

Numerical comparison of patch and sandwich piezoelectric transducers for transmitting ultrasonic waves

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ABSTRACT

Guided wave ultrasonic inspection is becoming an important method of non-destructive testing for long, slender structures such as pipes and rails. Often it is desirable to use transducers that can strongly excite a specific mode of wave propagation in the waveguide. Piezoelectric patch transducers are frequently employed, by researchers, for exciting waves in beam like structures. Sonar systems frequently make use of resonant transducers, such as sandwich transducers, for acoustic wave generation and this principle has been used to excite waves in a rail. This paper compares the two transduction approaches, for launching bending waves in rectangular waveguides, with numerical modeling. The numerical modeling combined a waveguide finite element model, of the waveguide, with conventional three-dimensional piezoelectric finite element models of the transducers. The waveguide finite elements were formulated using a complex exponential to describe the wave propagation along the structure and conventional finite element interpolation over the area of the element. Consequently, only a two-dimensional finite element mesh covering the cross-section of the waveguide is required. The harmonic forced response of the waveguide was used to compute a complex dynamic stiffness matrix which represented the waveguide in the transducer model. The effects of geometrical parameters of patch and sandwich transducers were considered before the comparison was made. It appears that piezoelectric patch transducers offer advantages at low frequencies while sandwich transducers are superior at high frequencies, where resonance can be exploited, at the cost of more complex design.

Keywords: finite element method, elastic waveguide, piezoelectric transducer

1. INTRODUCTION

Guided wave ultrasound is an attractive method for inspecting structures in certain situations. A review of guided wave ultrasonic inspection was presented by Rose [1]. When the structure is long and slender, such as a rail or a pipe, the method can be used to inspect large lengths of the structure, possibly even including inaccessible sections. If transducers are permanently attached to the structure, continuous monitoring is possible [2]. The design of an inspection system requires knowledge of the various modes of wave propagation in the structure, how these modes interact with damage and the ability to independently excite and sense particular modes of wave propagation. When large lengths are to be inspected it is desirable to be able to excite the modes strongly to permit greater lengths to be inspected.

Piezoelectric transducers are frequently used to excite the waves used in guided wave inspection systems. The excitation of low-frequency flexural and longitudinal waves in infinite beams of rectangular cross-section, by piezoelectric patch transducers, was analyzed by Gibbs and Fuller [3] and was reproduced in [4]. The analysis was based on Euler-Bernoulli beam theory and was limited to cases when the patch is thin compared to the beam. Giurgiutiu [5] analyzed Lamb wave generation, by piezoelectric patch actuators, in infinite plates. Guided wave inspection systems often operate at fairly low frequencies (up to 200 kHz) compared to conventional ultrasonic inspection techniques. Underwater sonar systems, operating at these frequencies, often make use of sandwich transducer configurations (sometimes referred to as Langevin or 'tonpiliz' transducers). A sandwich transducer was developed for transmitting waves, which were easily detected over 1 km away, in a rail monitoring system [2]. The piezoelectric

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patches principally apply a bending excitation while the sandwich transducer principally applies inertial forces along the axis of the transducer. The purpose of this study is to compare the excitation by piezoelectric patches to the excitation by sandwich transducers with a view to understanding which method is superior for a given application. The comparison is based on numerical modeling of the two transducers attached to a waveguide.

2. FINITE ELEMENT MODELING APPROACH

The excitation of low frequency waves in simple rectangular beams, by piezoelectric patch actuators, was analyzed by Gibbs and Fuller [3]. Their analysis is only valid for patches that are thin compared to the beam and only the longitudinal and flexural waves were considered. A general numerical modeling method for analyzing the excitation of complex waves in waveguides of complex (but constant) cross-section by arbitrary piezoelectric transducers has been developed [6]. This method was used in this research as it is general and can easily model both excitations by patch actuators and sandwich transducers. The method makes use of specially developed waveguide finite elements, which require only a two-dimensional mesh of the cross-section of the waveguide. The response of the waveguide to harmonic excitation is used to determine a complex boundary condition representing the waveguide in a finite element model of the piezoelectric transducer. This model allows computation of the frequency response of the transducer when attached to the waveguide. The forces at the interface between the transducer and waveguide are computed and used to determine the response of the waveguide. The response of each mode of the waveguide is computed and the contribution from each mode can be summed to provide the total response. This method has advantages over conventional time domain simulation of a length of waveguide as it requires only a two dimensional model of the waveguide, provides frequency response information directly (no Fourier transforms required) and provides the response of the individual modes, which can be difficult to extract from time domain simulations. This section provides an overview of the mathematical details of the method.

2.1 Formulation of the Waveguide Finite Elements

The finite elements used to describe the waveguide employ a complex exponential function to describe the variation of the displacement field along the waveguide axis and conventional interpolation functions over the area of the element. This means that only a two dimensional mesh of the cross-section of the waveguide is required. The displacement fields (u, v, w) in an elastic waveguide, extending in the z direction (as shown in Figure 1), can be written as:

$$\begin{aligned}
 u(x, y, z, t) &= u(x, y) \cdot e^{-j(\kappa z - \omega t)} \\
 v(x, y, z, t) &= v(x, y) \cdot e^{-j(\kappa z - \omega t)} \\
 w(x, y, z, t) &= j \cdot w(x, y) \cdot e^{-j(\kappa z - \omega t)}
 \end{aligned} \tag{1}$$

where, z is the coordinate in the direction along the waveguide, κ the wavenumber and ω the frequency.

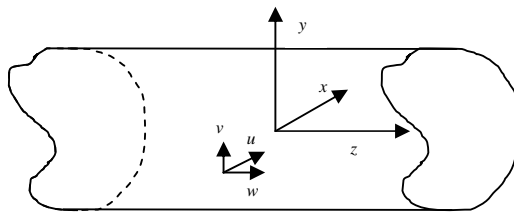


Figure 1: Waveguide coordinates and displacements.

The formulation used in this paper follows that presented by Gavrić [7]. The formulations of Hayashi [8] and Damljanić and Weaver [9] are similar to this formulation. Hayashi's formulation results in complex equations of motion while Damljanić and Weaver also derived complex equations of motion and then applied a transformation to obtain equations of motion comparable to those of Gavrić.

The strain (ε) and strain energy (k) of the waveguide can be separated into terms that are independent, linearly dependent or quadratically dependent on the wavenumber (* indicates complex conjugate transpose).

$$\varepsilon = \varepsilon_0 + \kappa \varepsilon_1$$

$$\varepsilon = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \kappa w \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} \\ -j\kappa v + j\frac{\partial w}{\partial y} \\ -j\kappa u + j\frac{\partial w}{\partial x} \end{Bmatrix}; \quad \varepsilon_0 = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ 0 \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} \\ j\frac{\partial w}{\partial y} \\ j\frac{\partial w}{\partial x} \end{Bmatrix}; \quad \varepsilon_1 = \begin{Bmatrix} 0 \\ 0 \\ w \\ 0 \\ -jv \\ -ju \end{Bmatrix}; \quad (2)$$

$$k_0 = \varepsilon_0 c \varepsilon_0^*$$

$$k_1 = \varepsilon_0 c \varepsilon_1^* + \varepsilon_1 c \varepsilon_0^*$$

$$k_2 = \varepsilon_1 c \varepsilon_1^* \quad (3)$$

Applying conventional finite element discretization to these terms yields the elemental mass and stiffness matrices.

$$k_0 = \int B_0 c B_0^* \det J d\xi d\eta$$

$$k_1 = \int [B_0 c B_1^* + B_1 c B_0^*] \det J d\xi d\eta$$

$$k_2 = \int B_1 c B_1^* \det J d\xi d\eta$$

$$m = \int N \rho N^* \det J d\xi d\eta \quad (4)$$

Assembly of the element matrices produces the system equations of motion for the waveguide.

$$M \ddot{u} + [\kappa^2 \cdot K_2 + \kappa \cdot K_1 + K_0] u = f \quad (5)$$

2.2 Combining waveguide and conventional finite element models

The approach adopted was to use the waveguide finite element model to calculate the receptance of the waveguide to point forces. The receptance is used as a boundary condition, representing the waveguide in a conventional finite element model [10] of the piezoelectric patch actuator. The forces applied to the waveguide are computed and then applied to the waveguide model to compute the response of the waveguide.

The forced response of the waveguide finite element model was developed by Damljanić and Weaver [11]. Their finite element formulation is slightly different to that used here but the method of solving the forced response still applies. The equations of motion may be solved by a method similar to that used for solving multi-degree-of-freedom damped oscillator systems. Equation 5 is complemented with an identity as follows,

$$\begin{bmatrix} K_0 - \omega^2 M & 0 \\ 0 & -K_2 \end{bmatrix} \begin{Bmatrix} u \\ \kappa u \end{Bmatrix} + \kappa \begin{bmatrix} K_1 & K_2 \\ K_2 & 0 \end{bmatrix} \begin{Bmatrix} u \\ \kappa u \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \quad (6)$$

so that it may be written in the form

$$A \bar{u} - \kappa B \bar{u} = \bar{f} \quad (7)$$

The free vibration problem ($f = 0$) is solved by performing an eigensolution. If a constant, real wavenumber (κ) is selected a real eigenproblem (5) must be solved for the frequencies (ω) and mode shapes (Ψ) of the propagating waves that correspond to this wavenumber. This solution is useful for determining the dispersion characteristics of a waveguide. Analysis of the transducer-waveguide frequency response requires that the frequency be specified and a complex eigenproblem must be solved. The wavenumbers that are obtained, by solving this problem, can be real, imaginary or complex and occur in pairs with opposite sign corresponding to waves traveling in opposite directions. If the number of nodes in the model is denoted N , the eigensolution results in $6N$ eigenvalue-eigenvector pairs κ_r and Ψ_r .

The solution to the forced harmonic vibration problem is found by applying a Fourier transform to (7) to obtain an equation in the wavenumber domain, solution in the wavenumber domain and inverse Fourier transform to obtain the solution in the spatial domain [11],

$$\bar{u}(z) = -j \sum_{r=1}^{3N} \Psi_r \frac{\Psi_r^T \bar{f}(z)}{\Psi_r^T B \Psi_r} e^{-j\kappa_r z} \quad (8)$$

where the summation is performed only over the positive real poles, negative imaginary poles and complex poles with negative imaginary parts.

The response of the waveguide to forces at each of the degrees of freedom (dof) in contact with the piezoelectric patch may be computed by (8) and the receptance r_{ij} is defined as the response at dof i due to a unit force applied at dof j , i.e. $u_i = r_{ij} f_j$. The displacements at the interface dof (u_{in}) due to loads at the interface dof can then be related by the receptance matrix,

$$u_{in} = R f_{in} \quad (9)$$

Recalling that the response of the waveguide was computed for a particular frequency of harmonic excitation, the inverse of this matrix (D_w) is the dynamic stiffness matrix of the interface dof's, at this frequency, i.e.

$$D_w u_{in} = f_{in} \quad (10)$$

The dynamic stiffness matrix of the waveguide is symmetric but fully populated. It is also complex representing the mass/stiffness loading and the damping due to energy being radiated along the waveguide.

A dynamic stiffness for the piezoelectric patch can be computed at the excitation frequency and this matrix (D_p) can be partitioned into degrees of freedom in contact with the waveguide (u_{in}) and degrees of freedom that are not in contact (u_n). The two dynamic stiffness matrices can then be combined to represent the piezoelectric patch attached to the waveguide.

$$\begin{bmatrix} D_{p_{nn}} & D_{p_{ni}} \\ D_{p_{in}} & D_{p_{in}} + D_w \end{bmatrix} \begin{Bmatrix} u_n \\ u_{in} \end{Bmatrix} = \begin{Bmatrix} f_n \\ f_{in} \end{Bmatrix} \quad (11)$$

The forces in this equation include electrically induced piezoelectric forces. This equation allows the computation of the forced harmonic response of the piezoelectric patch (attached to the waveguide). The forces applied to the waveguide can be computed by substituting the interface displacements into (10). The response of the waveguide can then be computed by substituting these interface forces into (8). Our interest will often be in the amplitude of response of a particular mode of wave propagation rather than the amplitude at a specific point on the waveguide. This response is written as,

$$p_r(z) = -j \frac{\Psi_r^T \bar{f}}{\Psi_r^T B \Psi_r} e^{-j\kappa_r z} \quad (12)$$

3. COMPARISON OF TRANSDUCERS

The comparison of the two transduction methods requires that certain restrictions are used. Firstly, the volume of piezoelectric ceramic material should be identical in both situations. Secondly, the applied electrical field should be the same as this rather than applied voltage is a limiting factor. While a compressive pre-stress is easily applied to the

piezoelectric ceramic in sandwich transducers this is not done in patch transducers. The higher drive levels allowed because of the pre-stress are not taken into account in this work. It is also assumed that the transducers are excited with short bursts and that heating effects are negligible. To simplify the modeling the sandwich transducer had a rectangular cross-section as shown in Figure 2. In practice such transducers usually have circular cross-sections.

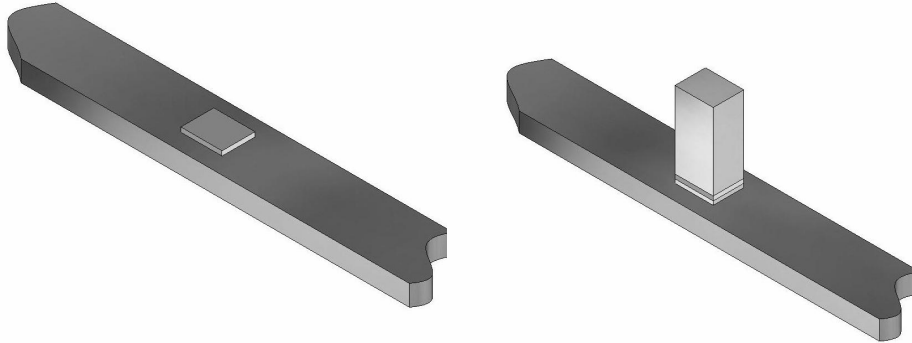


Figure 2: Geometry of piezoelectric patch and sandwich transducers.

The frequency range of operation has to be considered. In guided wave ultrasound applications this is often determined by the dispersion properties of the waves being excited. The wavenumber and group velocity of propagating waves in a 16 mm by 5 mm rectangular aluminum beam are shown in Figure 3. These curves were computed by setting the force to zero and the wavenumber to a real value in (5) and solving the real eigenproblem to find the frequencies of the propagating waves. The group velocity is computed by an analytical expression using the computed frequency and mode shape [12].

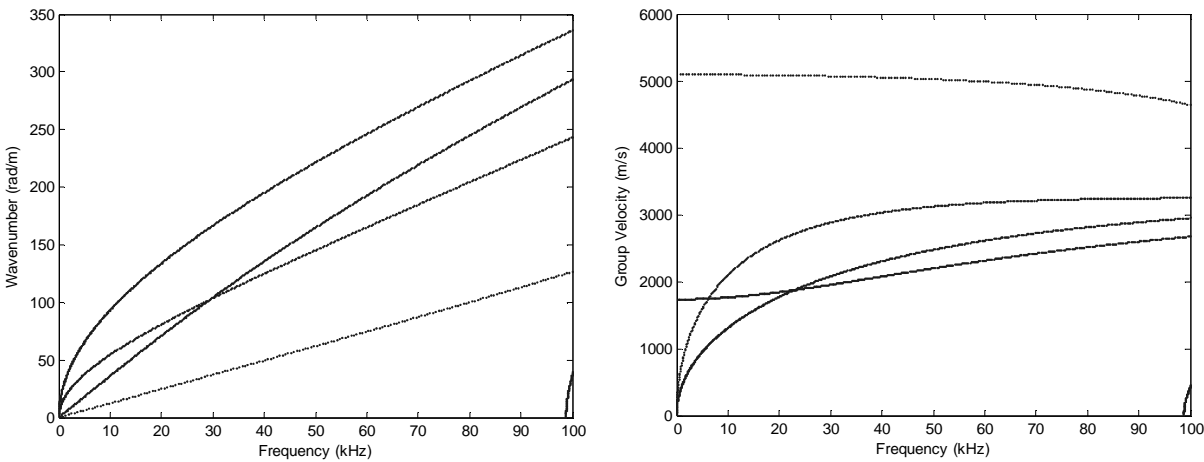


Figure 3: Dispersion of propagating waves in a rectangular waveguide.

In many situations it would be advantageous to work in the frequency region before the more complex waves begin to propagate. For this reason the frequency range considered was limited to 100 kHz.

The amplitude of the response of the first flexural mode of propagation was computed for the patch and sandwich transducers using the method described in Section 2. The effect of different geometry of patch transducer is shown in Figure 4. The patch was 8 mm wide and the thickness of the patch (0.5mm to 2mm) and the length of the patch were

varied to keep the volume of the patch constant at 80 mm^3 . The electric field applied in all cases was 1 V/mm . It is seen that the thinner (and longer) patch excites more strongly at the lower frequencies. The thinnest patch considered (0.5 mm thick and 20 mm long) could not excite the wave at 80 kHz as the wavelength of the wave is approximately 20 mm at this frequency. The minima observed around 98 kHz are due to bending of the cross-section of the beam. This is a feature that would not be predicted by models based on beam theory.

The sandwich transducer consisted of a $10 \times 8 \times 1 \text{ mm}$ piezoelectric ceramic section between two steel sections. The steel section between the piezoelectric and the waveguide is referred to as the front mass while the piece behind the piezoelectric is the back mass. Figure 5 shows the effects of varying the lengths of these two sections. Firstly, the length of the front mass was varied ($0.1, 1$ and 10 mm) while the length of the back mass was adjusted to maintain a total length of the transducer of 22 mm . The excitation of the flexural wave in the beam indicates resonant behavior of the transducer as desired. It was observed that the position of the piezoelectric ceramic along the length of the transducer does not have a very strong influence on the resonant excitation of the beam. However, the low frequency excitation is strongly influenced. It appears that when a thin front mass is used the lateral deflections of the piezoelectric ceramic cause bending moments, which excite the bending wave. This is the same mechanism as used in the patch transducer. If the front mass is thick the moment transfer is not achieved and excitation is achieved by inertial forces normal to the beam. The second parameter studied was the length of the back mass, which was varied from 5 mm to 40 mm while the front mass was a constant 1 mm thick. The effect of increasing the length of the sandwich transducer was to decrease the resonant frequency as would be expected. When the length was increased excessively two resonances occurred in the frequency range considered.

Finally, the excitation of flexural waves, in beams of different thicknesses, by the two transducers was considered. The results for beams with thicknesses of $5 \text{ mm}, 7.5 \text{ mm}$ and 10 mm are shown in Figure 6. At low frequencies it is evident that the patch actuator excites the flexural wave more strongly than the sandwich transducer. At higher frequencies the resonant behavior of the sandwich transducer produces a stronger excitation than that produced by the patch. The frequency at which the sandwich transducer becomes more effective depends on the thickness of the beam with this frequency being reached earlier for thicker beams.

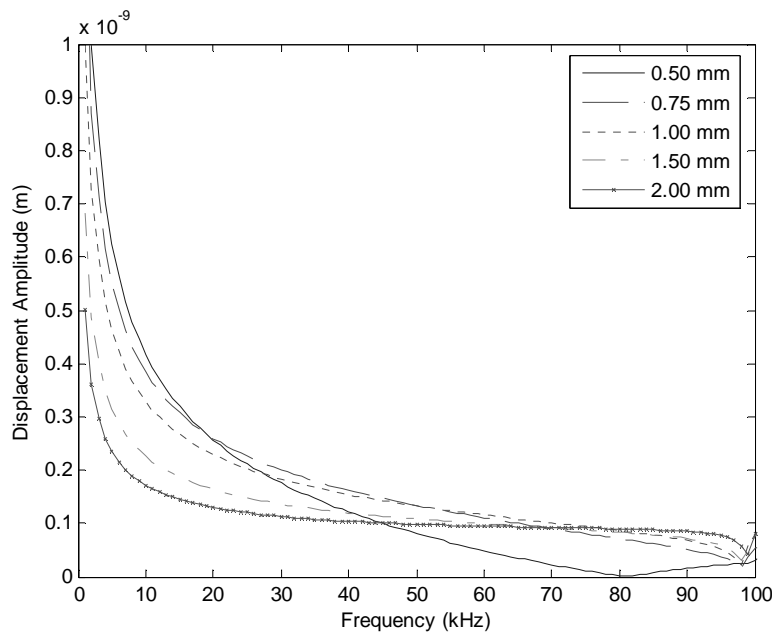


Figure 4: Effect of patch geometry on excited wave amplitude.

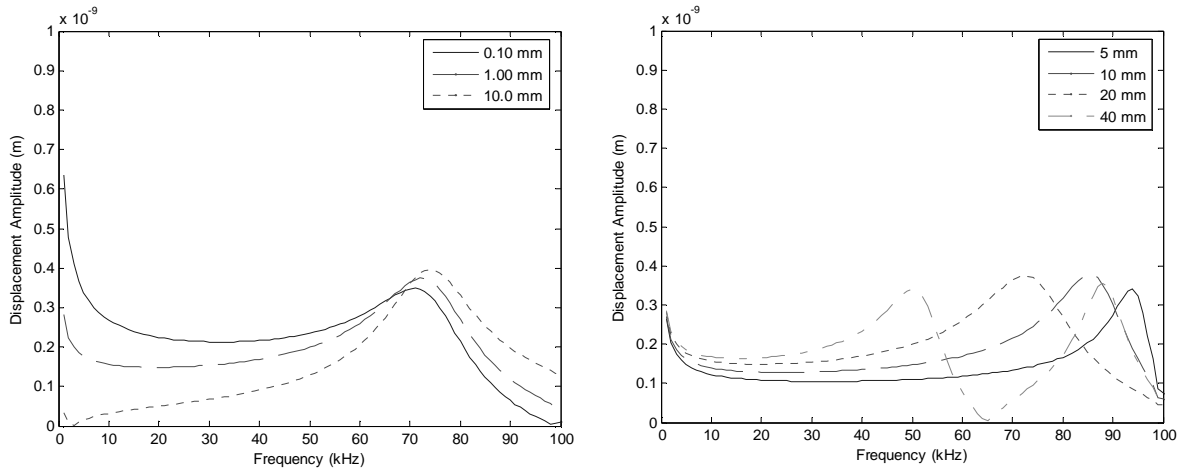


Figure 5: Effect of sandwich transducer geometry on excited wave amplitude.

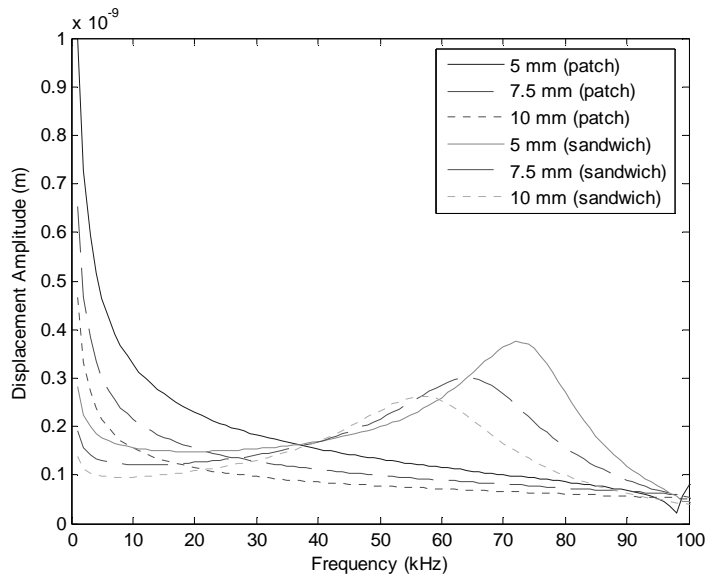


Figure 6: Excitation of different thickness beams with patch and sandwich transducers.

4. CONCLUSIONS

Patch and sandwich transducers, attached to a rectangular waveguide, were modeled numerically using a combination of conventional and waveguide finite elements. The modeling method showed some effects that would not be observed in analytical models based on beam theory.

The major design parameters of the two types of transducer were briefly investigated. A more complete optimization of the transducers could be performed in future.

The general conclusion is that the patch transducer excited the flexural wave effectively at low frequencies but less effectively at higher frequencies. The sandwich transducer could be designed to utilize its resonant nature at a desired frequency. This gives the sandwich transducer superior excitation ability at higher frequencies. The design of the sandwich transducer is more complicated but this could be worth the effort if it allows guided wave inspection over a longer range.

REFERENCES

1. J.L. Rose, Standing on the shoulders of giants: An example of guided wave inspection, *Materials Evaluation*, pp. 53-59, 2002.
2. P.W. Loveday, "Development of piezoelectric transducers for a railway integrity monitoring system," *Smart Structures and Materials 2000: Smart Systems for Bridges, Structures, and Highways, Proceedings of SPIE Vol. 3988*, pp.330-338, 2000.
3. G.P. Gibbs and C.R. Fuller, Excitation of thin beams using asymmetric piezoelectric actuators, *J. Acoust. Soc. Am.* 92(6), pp. 3221-3227, 1992.
4. C.R. Fuller, S.J. Elliot and P.A. Nelson, *Active Control of Vibration*, Academic Press, San Diego, 1996, pp. 27-35.
5. V. Giurgiutiu, Lamb wave generation with piezoelectric wafer active sensors for structural health monitoring, *Smart Structures and Materials 2003: Smart Structures and Integrated Systems*, SPIE Vol. 5056, pp. 111-122, 2003.
6. P.W. Loveday, "Analysis of piezoelectric ultrasonic transducers attached to waveguides using waveguide finite elements", submitted to *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*.
7. L. Gavrić, "Computation of propagative waves in free rail using a finite element technique", *Journal of Sound and Vibration* **185**(3), pp531-543, 1995.
8. T. Hayashi, W-J. Song and J.L. Rose, "Guided wave dispersion curves for a bar with an arbitrary cross-section, a rod and rail example", *Ultrasonics* **41**, pp175-183, 2003.
9. V. Damljanović and R.L. Weaver, "Propagating and evanescent elastic waves in cylindrical waveguides of arbitrary cross section", *Journal of the Acoustical Society of America* **115**(4), pp1572-1581, 2004.
10. H. Allik and T. J. R. Hughes, "Finite Element Method for Piezoelectric Vibration", *Int. J. Numer. Methods Engrng.*, 2, pp151-157, 1970.
11. V. Damljanović and R.L. Weaver, "Forced response of a cylindrical waveguide with simulation of the wavenumber extraction problem," *J. Acoust. Soc. Am.*, vol. 115, no. 4, pp. 1582-1591, 2004.
12. P.W. Loveday and Michael Y. Shatalov, "Finite element computation of dispersion in piezoelectric waveguides", *Smart Structures and Materials 2005: Modeling, Signal Processing and Control, Proceedings of SPIE Vol. 5757*, pp.432-440, 2005.