CHAMBER OF MINES OF SOUTH AFRICA

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REPORT ON


# SOME CONSIDERATIONS REGARDING THE CHOICE OF A MATCHING NETWORK FOR ELECTRICALLY SMALL LOOP ANTENNAS


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## TECHNICAL SUMMARY

This report presents a comparison of the three capacitive matching networks most commonly used with electrically small loop antennas. The three networks are considered as special cases of a universal capacitive matching network which together with the antenna is analysed by means of a desk-top calculator. Graphical plots of the important parameters indicate the simple series tuned circuit to have many important advantages. Furthermore, narrowband radio frequency modulation techniques are shown to be desireable since the antenna gain and selectivity increase with antenna Q factor and hence causes a reduction of bandwidth.

1. Introduction

Electrically small loop antennas are used where medium and high frequency radio transmission is required using an antenna having a minimum volume. These antennas are suited to underground mining situations and are being used for voice, telemetry and remote control applications. However, the small electrical size of the antenna leads to a relatively large reactive input impedance which is equalised with a matching network in order to optimise the efficiency of the transmission system. Generally, the matching network is designed to operate at resonance but frequently the transmitted signal spectrum requires the antenna and matching network characteristics to be known over a range of frequencies centred on the resonant frequency. This is of particular interest for the transmission of digital telemetry data.

This report presents an analysis of an antenna and matching network system in order to assist in the design of such a system. The analysis is extended to the three capacitive matching networks most commonly used with electrically small loop antennas whose properties are compared to establish which is the most suitable. Use is made of a desk-top calculator to perform the calculations and plot the results and the program developed is listed in the Appendix.

2. A universal capacitive matching network

Most matching networks use the simple two element "L circuit" which can transform a large range of load impedances to a real input impedance. The eight forms of this circuit consist of various combinations of inductors and capacitors and have been examined by P.H. Smith[1], who presented a series of graphs to enable the components to be selected. However, G.S. Smith[2], has shown that the most efficient coupling networks for an inductive antenna consists only of capacitors and a network which incorporates all the various capacitive matching circuits is shown in Figure 1.

2.1 Circuit analysis

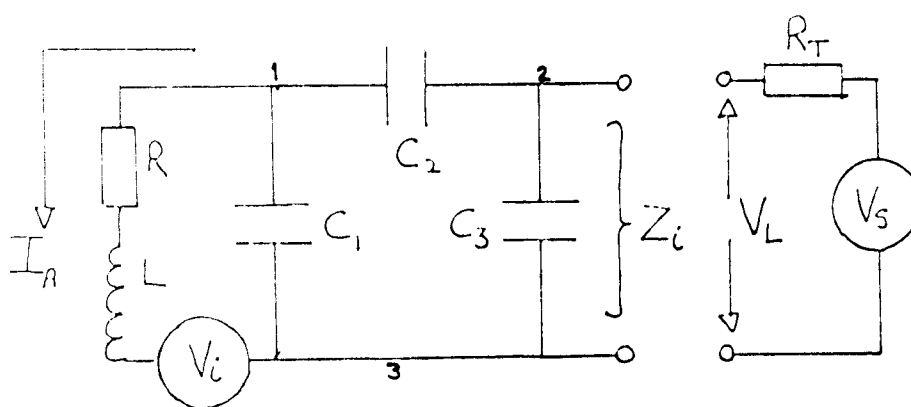Three cases of the network in Figure 1 are of importance, these are:

**Figure 1**

**Universal capacitive matching network**

(A) $C_1 = 0$

(B) $C_3 = 0$

(C) $C_1 = C_3 = 0$

The matching networks corresponding to the above are denoted by (A), (B) and (C) and it is evident that (A) and (B) are two of the eight forms of "L circuits". Network (C) has not been considered by P.H. Smith but it will be instructive to analyse this series tuned circuit since it has many advantages. Generally, a matching transformer is used with (C) since its input impedance at resonance is low. However, the transformer will be omitted for the present calculations since its only effect is to change the magnitudes of the source impedance and source voltage to the equivalent values shown in Figure 1.

When the antenna is in the receiving mode, $R_T$ is the load impedance ($R_L$) "seen" by the matching network, $V_i$ is the induced voltage and $V_s$ is zero. In the transmit mode $V_i$ is zero and $R_T$ is equal to the source impedance ($R_S$).

The most important parameters in a matching network are the magnitude and phase of the antenna current ($I_A$), the loaded output voltage ($V_L$) and the input impedance ($Z_i$). Each of these is derived in the Appendix and can be expressed by the following equations:

$$\frac{I_A}{V_s} = \frac{Z_3 + jZ_4}{\left\{\left[Z_1 + R_s(Z_3 - Y_1)\right] + j\left[(Z_2 + R_s(Z_4 + Y_2))\right]\right\} \left\{Z_6 + jZ_7\right\}} \qquad (14)$$

$$\frac{V_L}{V_i} = \left\{\frac{Y_3}{Y_4 + jY_5}\right\} \left\{\frac{\left[Z_3 - Y_1 + jR_L(Z_4 + Y_2)\right]}{\left[R_L(Z_3 - Y_1) + Z_1\right] + j\left[R_L(Z_4 + Y_2) + Z_2\right]}\right\} \qquad (15)$$

$$Z_i = \frac{Z_1 + jZ_2}{(Z_3 - Y_1) + j(Z_4 + Y_2)} = R_i + jX_i \qquad (16)$$

where $Z_1 = 1 - \omega^2 L(C_1 + C_2)$      $Z_2 = \omega R(C_1 + C_2)$

$Z_3 = -\omega^2 C_1 C_2 R$      $Z_4 = \omega C_2(1 - \omega^2 L C_1)$

$Z_6 = 1 - \omega^2 L C_1$      $Z_7 = \omega C_1 R$

$Y_1 = \omega C_3 Z_2$      $Y_2 = \omega C_3 Z_1$

$Y_4 = 1 - \omega^2 L C_4$      $Y_5 = \omega C_4 R$

$Y_3 = \dfrac{C_2}{C_2 + C_3}$      $C_4 = C_1 + \dfrac{C_2 C_3}{C_2 + C_3}$

$\omega = 2\pi f$

## 3.  The selection of the matching network components

Graphs, which enable the matching components to be selected when the magnitude of the antenna impedance is of the same order as the terminal impedance, have been presented previously by P.H. Smith[1]. However, when the antenna resistance is a small fraction of the required terminal impedance, other methods of selection are necessary.

For matching network (A) the component selection procedure is done in two steps. The first step requires the antenna loss (R), antenna inductance (L) and antenna capacitance ($C_1$) to be known together with the required output impedance at resonance ($f_o$). The network output impedance at ($f_o$) can be derived from equation (3), which becomes:

$$\frac{\left(2\pi f_o L\right)^2}{R} \left(\frac{C_2}{C_2 + C_3}\right)^2 + \frac{1}{j2\pi f_o C} \tag{4}$$

where $C = C_3 \left[ 1 + \frac{C_1 C_2}{C_3(C_1 + C_2)} \right]$.

For many applications the reactive term can be ignored and the output impedance becomes:

$$\left\{\frac{(2\pi f_o L)^2}{R}\right\} \left(\frac{C_2}{C_2 + C_3}\right)^2 = R_o \tag{5}$$

The second step in the selection of $C_2$ and $C_3$ makes use of the resonance equation, that is,

$$\left[ (2\pi f_o)^2 L \right]^{-1} = C_1 + \frac{C_2 C_3}{C_2 + C_3} = C_r \tag{6}$$

Hence the values of $C_2$ and $C_3$ can be found by solving equations (5) and (6).

The component selection for matching network (B) uses the variation of the effective inductance ($L_e$) and loss ($R_e$) of a parallel tuned circuit. As the frequency approaches resonance both $L_e$ and $R_e$ increase. The component values are selected such that $R_e$ equals the desired output resistance and $C_2$ is chosen to resonate the resulting value of $L_e$. The selection of $C_1$ and $C_2$ is therefore done in three steps. The first step requires $R, L, f_o$ and $R_o$ to be

known and hence $C_1$ is solved using:

$$R_o = \frac{R}{[1 - (2\pi f_o)^2 LC_1]^2 + (2\pi f_o C_1 R)^2} \tag{7}$$

The value for $\bar{C}_1$ is easily calculated by using a computer to plot $R_o$ against $C_1$. It should be noted that $C_1$ includes the antenna self-capacitance.

The second step evaluates $L_e$ from:

$$L_e = \frac{L [1 - (2\pi f_o)^2 LC_1] - C_1 R^2}{[1 - (2\pi f_o)^2 LC_1]^2 + (2\pi f_o C_1 R)^2} \tag{8}$$

The third part resonates $L_e$ with $C_2$ and hence:

$$C_2 = \frac{1}{(2\pi f_o)^2 L_e} \tag{9}$$

The selection of the components for matching network (C) requires the effective antenna inductance ($L_e$) and loss ($R_e$) to be known, whereupon, $C_2$ is chosen to resonate with ($L_e$) and the value for ($R_e$) determines the turns ratio for the matching transformer.

4. Basic properties of various capacitive matching networks

In this section the magnitude and phase of the antenna current, loaded output voltage and input impedance are compared for the three matching networks under consideration to establish the most suitable network. The networks are analysed using the computer program which is listed in Appendix 9.4. The program is written in H.P. Basic for a Hewlett-Packard desk-top calculator (9830A). The results are drawn by means of a Hewlett Packard plotter (9862A).

## 4.1 The network components ($C_1$, $C_2$, $C_3$)

The procedures outlined in the previous section were used to calculate $C_1$, $C_2$, and $C_3$, for each of the matching networks to resonate an antenna at 913 kHz.

The output impedance required from (A) and (B) is 50 $\Omega$ and from (C) 1,23 $\Omega$, which is equal to the loss resistance (R) of a particular antenna having an inductance of 21,4 $\mu$H; the antenna is assumed to have no self capacitance. Hence the antenna Q factor $\frac{\omega L}{R}$ is 100.

(A)  $C_1 = 0; C_2 = 1,52$ nF;  $C_3 = 22,30$ nF

(B)  $C_1 = 1,20$ nF;  $C_2 = 221$ pF;  $C_3 = 0$.

(C)  $C_1 = 0$;  $C_2 = 1,419$ nF;  $C_3 = 0$.

## 4.2 The normalised antenna current ($I_A$)

The variation of the antenna current with normalised frequency ( $\frac{f}{f_o} = 913$ kHz) for the three networks when driven by matched source impedance is shown in Figure 2. The curves for networks (B) and (C) have identical shapes although (C) is displaced 16 dB above (B). This result is not surprising since no matching transformer is used with (C) which has an input resistance ($R_i$) at resonance of 1,23 $\Omega$. If all the input power is dissipated in the antenna resistance (R), and $Z_i$ at resonance is purely resistive then:
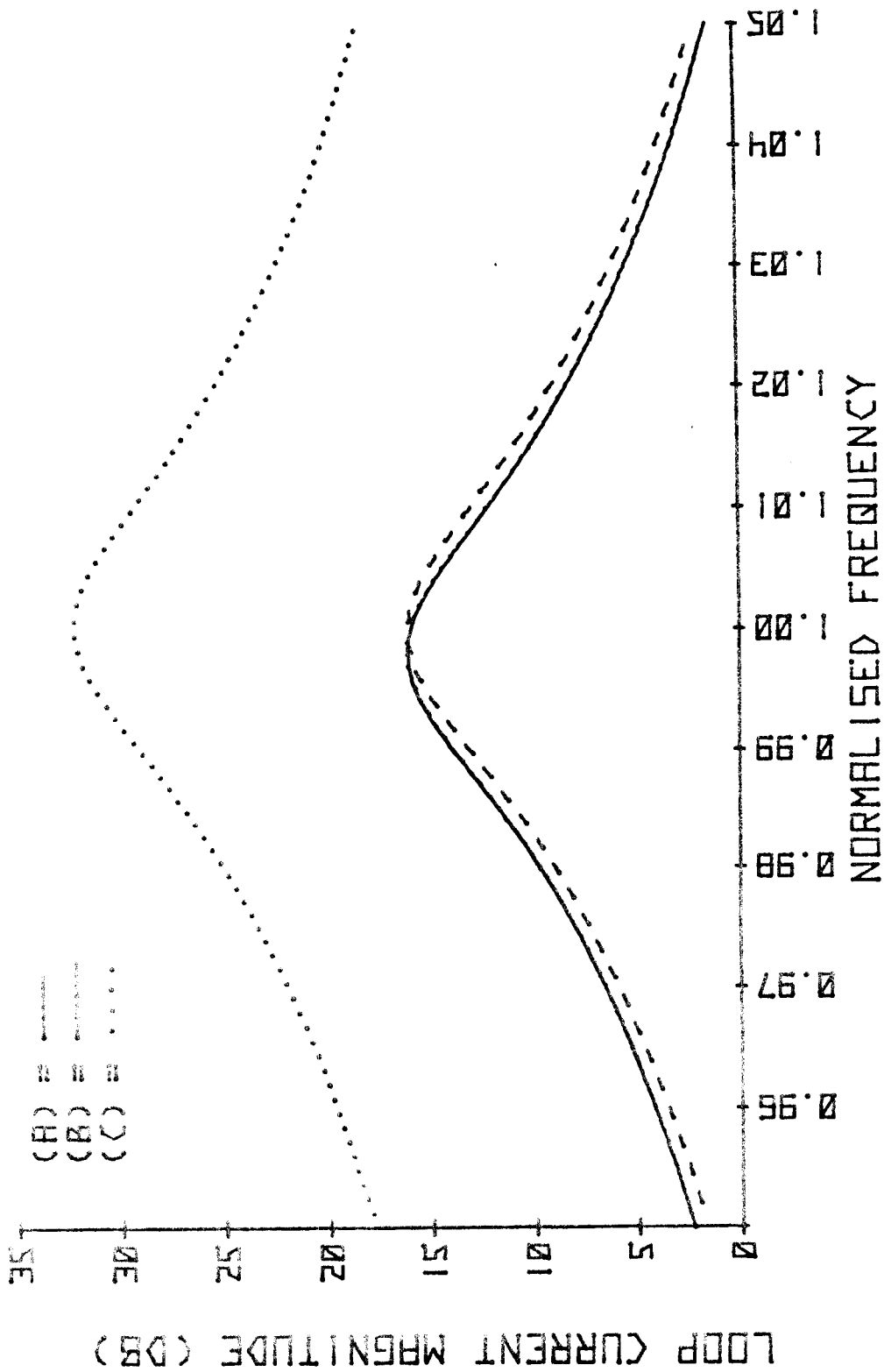
$$\frac{V_s^2}{4R_i} = I_A^2 R$$

Figure 2

The variation of antenna current with frequency

Hence for identical antennas and voltage
sources the ratio of the resonant currents in (B)
and (C) are:                    ,

$$\left( \frac{R_i(B)}{R_i(C)} \right)^{\frac{1}{2}}$$

which, expressed logarithmically is equal to 16,1 dB;
which is the difference between the two responses
in Figure 2.

Network (C) with a perfect matching transformer
therefore produces an equivalent maximum antenna
current as in (A) and (B). However, the frequency
response using network (A) is not quite the same,
being displaced to a lower frequency range. A factor
common to all curves is that the frequency difference
at the 3 dB points $(f_2 - f_1)$ is given by half the an-
tenna Q factor, i.e.

$$f_2 - f_1 = \frac{2f_o}{Q}$$

The above relationship holds only for a source impe-
dance equal to the matching network input impedance
at the desired resonant frequency. The effect of
other values of source impedance is treated later in
this report.

4.3  The normalised loaded output voltage $\left( \frac{V_L}{V_i} \right)$

The shape and position relative to the frequency axis
of the normalised output voltage frequency characteristics
for networks (B) and (C) are identical to those in Figure
2 if $R_L = R_s$. The shape of the characteristic for net-
work (C) is also the same as that in Figure 2 but the
maximum output is 16 dB less than (B) or (C). This is
due to the difference of terminal impedances, since no
matching transformer is used for the calculations. Again,
the response from (A) is shifted to a lower frequency
range than either (B) or (C) and the bandwidth of all
three is determined by Q/2.

Electrically small, antennas therefore exhibit the property that the system gain and selectivity can be controlled by means of the antenna Q factor.

The effect of Q on the system output can be deduced easily if it is assumed that each network transfers the same input power to the load. Using these considerations then $I_A$ and $V_L$ are each proportional to $Q^{\frac{1}{2}}$ and for identical receiving and transmitting antennas the total system voltage gain is proportional to Q.

The frequency response of (A) and (B) over a wide frequency range is shown in Figure 3. The response from (C) is not shown since it is identical to (B). It is apparent that (A) exhibits more outband attenuation than (B) above the resonant frequency but below the resonant frequency (B) is superior. For receiver applications these points are important since the noise performance of the system is influenced by the matching network selectivity.

The ideal frequency responses obtained for $I_A$ and $V_L$ imply that $I_A$ and $V_L$ are the reciprocals of each other when the terminal impedances are the same. Complete reciprocity also requires the phase responses to be identical and this has been found to be true. The phase response for (A) and (B) is shown in Figure 4. The phase response of (b) is identical to that of (C). The phase response for all three networks is linear over the system bandwidth which indicates that these antennas should give good performance in digital transmission systems.
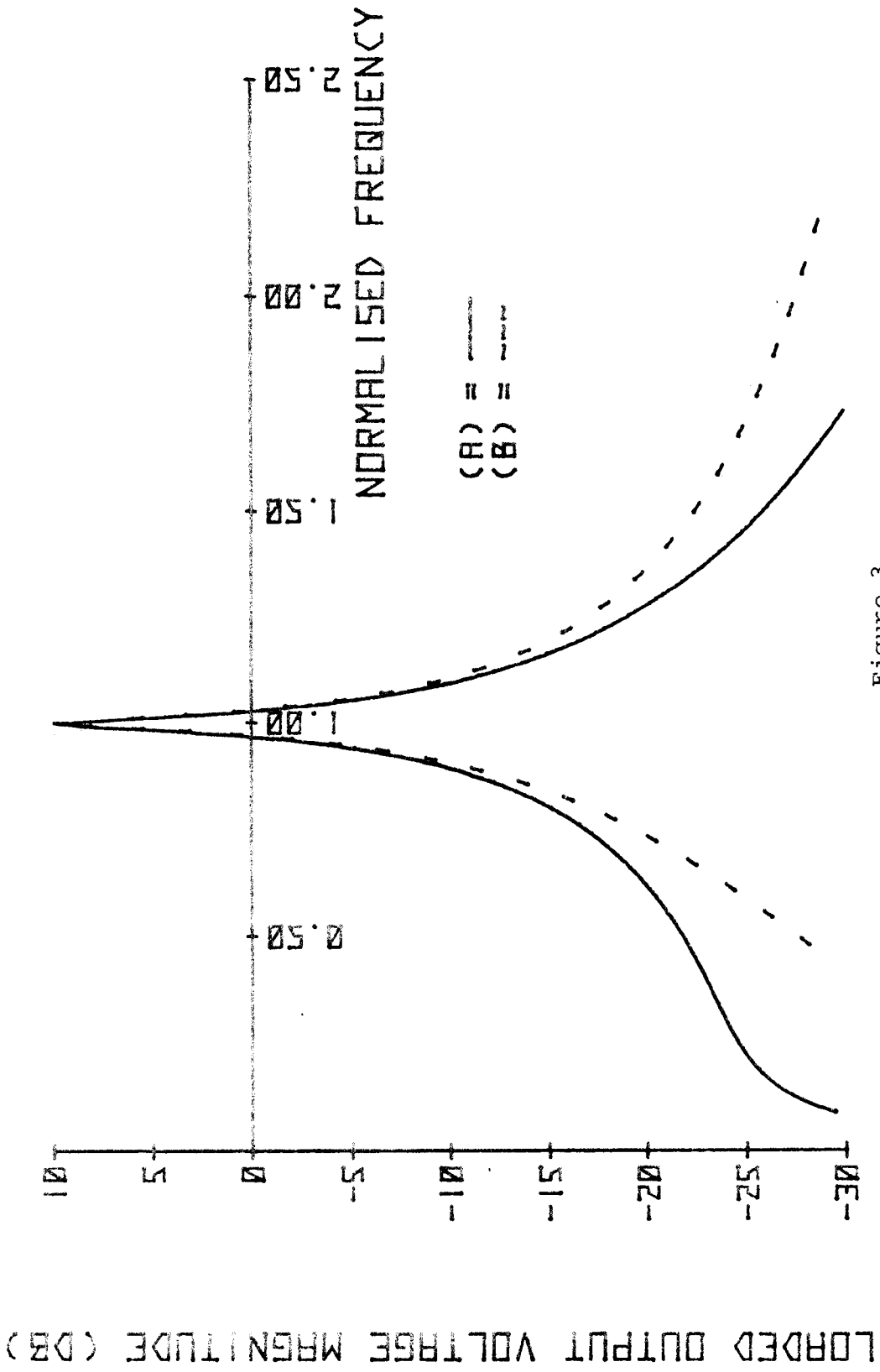
Figure 3
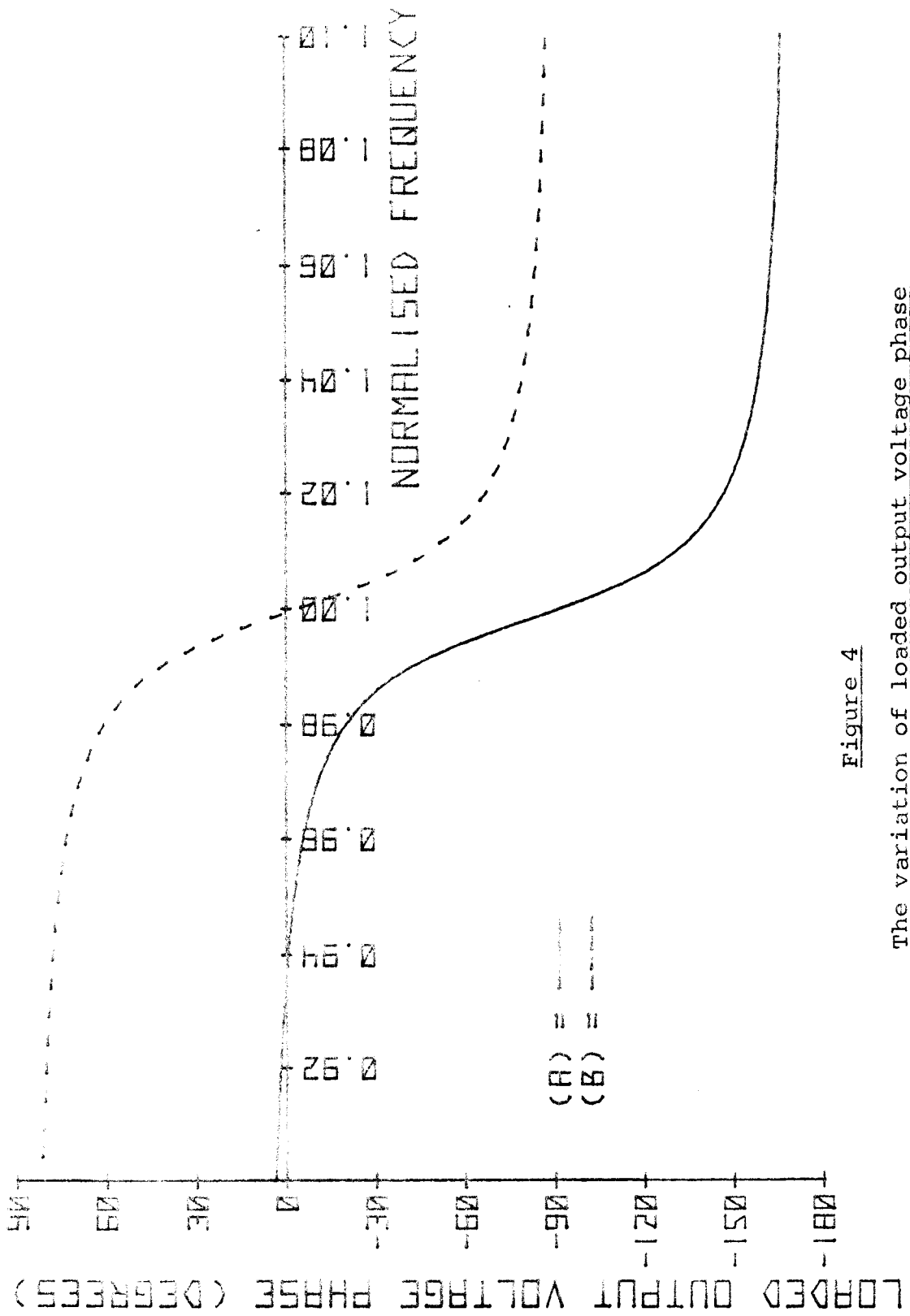
The loaded output voltage over a wide frequency range

Figure 4

The variation of loaded output voltage phase
with frequency

## 4.4    The input impedance $(Z_i)$

The variation of $Z_i$ with frequency for (A) and
(B) and for (C) with a perfect matching transformer
is shown in Figure 5, the phase responses are shown
in Figure 6. It is apparent that (B) and (C) are
similar since the impedance increases for frequencies
away from resonance while the impedance for (A) de-
creases. An extended plot reveals that although (B)
and (C) are similar within the system bandwidth, the
difference between the two increases as the frequency
deviates further from resonance. The phase responses
for (B) and (C) are also different, both being capa-
citive below resonance, with (C) becoming inductive
above resonance while (B) is at first inductive and
then capacitive. Network (A) behaves in a similar
manner to (B) but the inductive region is below re-
sonance.

It is important to bear in mind that the impedance
curves in Figure 5 and Figure 6 refer to one set of
antenna specifications and that different results can
be obtained for other antenna systems. In particular
it is not always possible to obtain a real value of
input impedance and Smith[1], has shown that it is not
possible to match all types of antenna impedance with
networks (A) and (B). For example, if network (A) is
designed for an output impedance of 5 Ω with the an-
tenna in Section 4.1, the phase response is shifted
such that the peak just reaches zero for $Q = 100$ at a
normalised frequency of 0,995. For all other frequen-
cies the input impedance is capacitive. If the out-
put impedance is 50 Ω the critical Q factor is 10.
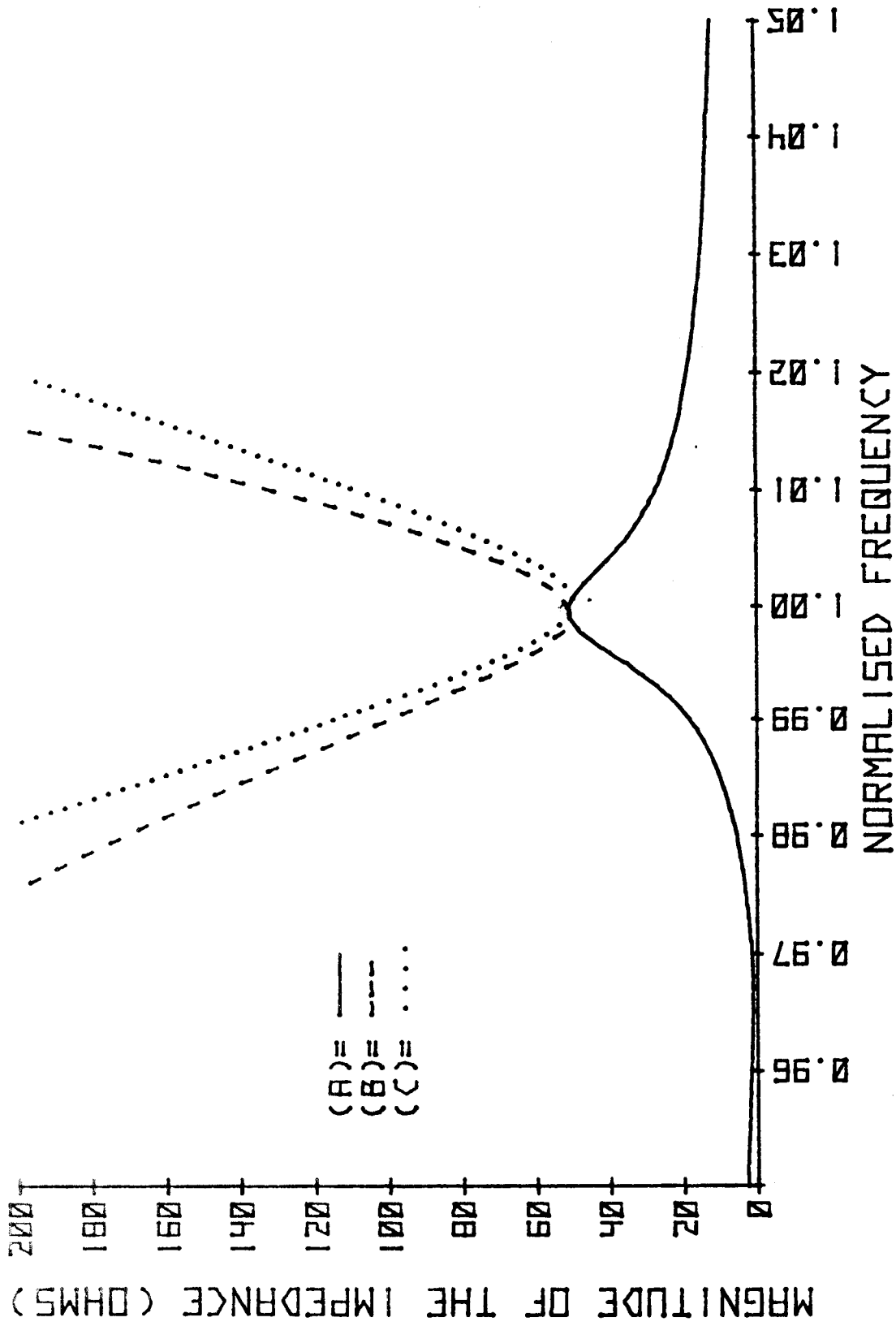In these cases the capacitive term in equation (4)

Figure 5

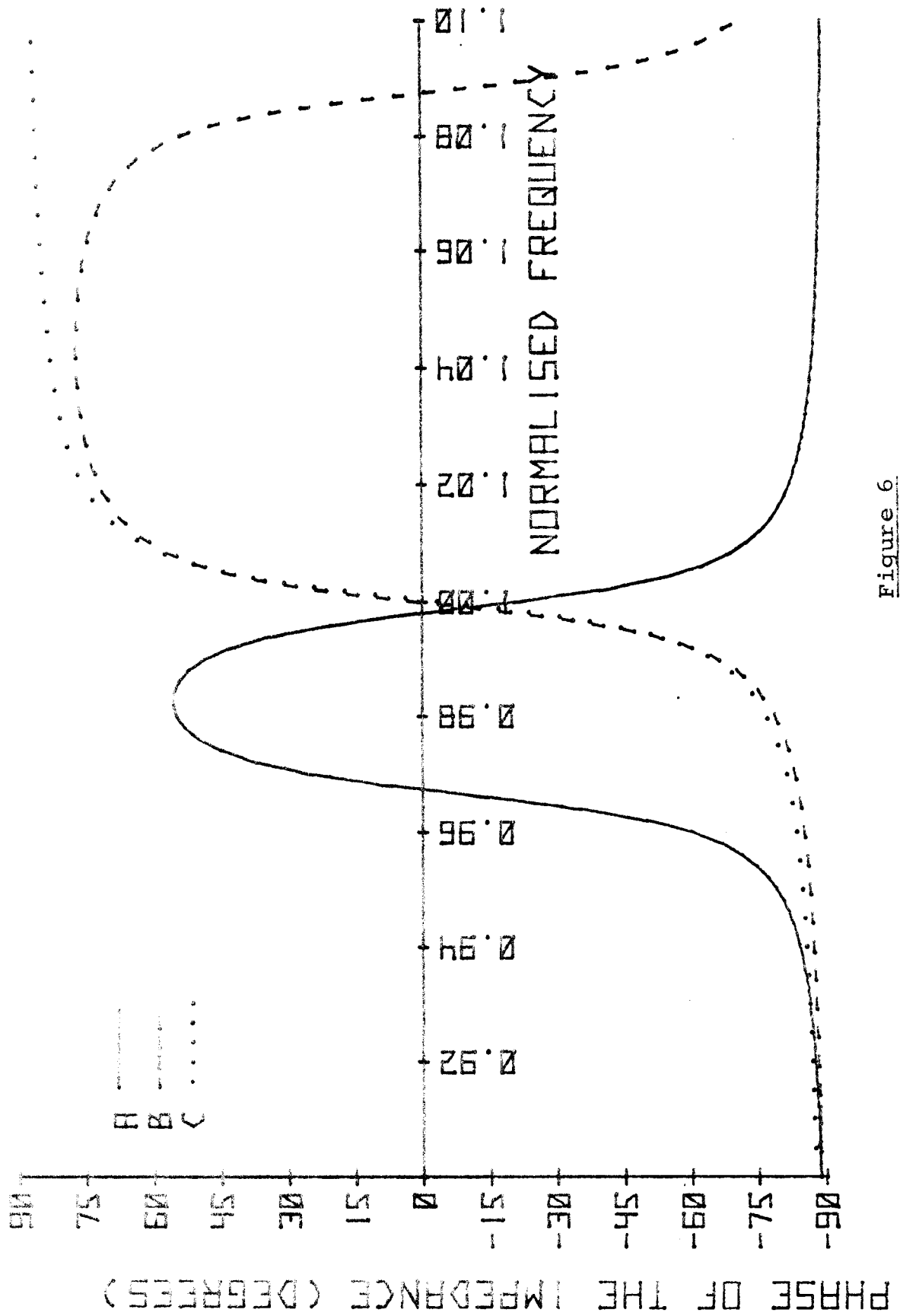The variation of matching network input impedance with frequency

Figure 6

The variation of the phase of the matching network input impedance
with frequency

has a significant effect. If the Q factor for this
antenna is reduced from 100 for an output impedance
of 5 Ω the impedance is always capacitive and addi-
tional components are required to produce a real in-
put impedance and hence obtain efficient power trans-
fer.

A restriction on the range of antenna impedance
that can be matched is also imposed by network (B).
However, network (C) can always provide a real in-
put impedance and furthermore, its impedance increases
away from the resonant frequency. This is a desirable
property for transmitter applications since a large part
of the signal spectrum can be at the extremes of the
system bandwidth and the supply current will be less
for an impedance characteristic which increases rather
than decreases. Network (A) however, exhibits a 60%
decrease in impedance at the lower edge of the band with
a consequent increase in current consumption.

The asymmetric decrease of impedance about resonance
for network (A) is due to the series resonance of L and
$C_2$ which can cause the input impedance to decrease to a
very low value within the system bandwidth. The rate of
decrease increases as the ratio $\dfrac{C_2}{C_3}$ decreases, that is,
for high inductance antennas or networks with a low out-
put impedance. With some antenna systems a decrease of
80% at the lower 3 dB point has been calculated. The
change of $Z_i$ with frequency can cause a problem with
portable equipment such as speech transceivers if varia-
tions in the matching or antenna components cause the
resonant frequency to increase by greater than half the
system bandwidth. In this situation the operating fre-
quency could coincide with the impedance mimimum and the

operation of the radio transmitter and the RF filter
of the radio receiver may be adversely affected.

A close examination of Figure 2 and Figure 5 re-
veals that the impedance peak for network (A) is at
a higher frequency than the peak of the loop current.
As the antenna Q factor reduces, the frequency separa-
tion between the two peaks increases and with the
antenna used for this example, if the Q factor is less
than 20, the loop current at the frequency of the im-
pedance maximum is less than the resonant peak by more
than 2 dB. For an output impedance of 5 Ω the critical
Q factor is 50. Consequently the output impedance peak
is not an indication of resonance and if used for tuning
the antenna it can cause misleading results. Conversely
the output impedance may be significantly less at re-
sonance than the maximum value and hence overload the
radio transmitter.

These factors are important for intrinsically safe
equipment since in these applications the antenna voltage
is limited and a low impedance is often used to obtain
higher transmission powers.

5.   The effect of component tolerances on the frequency
     response

The previous sections show that a high antenna Q
factor leads to a higher input signal at the radio re-
ceiver and to better selectivity. However, before a
value for the Q factor can be determined, it is necessary
to consider the sensitivity of the system parameters to
practical variations in the components of the antenna
and matching network.

The maximum change of frequency response allowable depends on the type of transmission system. If the modulation bandwidth is much less than the transmission bandwidth then the maximum change of resonant frequency is generally taken as half the transmission bandwidth. If the two bandwidths are comparable then the resonant frequency drift must be much less and depends on the type of modulation. For example, the minimum transmission bandwidth for a digital system is comparable to the baud rate[3]. In this case the frequency stability should be better than one tenth[4] of the data rate and for a resonant frequency tolerance of $\pm$ 0,5% the antenna Q factor must be less than 40. For the purposes of this paper the component tolerances are considered for a system whose modulation bandwidth is much less than the transmission bandwidth.

## 5.1  Capacitance effects

Generally the antenna capacitance is much smaller than the capacitance of the matching components and hence its variation due to temperature is a second order effect. The antenna capacitance is also influenced by the presence of conducting objects which affect the antenna's inductance as well. However, below the antenna self-resonant frequency the antenna capacitance can be absorbed in the inductance and the effect of the two are treated in the next section.

The values of the matching network components, however, change with temperature, the environment and age by amounts which depend on the type of components used. It is recommended that silver mica capacitors be used for receiver applications and polypropylene capacitors for transmitter matching networks. Silver mica capacitors have a long-term stability of about 0,1% and a temperature coefficient of $\pm$ 50 ppm per $^{\circ}$C.

Hence a typical design tolerance for these components is $\pm$ 0,2%. Polypropylene is not as stable (0,5%) as silver mica and has a higher temperature coefficient (-200 ppm per $^{\circ}$C), and hence a typical design tolerance for these components is about $\pm$ 1%.

The effect of capacitance variations on matching network (A) depends on the relative magnitudes of the components. For most applications the antenna impedance at resonance is much greater than the network output impedance and hence $C_3$ is significantly greater than $C_2$.

Therefore, the resonance condition will be influenced primarily by $C_2$ rather than $C_3$.

For reasons mentioned previously this network is more suitable for receiver applications in which case silver mica capacitors would be used. Assuming a tolerance of $\pm$ 0,2%, the resonant frequency changes by $\pm$ 0,1% (since $\omega_o = (LC)^{-\frac{1}{2}}$) which implies that for this change in component value to be serious the antenna Q factor would be in the region of 1 000, an impractical situation. Furthermore, the output impedance of the network is proportional to $\left(\dfrac{C_2}{C_3}\right)^2$ and hence a worst case error of $\pm$ 0,8% is expected and this is also generally insignificant.

With network (B), however, the component sensitivity is not quite so clear since the parallel capacitance ($C_1$) has a complex effect on $L_e$ and $R_e$. If $R_e$ is plotted against $C_1$ at 913 kHz then Figure 7 shows that large values of $R_e$
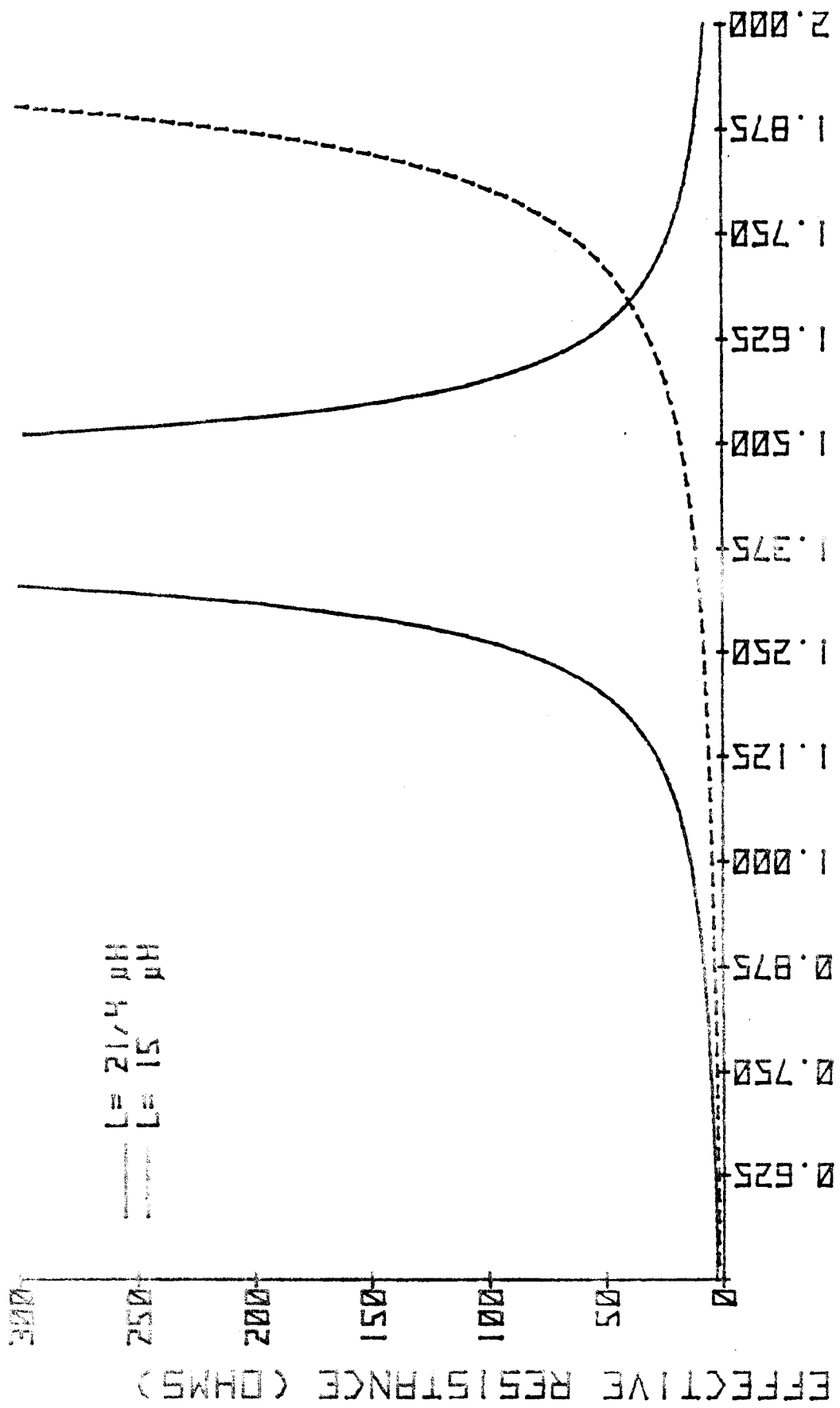
VALUE OF $C_1$ (NANO-FARADS)

Figure 7

The effects of $C_1$ on the effective resistance of a parallel tuned circuit

are very sensitive to $C_1$. Even when $R_e = 50 \, \Omega$, the effective resistance changes by about 10% for a 1% change of $C_1$. The change of $L_e$ is not quite so dramatic, being about half the change of $R_e$ and hence for a 1% change of $C_1$, the series resonant frequency will change by about 2,5%. Hence the Q factor should be less than 40 in order to achieve a stable system.

The effect of $C_2$ using network (C) is straight forward. In practice this network is suited to transmitter applications and hence the maximum change of resonant frequency with polypropylene capacitors should not exceed 0,5% and the Q factor must be less than 200.

## 5.2 Inductance effects

The antenna inductance is hardly affected by changes in temperature. However, its value is affected by other factors such as the presence of conducting objects and by physical movement if the antenna is flexible and is moved. Measurements at 200 kHz indicate that the inductance can be reduced by 7% when indirectly coupled within a few centimetres to underground power-line cables. The inductance of a portable antenna worn across the chest may change by $\pm$ 2% due to movement of the body.

If the equipment is portable then the maximum change of frequency response must lie within the transmission bandwidth. With network (A) the antenna Q factor should be less than 100 for the majority of mining applications and the transmitter must be able to deliver power into the reduced matching network input impedance as the impedance response drifts. The antenna Q factor using network (C) should also be less than 100 to avoid retuning but, although the transmitter will generally not be harmed by a

load impedance higher than nominal, it would limit
the maximum output voltage with a consequent in-
crease in transmitted harmonics. The effect of an
inductance change using network (B) is far more
serious and Figure 7 indicates that for a 30% re-
duction of L, $R_e$ falls from 50 Ω to 6 Ω. For a
2% reduction of L both $R_e$ and $L_e$ reduce by almost
a half and complete retuning is necessary.

In some permanent installations, such as the
transmission of telemetry data in shafts, indirect
coupling into power-line cables is used to achieve
a considerable increase in received signal strength.
The received signal can be increased further by tuning
the antenna in-situ and practical measurements in-
dicate that 100% improvement can be obtained. Using
network (A), $C_2$ is adjusted to bring the resonant peak
for either $I_A$ or $V_L$ coincident with the frequency of
operation. Using this procedure $Z_o$ does not change
since its real part reduces to:

$$\frac{1}{\omega^2 C_3^2 R^2}$$

However, with L smaller than the free space value, the
antenna Q factor is reduced and the transmission band-
width is increased. Retuning using network (C) is
also simple but with network (B) both matching components
must be adjusted by a fairly complex procedure.

5.3     <u>Source and load impedance effects</u>

The effect of source impedance ($R_s$) on the antenna
current ($I_A$) using network (A) is shown in Figure 8.
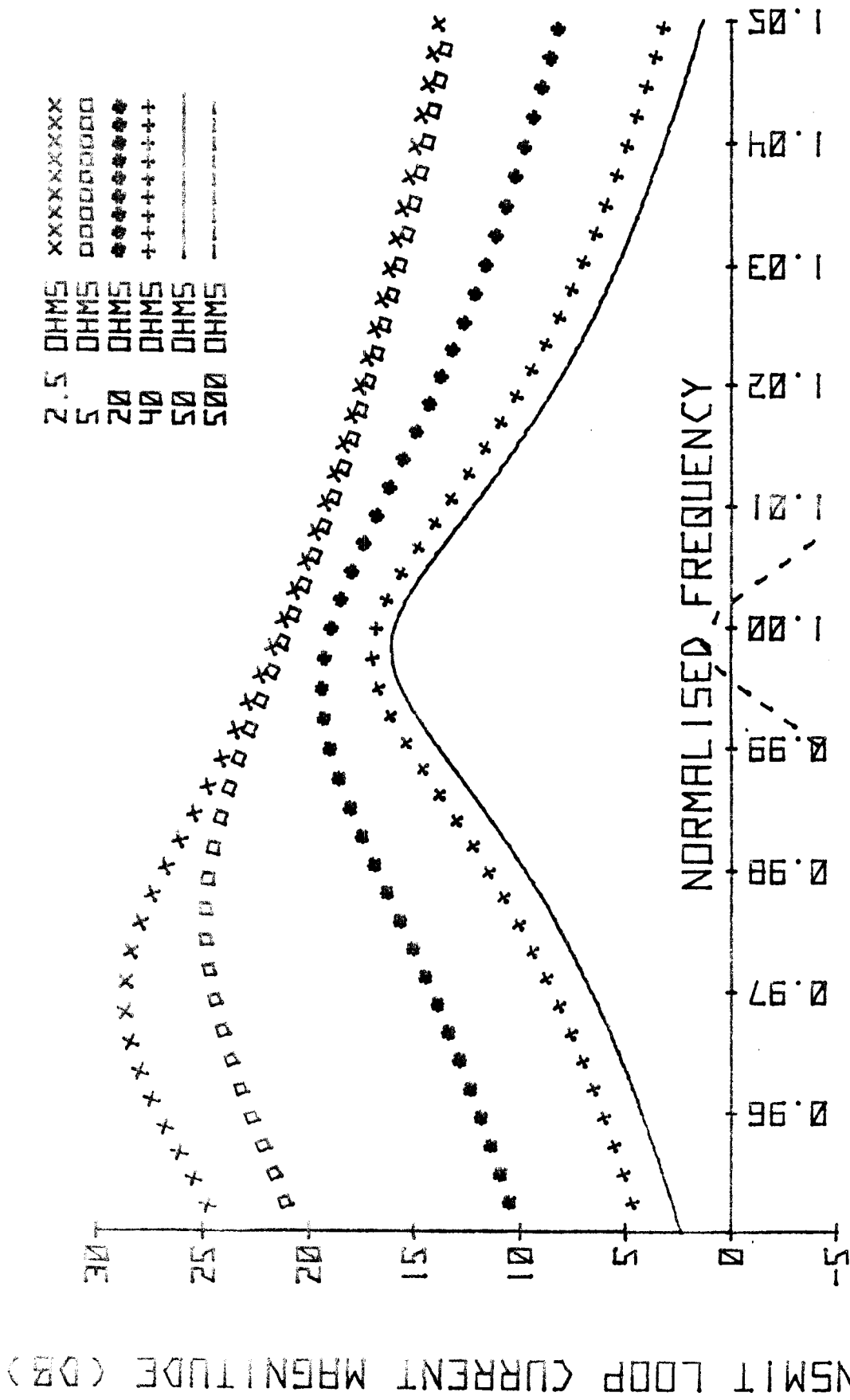
Figure 8

The effect of source resistance on the loop current frequency response
using network (A).

As the source impedance decreases from 500 Ω the
bandwidth widens with the response at first being
centred just below 913 kHz and then moving to lower
frequencies. For values of source impedance less
than 2 Ω, the current maximum occurs at the series
resonant frequency of the network. The bandwidth
increases until the source impedance is about 5 Ω,
whereafter it decreases. When $R_s$ = 5 Ω the band-
width is double the value for a matched system.
The real and imaginary parts of the antenna input
impedance at the maximum value are given in Table 1.

TABLE 1

| $R_s (\Omega)$ | $R_{iA} (\Omega)$ | $X_{iA} (\Omega)$ |
|---|---|---|
| 2,5 | 1,41 | 0,51 |
| 5,0 | 2,44 | 2,91 |
| 20 | 31,18 | 18 |
| 50 | 51 | 0 |
| 500 | 51 | 0 |

From this information it is possible to calcu-
late, for a lossless matching network, how much extra
reactive power must be supplied for the same power into
$R_{iA}$ and hence the same antenna current. The calculated
values are given in Table 2.

TABLE 2

| $R_s (\Omega)$ | Extra transmitter power (dB) |
|---|---|
| 2,5 | 0,27 |
| 5 | 1,92 |
| 20 | 0,62 |
| 50 | 0 |
| 500 | 0 |

With some transmitter circuits the output
impedance can be adjusted independently of the out-
put power. If the output impedance of such a trans-
mitter is increased above 50 $\Omega$ while keeping the out-
put power constant, the system bandwidth reduces but
the maximum antenna current remains the same. This
is not a desirable mode of operation since the re-
duced bandwidth could be obtained with a higher antenna
Q factor with a consequent increase in the maximum
antenna current. It is therefore concluded that the
optimum source impedance for network (A) is equal to
the network input impedance.

The effect of source impedance variations on net-
works (B) and (C) is to change only the system Q factor;
the resonant peak remains at the same frequency. As
the source impedance is reduced, the responses become
narrower and hence the effective antenna Q factor in-
creases. For an antenna of a defined number of turns
this property can be used to advantage to increase the
antenna current[5] while keeping the system Q factor con-
stant. In order to achieve this it is necessary to have
the greater part of the total series resistance which
determines the system Q factor in the output impedance
of the transmitter where, by suitable circuitry[6], it ab-
sorbs little power. As the antenna loss (R) decreases,
the transmitter output impedance must increase to keep
the system Q factor constant, but for a transmitter of
defined output power ($P_o$) the antenna current is given
by:

$$I = \left(\frac{P_o}{R}\right)^{\frac{1}{2}}$$

Hence by using a low loss antenna and a matching trans-
former with a high turns ratio, it is possible to obtain

a high value of antenna current. An antenna using
these principles has been built and has produced a
200 $\mu$V (p.d.) signal at a receiver antenna when in-
directly coupled to 2,5 km of 6,6 kV power cable.
The transmitter had an output power of 10 W.

6.  Matching network comparison

Network (A) is used frequently for matching loop
antennas since it is simple to design and being only
capacitive, is highly efficient. However, for trans-
mitters it has the disadvantage of a sharp reduction
of input impedance on the low-frequency side of the
resonance curve and thereby increases the consumption
of current by the transmitter, if the modulation band-
width is comparable to the transmission bandwidth. In
radio receiving systems it has the disadvantage of lower
outband attenuation compared to the other matching net-
works for frequencies below resonance which is a region
that generally has a greater noise content. In situa-
tions where the antenna Q factor is low it is difficult
to provide a terminal impedance which is real and which
has a low value. The optimum source impedance is equal to
the input impedance of the network and its value should
be tightly controlled if a real input impedance is re-
quired at the system resonant frequency.

Matching network (B) produces an identical frequency
response to network (C). However, network (B) is restric-
ted in cases of antenna detuning since both $L_e$ and $R_e$ are
sensitive to capacitance and inductance variations. Typi-
cal long-term stability and temperature variations of the
matching components restrict the antenna Q factor to less
than 40.

If retuning is necessary then both matching components must be adjusted which is a difficult task in the field. However, this network has an increasing impedance characteristic away from resonance which is a desireable characteristic for transmitter applications.

The problems existing with matching networks (A) and (B) are not present with (C). Since only one resonating component is used, the task of retuning is much simpler than with (A) or (B). The impedance characteristics increases at frequencies away from resonance and its lower outband attenuation is better than that of (A). Furthermore, it is theoretically possible to match the antenna to any real value of terminal impedance. However, it has the disadvantage of requiring a transformer to match the low-impedance antenna to a transmission line and for a high turns-ratio the design of a low-loss transformer is no easy task.

## 7. Conclusion

This report has presented an analysis of an electrically small loop antenna together with a universal capacitive matching network. Various matching networks have been compared and it has been shown that the simple series resonant circuit possesses many important features.

The resulting frequency responses indicate that the best performance is achieved with a narrow band transmission system using an antenna with a high Q factor. The maximum Q factor is determined partly by the effect of the mine environment on the maximum change of all the components of the antenna system, by the distortion which may be introduced into the demodulated radio frequency signal and by the decreasing noise performance of some modulation methods which have a narrow bandwidth.

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APPENDIX

1.1 The universal matching network input impedance $(Z_i)$

In Figure 1 the impedance between 2 and 3 without $C_3$ and the termination is given by:

$$Z_{23} = \frac{(R + j\omega L)\dfrac{1}{j\omega C_1}}{R + j\omega L + \dfrac{1}{j\omega C_1}} + \frac{1}{j\omega C_2}$$

$$= \frac{R + j\omega L}{1 - \omega^2 L C_1 + j\omega C_1 R} + \frac{1}{j\omega C_2}$$

$$= \frac{1 - \omega^2 L(C_1 + C_2) + j\omega R(C_1 + C_2)}{-\omega^2 C_1 C_2 R + j\omega C_2(1 - \omega^2 L C_1)}$$

$$= \frac{Z_1 + jZ_2}{Z_3 + jZ_4} \tag{1}$$

where $Z_1 = 1 - \omega^2 L (C_1 + C_2)$; $Z_2 = \omega R(C_1 + C_2)$

$$Z_3 = -\omega^2 C_1 C_2 R; \quad Z_4 = \omega C_2 (1 - \omega^2 L C_1)$$

The impedance between 2 and 3 including $C_3$ but without the termination is given by:

$$Z_i = \frac{Z_{23}\dfrac{1}{j\omega C_3}}{Z_{23} + \dfrac{1}{j\omega C_3}} \tag{2}$$

Substituting for $Z_{23}$ from (1) into (2) gives:

$$Z_i = \frac{Z_1 + jZ_2}{(Z_3 - \omega C_3 Z_2) + j(Z_4 + \omega C_3 Z_1)} \tag{3}$$

$$OR \quad Z_i = \frac{Z_1 + jZ_2}{(Z_3 - Y_1) + j(Z_4 + Y_2)} \tag{4}$$

where $Y_1 = \omega C_3 Z_2$ ; $Y_2 = \omega C_3 Z_1$

## 1.2  The loaded output voltage ($V_L$)

The loaded output voltage can be derived by using Thevenin's equivalent circuit.  First the open circuit output voltage ($V_{oc}$) and output impedance ($Z_i$) are found and then used to obtain the loaded output by means of the potential divider formed by $Z_i$ and $R_L$.  Using Figure 1 with no load, then:

$$\frac{V_{13}}{V_i} = \frac{\frac{1}{j\omega C_4}}{R + j\omega L + \frac{1}{j\omega C_4}} = \frac{1}{1 - \omega^2 LC_4 + j\omega C_4 R}$$

where,  $C_4 = C_1 + \dfrac{C_2 C_3}{C_2 + C_3}$

therefore,  $\dfrac{V_{oc}}{V_i} = \dfrac{C_2}{C_2 + C_3} \dfrac{V_{13}}{V_i} = \dfrac{C_2}{C_2 + C_3} \left[ \dfrac{1}{1 - \omega^2 LC_4 + j\omega C_4 R} \right]$

$$= \frac{Y_3}{Y_4 + jY_5}$$

where,  $Y_3 = \dfrac{C_2}{C_2 + C_3}$ ;  $Y_4 = 1 - \omega^2 LC_4$ ;  $Y_5 = \omega C_4 R$

From equation 4 in Appendix 1.1.

$$Z_i = \frac{Z_1 + jZ_2}{Z_3 - Y_1 + j(Z_4 + Y_2)}$$

Hence if:  $\dfrac{V_L}{V_{oc}} = \dfrac{R_L}{R_L + Z_i}$

then:  $\dfrac{V_L}{V_i} = \left[ \dfrac{Y_3}{Y_4 + jY_5} \right] \left\{ \dfrac{(Z_3 - Y_1) + jR_L(Z_4 + Y_2)}{R_L(Z_3 - Y_1) + Z_1 + j[R_L(Z_4 + Y_2) + Z_2]} \right\}$

## 1.3 The antenna current($I_A$)

The input voltage to the matching network of Figure 1 in the "transmit" mode is given by:

$$V_{23} = V_s \frac{Z_i}{Z_i + R_s} \tag{5}$$

The current flowing through $C_2$ ($I_{12}$) can be found using the value for $Z_{23}$ derived in Appendix 1.1. that is:

$$I_{12} = \frac{V_{23}}{Z_{23}}$$

$$= V_s \left(\frac{Z_i}{Z_i + R_s}\right) \left(\frac{Z_3 + jZ_4}{Z_1 + jZ_2}\right) \tag{6}$$

Substituting the value for $Z_i$ derived in Appendix 1.1., equation (6) becomes:

$$\frac{I_{12}}{V_s} = \frac{(Z_3 + jZ_4)}{Z_1 + jZ_2 + R_s\left[(Z_3 - Y_1) + j(Z_4 + Y_2)\right]} \tag{7}$$

Using $I_{12}$, then $I_A$ is given by:

$$I_A = \frac{I_{12} \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + R + j\omega L} = \frac{I_{12}}{1 - \omega^2 L C_1 + j\omega C_1 R}$$

$$= \frac{I_{12}}{Z_6 + jZ_7} \tag{8}$$

where $Z_6 = 1 - \omega^2 L C_1$; $Z_7 = \omega C_1 R$

Hence by substituting equation 7 into equation 8 the antenna current is given by:

$$\frac{I_A}{V_s} = \left( \frac{Z_3 + jZ_4}{Z_6 + jZ_7} \right) \frac{1}{[R_s(Z_3 - Y_1) + Z_1] + j[R_s(Z_4 + Y_2) + Z_2]}$$

1.4    Antenna parameter programme

## AERIAL PARAMETER PROGRAMME
=============================

BY
C.W.F.ATTWELL
B.A.BOWLES

```
10 REM
20 REM
30 REM
40 REM
50 REM
60 REM
70 REM
80 REM
90 REM
100 REM*******************INPUT OF PLOTTING PARAMETERS SECTION********
110 DIM A$[100],B$[100],C$[20]
120 DISP "XMIN,XMAX,SCALE INC.";
130 INPUT A,B,C
140 DISP "PLOTTING STEP";
150 INPUT S
160 DISP "YMIN,YMAX,SCALE INC.";
170 INPUT A1,B1,C4
180 DISP "X-OFFSET,Y-OFFSET";
190 INPUT D1,D2
200 DISP "ASPECT RATIO (H/W)";
210 INPUT A9
220 DISP "X-AXIS LABEL";
230 INPUT A$
240 DISP "Y-AXIS LABEL";
250 INPUT B$
260 REM----------------------------------------------------------------
270 REM*****************PLOTTING AND LABELING OF AXIS SECTION********
280 A=A-(2*C)
290 B=B+C
300 A1=A1-2*C4
310 B1=B1+C4
320 SCALE A,B,A1,B1
330 A1=A1+2*C4  .
340 B1=B1-C4
350 P=3
360 A=A+(2*C)
370 B=B-C
380 XAXIS D2,C,A,B
390 YAXIS D1,C4,A1,B1
400 LABEL (*,2,1.7,0,A9)
410 FOR Y=A1 TO B1 STEP C4
420 PLOT A,Y,1
430 CPLOT -5.5,-0.3
440 LABEL (460)Y;
450 NEXT Y
460 FORMAT F5.0
470 LABEL (*,2,1.7,PI/2,A9)
480 FOR X=A TO B STEP C
490 IF X=A THEN 530
500 PLOT X,+D2,1
510 CPLOT -5.5,-0.3
520 LABEL (540)X;
530 NEXT X
540 FORMAT F5.2
550 PLOT (A+(B-A)/2),D2,3
560 LABEL (*,2.5,1.7,0,A9)
570 CPLOT -(LEN(A$)/2),+0.5
580 LABEL (*,2.5,1.7,0,A9)A$[1,(LEN(A$))];
590 LABEL (*,2.5,1.7,PI/2,A9)
600 PLOT D1,A1+(B1-A1)/2,1
```

```
     CPLOT -(LEN(B$)/4),+3
     PLOT -(LEN(B$)/4),0
650  LABEL (*,2.5,1.7,PI/2,A9)B$[1,(LEN(B$))];
640  PLOT B,B1,1
645  DISP
650  REM-------------------------------------------------------------
660  REM****************************INPUT OF VARIABLES SECTION*************
670  DISP "INSERT VARIABLES C1,C2,C3";
680  INPUT C1,C2,C3
690  DISP "Q FACTOR";
700  INPUT Q
710  DISP "FREQUENCY (HZ)";
720  INPUT W
730  DISP "INDUCTANCE";
740  INPUT L
750  DISP "LOAD RESISTANCE";
760  INPUT R
770  REM-------------------------------------------------------------
780  REM*********************PROGRAMME SELECTOR SECTION************
790  PRINT "FUNCTION NAME                    DESCRIPTION"
800  PRINT
810  PRINT "IMPMAG . . . . . . . . . . .IMPEDANCE MAGNITUDE"
820  PRINT "IMPHASE  . . . . . . . . . .IMPEDANCE PHASE"
830  PRINT "LOADOUTMAG . . . . . . . .LOADED OUTPUT MAGNITUDE"
840  PRINT "OUTPHASE . . . . . . . . .LOADED OUTPUT PHASE"
850  PRINT "LOOPMAG  . . . . . . . . .TRANSMIT LOOP CURRENT MAGNITUDE"
860  PRINT "LOOPHASE . . . . . . . . .TRANSMIT LOOP CURRENT PHASE"
870  PRINT
880  DISP "WHICH FUNCTION REQUIRED";
890  INPUT C$
900  IF C$="IMPMAG" THEN 1200
910  IF C$="IMPHASE" THEN 1380
920  IF C$="LOADOUTMAG" THEN 1640
930  IF C$="OUTPHASE" THEN 1890
940  IF C$="LOOPMAG" THEN 2190
950  IF C$="LOOPHASE" THEN 2430
960  PRINT
970  GOTO 880
980  STOP
990  REM-------------------------------------------------------------
1000 REM*****************DEFINITION OF ARCTAN ABSOLUTE FUNCTION*****
1010 DFC "ARCTAN"(Z)
1020 S1=SGN(V1)
1030 S2=SGN(V2)
1040 DEG
1050 IF S1=1 AND S2=1 THEN 1090
1060 IF S1=1 AND S2=-1 THEN 1110
1070 IF S1=-1 AND S2=-1 THEN 1130
1080 IF S1=-1 AND S2=1 THEN 1150
1090 Z=ATN(ABS(V1/V2))
1100 RETURN Z
1110 Z=180-ATN(ABS(V1/V2))
1120 RETURN Z
1130 Z=180+ATN(ABS(V1/V2))
1140 RETURN Z
1150 Z=360-ATN(ABS(V1/V2))
1160 RETURN Z
1170 STOP
1180 REM-------------------------------------------------------------
1190 REM******************MAGNITUDE OF THE IMPEDANCE SECTION*******
1200 F=3
```

```
1210 FOR X=A TO B STEP S
1220 PLOT X,FC"IMPMAG"(X),P
1230 P=2
1240 NEXT X
1250 PEN
1260 GOTO 2720
1270 DFC "IMPMAG"(X)
1280 Z1=1-((2*PI*X*W)^2)*L*(C1+C2)
1290 Z2=(((2*PI)^2)*(W^2)*X*L*(C1+C2))/Q
1300 Z3=((-1)*((2*PI)^3)*(W^3)*(X^2)*C1*C2*L)/Q
1310 Z4=2*PI*X*W*C2*(1-(((2*PI)^2)*(X^2)*(W^2)*L*C1))
1320 Y1=2*PI*W*X*C3*Z2
1330 Y2=2*PI*W*X*C3*Z1
1340 Y3=(((Z1^2)+(Z2^2))/((Z3-Y1)^2+(Z4+Y2)^2))^0.5
1350 RETURN Y3
1360 STOP
1370 REM--------------------------------------------------------------
1380 REM*********************IMPEDANCE PHASE SECTION******************
1390 P=3
1400 FOR X=A TO B STEP S
1410 PLOT X,FC"IMPHASE"(X),P
1420 P=2
1430 NEXT X
1440 PEN
1450 GOTO 2720
1460 DFC "IMPHASE"(X)
1470 Z1=1-((2*PI*X*W)^2)*L*(C1+C2)
1480 Z2=(((2*PI)^2)*(W^2)*X*L*(C1+C2))/Q
1490 Z3=((-1)*((2*PI)^3)*(W^3)*(X^2)*C1*C2*L)/Q
1500 Z4=2*PI*X*W*C2*(1-(((2*PI)^2)*(X^2)*(W^2)*L*C1))
1510 Y1=2*PI*W*X*C3*Z2
1520 Y2=2*PI*W*X*C3*Z1
1530 V1=Z2
1540 V2=Z1
1550 F1=FC"ARCTAN"(Z)
1560 V1=Z4+Y2
1570 V2=Z3-Y1
1580 F2=FC"ARCTAN"(Z)
1590 F3=F1-F2
1600 RETURN F3
1610 STOP
1620 REM--------------------------------------------------------------
1630 REM******************MAGNITUDE OF LOADED OUTPUT VOLTAGE SECTION******
1640 P=3
1650 FOR X=A TO B STEP S
1660 PLOT X,FC"LOADOUTMAG"(X),P
1670 P=2
1680 NEXT X
1690 PEN
1700 GOTO 2720
1710 DFC "LOADOUTMAG"(X)
1720 Z1=1-((2*PI*X*W)^2)*L*(C1+C2)
1730 Z2=(((2*PI)^2)*(W^2)*X*L*(C1+C2))/Q
1740 Z3=((-1)*((2*PI)^3)*(W^3)*(X^2)*C1*C2*L)/Q
1750 Z4=2*PI*X*W*C2*(1-(((2*PI)^2)*(X^2)*(W^2)*L*C1))
1760 Y1=2*PI*W*X*C3*Z2
1770 Y2=2*PI*W*X*C3*Z1
1780 C4=C1+((C2*C3)/(C2+C3))
1790 Y9=C2/(C2+C3)
1800 Y4=1-(((X*2*PI*W)^2)*L*C4)
```

```
1810 Y5=((-2*PI*W)↑2)*X*C4*L)/Q
1820 Y6=((R*(Z3-Y1)+Z1)↑2)+((R*(Z4-Y2)+Z2)↑2)
1830 Y7=((Y9*R)↑2)*(((Z3-Y1)↑2)+((Z4+Y2)↑2))
1840 Y8=10*LGT(Y7/(Y6*((Y4↑2)+(Y5↑2))))
1850 RETURN Y8
1860 STOP
1870 REM------------------------------------------------------------
1880 REM*******************LOADED OUTPUT VOLTAGE PHASE***********
1890 P=3
1900 FOR X=A TO B STEP S
1910 PLOT X,FC"OUTPHASE"(X),P
1920 P=2
1930 NEXT X
1940 PEN
1950 GOTO 2720
1960 DFC "OUTPHASE"(X)
1970 Z1=1-((2*PI*X*W)↑2)*L*(C1+C2)
1980 Z2=(((2*PI)↑2)*(W↑2)*X*L*(C1+C2))/Q
1990 Z3=((-1)*((2*PI)↑3)*(W↑3)*(X↑2)*C1*C2*L)/Q
2000 Z4=2*PI*X*W*C2*(1-(((2*PI)↑2)*(X↑2)*(W↑2)*L*C1))
2010 Y1=2*PI*W*X*C3*Z2
2020 Y2=2*PI*W*X*C3*Z1
2030 C4=C1+((C2*C3)/(C2+C3))
2040 Y4=1-(((X*2*PI*W)↑2)*L*C4)
2050 Y5=(((2*PI*W)↑2)*X*C4*L)/Q
2060 V1=Z4+Y2
2070 V2=Z3-Y1
2080 P1=FC"ARCTAN"(Z)
2090 V1=Y5
2100 V2=Y4
2110 P2=FC"ARCTAN"(Z)
2120 V1=R*(Z4+Y2)+Z2
2130 V2=R*(Z3-Y1)+Z1
2140 P3=FC"ARCTAN"(Z)
2150 P4=P1-P2-P3
2160 RETURN P4
2170 STOP
2180 REM------------------------------------------------------------
2190 REM*******************TRANSMIT LOOP CURRENT SECTION***********
2200 P=3
2210 FOR X=A TO B STEP S
2220 PLOT X,FC"LOOPMAG"(X),P
2230 P=2
2240 NEXT X
2250 PEN
2260 GOTO 2720
2270 DFC "LOOPMAG"(X)
2280 Z1=1-((2*PI*X*W)↑2)*L*(C1+C2)
2290 Z2=(((2*PI)↑2)*(W↑2)*X*L*(C1+C2))/Q
2300 Z3=((-1)*((2*PI)↑3)*(W↑3)*(X↑2)*C1*C2*L)/Q
2310 Z4=2*PI*X*W*C2*(1-(((2*PI)↑2)*(X↑2)*(W↑2)*L*C1))
2320 Z6=1-(((2*PI*W*X)↑2)*L*C1)
2330 Z7=(((2*PI*W)↑2)*X*C1*L)/Q
2340 Y1=2*PI*W*X*C3*Z2
2350 Y2=2*PI*W*X*C3*Z1
2360 L1=(Z3↑2)+(Z4↑2)
2370 L2=((Z1+(R*(Z3-Y1)))↑2)+((Z2+(R*(Z4+Y2)))↑2)
2380 L3=(Z6↑2)+(Z7↑2)
2390 L4=10*LGT((L1*10000)/(L2*L3))
2400 RETURN L4
```