concrete, macadam, and subgrade soil, respectively. Because of the unidirectional compaction during the construction process, it is assumed that property of the second layer, which is made of granular materials, will be cross-anisotropic, while the first and third layers are considered isotropic.

Elastic modulus in vertical direction E_{2v} = 400 MPa for the second layer was held constant. In order to evaluate the effect of cross-anisotropic material property, a total of 5 different elastic moduli in horizontal direction were considered as shown in **Table 1**. Shear modulus is another independent parameter. However the value obtained from equation (4) is herein used.

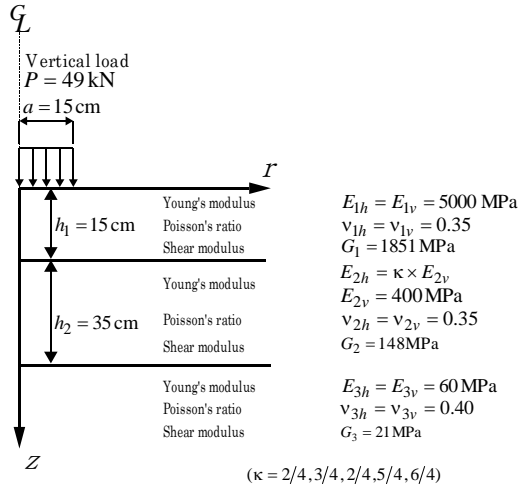$$G_2 = E_{2v}/2(1 + v_{2v}) \tag{4}$$

Figure 1 Three-layered pavement model

κ in **Table 1** is the ratio between E_{2h} and E_{2v}, whereby $\kappa = 4/4$ represents isotropic material property.

$$\kappa = E_{2h}/E_{2v} \qquad (5)$$

Table 1 Layer material properties

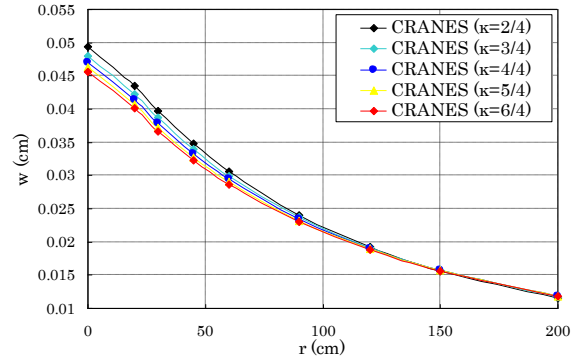|		E_h	E_v	G	v_h	v_v
Unit		MPa	MPa	MPa		
1 Layer		5000.0	5000.0	1851.0	0.35	0.35
2 Layer	$\kappa=2/4$	200.0	400.0	148.1	0.35	0.35
	$\kappa=3/4$	300.0	400.0	148.1	0.35	0.35
	$\kappa=4/4$	400.0	400.0	148.1	0.35	0.35
	$\kappa=5/4$	500.0	400.0	148.1	0.35	0.35
	$\kappa=6/4$	600.0	400.0	148.1	0.35	0.35
3 Layer		60.0	60.0	21.4	0.40	0.40

Figure 2 Surface deflection

CRANES was used to determine surface deflections for all the pavement layer parameters shown in **Figure 1** and **Table 1**. Surface deflections were determined at positions corresponding to the Falling Weight Deflectometer (FWD) sensors positions (i.e. 0, 20, 30, 45, 60, 90, 120, 150, 200 (cm)) relative to the center of the loading plate.

Surface deflection
Figure 2 shows results of surface deflections, w. As the ratio of elastic moduli becomes smaller, deflections near the point of loading become bigger but the differences decrease for points far from the point of loading. Differences in deflections for points further than 150 cm from the point of loading, are negligible irrespective of elastic moduli ratio. Since the only difference between analyses for isotropic and cross-anisotropic material properties used here is the elastic modulus in horizontal direction, this is the only factor that contributes to the differences in surface deflections. Differences in surface deflections at the center of the load ($r = 0$) for $\kappa = 2/4$ and $6/4$ is 0.0037 cm, which is about 8% of the deflection for $\kappa = 6/4$. In this case, cross-anisotropic material property for the second layer may not be neglected.

Backcalculation
Surface deflections obtained by CRANES software for the five combinations of cross-anisotropic material properties of the second layer were used in BALM'2003 software to backcalculate equivalent layer moduli. Except for the layer moduli, all other material properties shown in **Figure 1** were used in backcalculation analyses.

Since backcalculation results depend on the seed values, uniform random numbers were used to generate 100 different seed values based on the boundaries and analytical model shown in
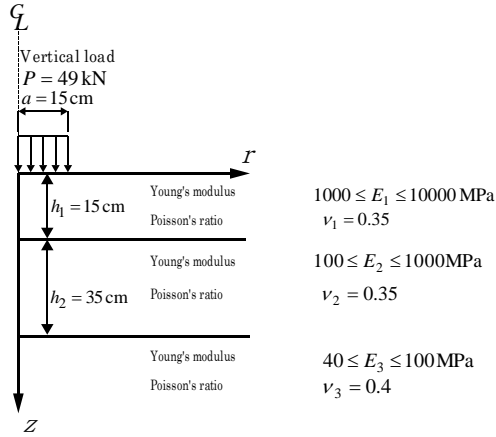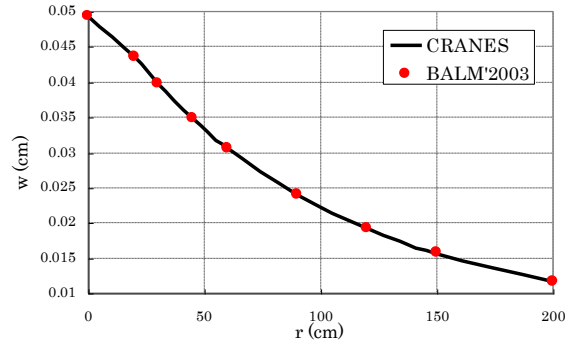
Figure 3 Back-calculation model

Figure 4 Surface deflection ($\kappa = 2/4$)

Table 2 Layer moduli

			Layer moduli (MPa)		
			Layer 1	Layer 2	Layer 3
κ=2/4	Back-calculation	Averege	5383.8	314.4	59.9
		Standard Deviation	1.34	8.16E-02	1.43E-14
	Cross-anisotropy	Ev	5000.0	400.0	60.0
		Eh	5000.0	200.0	60.0
		G	1851.0	148.1	21.0
κ=3/4	Back-calculation	Averege	5176.7	361.7	59.9
		Standard Deviation	0.58	7.16E-02	1.43E-14
	Cross-anisotropy	Ev	5000.0	400.0	60.0
		Eh	5000.0	300.0	60.0
		G	1851.0	148.1	21.0
κ=4/4	Back-calculation	Averege	4997.0	400.1	60.0
		Standard Deviation	1.27	8.87E-02	6.39E-03
	Cross-anisotropy	Ev	5000.0	400.0	60.0
		Eh	5000.0	400.0	60.0
		G	1851.0	148.1	21.0
κ=5/4	Back-calculation	Averege	4839.5	432.4	60.2
		Standard Deviation	1.27	0.18	7.57E-03
	Cross-anisotropy	Ev	5000.0	400.0	60.0
		Eh	5000.0	500.0	60.0
		G	1851.0	148.1	21.0
κ=6/4	Back-calculation	Averege	4755.1	458.1	60.3
		Standard Deviation	1.03	0.13	9.03E-03
	Cross-anisotropy	Ev	5000.0	400.0	60.0
		Eh	5000.0	600.0	60.0
		G	1851.0	148.1	21.0

Figure 3. **Figure 4** is a plot of surface deflections determined by CRANES for $\kappa = 2/4$ as well as surface deflections from backcalculated layer moduli determined by BALM'2003. This figure shows very good agreement between surface deflections considering cross-anisotropic material property and equivalent layer moduli.

Table 2 shows backcalculated layer moduli and standard deviation values of the results. The highest standard deviation of the results is 1.34MPa, which is for the first layer when $\kappa = 2/4$, this is an indication that good and stable backcalculated results were obtained.

With regards to equivalent layer moduli results, the following observations were made:

Equivalent elastic modulus of the first layer was relatively lower for lower elastic modulus in horizontal direction ($\kappa = 2/4$) and relatively higher for higher elastic modulus in horizontal direction ($\kappa = 6/4$). This is influenced by the magnitude of surface deflections used in backcalculation analysis as shown in **Figure 2**, where bigger deflections for $\kappa = 2/4$ resulted in lower equivalent elastic modulus and smaller deflections for $\kappa = 6/4$ resulted in higher equivalent elastic modulus.

With regards to the second layer, backcalculation analyses converged to values within elastic moduli in vertical (E_{2v}) and horizontal (E_{2h}) directions. Since elastic modulus in vertical direction and Poisson's ratio for effect of vertical stress on vertical strain were used to determine shear modulus, this may be the reason why backcalculated results were closer to the elastic modulus in the vertical direction.

Results for the subgrade soils were very close to the theoretical values used in CRANES. This shows that equivalent elastic modulus for the third layer is not affected by cross-anisotropic material property of the second layer.

Comparison of Cross-Anisotropy with back-calculated results
Strains ε_x at the bottom of surface layer and ε_z at the top of subgrade layer were computed for both cross-anisotropic and backcalculated elastic layer moduli. Pavement performances were determined using Asphalt Institute (AI) models shown in equations (6) and (7).

$$N_{fa} = \beta_{a1} \cdot C \cdot (6.167 \times 10^{-5} \cdot \varepsilon_t^{-3.291\beta_{a2}} \cdot E^{-0.854\beta_{a3}} \tag{6}$$

where, $\beta_{a1}, \beta_{a2}, \beta_{a3}$: are Japanese correction factors for AI model
$\quad\quad\quad C$: parameter for asphalt mix volumetric properties
$\quad\quad\quad N_{fa}$: allowable number of 49kN axles
$\quad\quad\quad \varepsilon_t$: tensile strain at the bottom of asphalt layer
$\quad\quad\quad E$: elastic modulus

$$N_{fs} = \beta_{s1} \cdot (1.365 \times 10^{-9} \cdot \varepsilon_z^{-4.477\beta_{s2}}) \tag{7}$$

where, β_{s1}, β_{s2} : are Japanese correction factors for AI model
$\quad\quad\quad N_{fs}$: allowable number of 49kN axles
$\quad\quad\quad \varepsilon_z$: compressive strain at the top of subgrade layer

Equation (6) is asphalt damage model that gives allowable number of repetitions of 49 kN axles for cracks to occur at the bottom of asphalt layer and propagate to the surface until 20% of the surface is covered with cracks. Equation (7) is damage model, which gives allowable number of repetitions of 49 kN axles until 15 mm rutting occurs.

Figure 5 shows ε_x at the bottom of the first layer. In this figure, (a) and (b), are the results for different κ values. Very good agreement between strains were obtained for each κ value. This is an indication that equivalent backcalculated layer moduli may be used with good

(a) $\kappa = 2/4$ (b) $\kappa = 6/4$
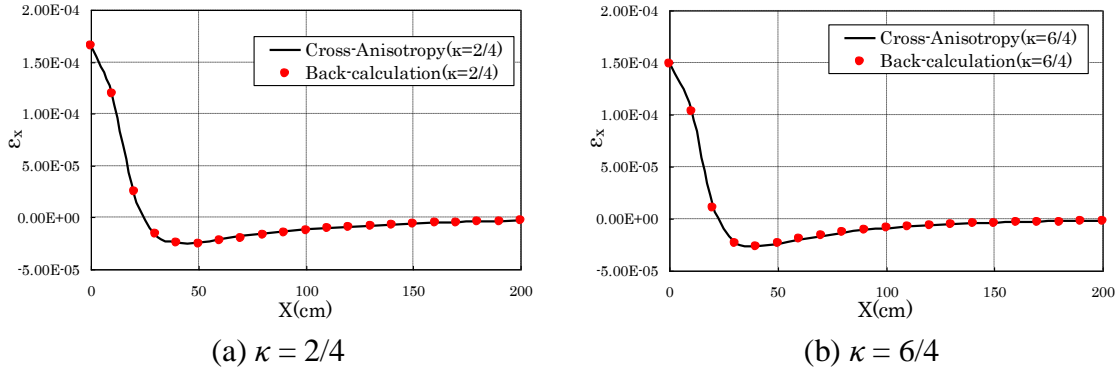
Figure 5   Strain ε_x at bottom of first layer

(a) $\kappa = 2/4$ (b) $\kappa = 6/4$
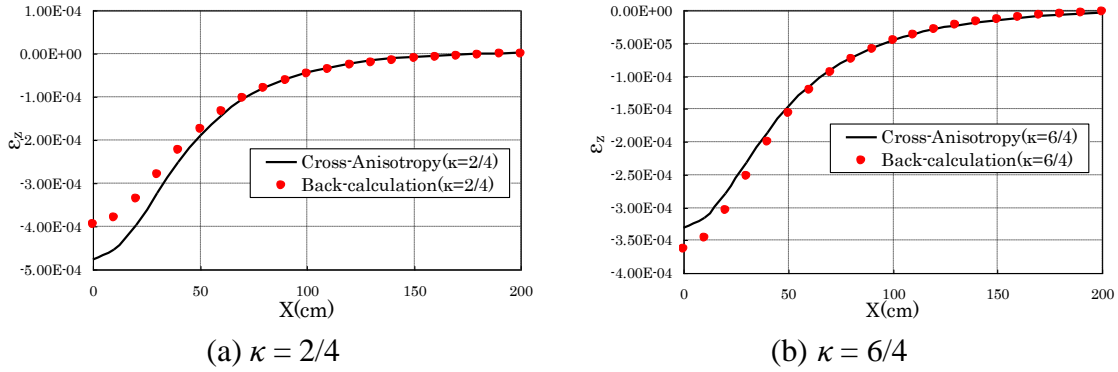
Figure 6   Strain ε_z at top of subgrade layer

Table 3   Comparison of pavement performance

κ	Asphalt mixture		Subgrade	
	cross-anisotropic	isotropic	cross-anisotropic	isotropic
$\kappa=2/4$	4.76335E+07	4.00619E+07	4.43839E+06	8.75649E+06
$\kappa=3/4$	6.08862E+07	5.17216E+07	7.39462E+06	9.70471E+06
$\kappa=4/4$	6.34375E+07	6.34817E+07	1.05433E+07	1.05392E+07
$\kappa=5/4$	6.93598E+07	7.54432E+07	1.37267E+07	1.12835E+07
$\kappa=6/4$	7.56256E+07	8.60771E+07	1.68418E+07	1.20047E+07

accuracy to determine tensile strains at the bottom of asphalt layer.

Figure 6 shows strain ε_z at the top of the subgrade layer. Results show good agreement for points which are more than 100 cm from the center of loading and poor agreement for points within 100 cm from the center of loading. Results with worst agreement was for $\kappa = 2/4$, where at the center of load (x = 0), the absolute difference was 8.05E-05. Further, strain with anisotropic value is larger that that with isotropic value in case of $\kappa = 2/4$ and smaller in case of $\kappa = 6/4$. This tendency may be influenced by layer moduli obtained from backcalculation analyses. Good load dispersion to the underlying layers is obtained when elastic modulus of the first layer is big enough, which will result in smaller ε_z at the top of subgrade layer. And for smaller elastic moduli of the first layer, bigger values of ε_z at the top of subgrade layer will be obtained. This trend was observed for all results obtained in this study.

Comparison of pavement performance using equations (6) and (7) are as shown in **Table 3**. "Isotropic" means backcalculation results were used to determine pavement performance.

Asphalt concrete was underestimated by 15% when $\kappa = 2/4$ and overestimated by 11% when $\kappa = 6/4$. For the case of subgrade layer, performance prediction using backcalculated layer moduli was twice as much as performance for $\kappa = 2/4$.

CONCLUSIONS
Comparison of strains obtained from multi-layered linear elastic analyses considering cross-anisotropic and isotropic material properties, the following conclusions were reached:

(1) Deflections near the load application points are different when isotropic and cross-anisotropic base course material properties are considered, however good match of deflections is obtained for points far from the point of loading.

(2) Anisotropic material property of the second layer does not influence elastic modulus of the third layer if backcalculation analysis is performed assuming isotropic material properties,. However, elastic modulus of the first layer is overestimated for $\kappa < 4/4$ and underestimated for $\kappa < 4/4$.

(3) Differences of strains ε_x at the bottom of the first layer between isotropic and anisotropic material properties are negligible.

(4) Strains ε_z at the top of the third layer directly underneath the load are different for isotropic and cross anisotropic properties but agree well for points far from the load point.

(5) Performance prediction for the first layer using isotropic property compared to cross-anisotropic property is relatively shorter for $\kappa < 4/4$ and longer for $\kappa > 4/4$.

(6) Performance prediction for the third layer using isotropic property compared to cross-anisotropic property is very long for $\kappa < 4/4$ and very short for $\kappa > 4/4$.

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