Development of a Piezoelectric Adaptive Mirror for Laser Beam Control

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Abstract:
An intra-cavity adaptive mirror is developed for compensation of time-dependent phase aberrations to a laser beam, such as those caused by thermal lensing. The unimorph-type device consists of a metallic disc, with a mirror finish, bonded to a piezoelectric disc, providing a small, low-cost deformable mirror for this application. The mirror is required to be able to deform in the shape of each of the lower order Zernike polynomials, which describe aberrations in optical systems. Numerical modelling is employed to predict the deformation shapes that can be achieved by a unimorph mirror with a particular electrode pattern. The results from a Rayleigh-Ritz model and a finite element model, employing elements including rotational degrees of freedom, are compared to results from a conventional finite element model. The finite element model is applied to model a prototype deformable mirror and produced good agreement with experimental results. Finally, a mathematical optimization routine is proposed to predict the optimal electrode configuration on the free electrode.

Keywords: deformable mirror, laser beam control, piezoelectric unimorph

Introduction

A compact, low-cost deformable mirror is developed for intra-cavity laser beam control. This mirror can be used to correct for time-dependent phase aberrations to the laser beam, such as those caused by thermal lensing. A piezoelectric unimorph design is suitable for this application [1]. The device consists of a copper disc, with a mirror finish, bonded to a piezoelectric disc. When a voltage is applied to the piezoelectric disc the induced strains in the plane of the disc cause bending of the unimorph. The electrode on the free surface of the piezoelectric disc can be divided into segments, which can each have a different voltage applied. In this way the mirror can be deformed into relatively complex shapes.

In adaptive optics the deformations that the mirror is required to perform are described by Zernike polynomials, which are a complete set of orthogonal functions defined on a unit circle. The form of the (even) polynomials is given by:

$$Z_n^m(\rho,\varphi) = R_n^m \cos(m\varphi), \qquad (1)$$

with $0 \leq \rho \leq 1$, $0 \leq \varphi \leq 2\pi$, and where

$$R_n^m = \sum_{k=0}^{(n-m)/2} (-1)^k \rho^{n-2k} \frac{(n-k)!}{k!\left(\left(n+m/2-k\right)!\left(n-m/2-k\right)!\right)!}.$$

Due to the symmetry of the problem, only axisymmetric polynomials are considered (i.e. m=0). For brevity, we will denote this subset of Zernike polynomials as:

$$Z_i \equiv Z_{n=2i}^{m=0}, \quad i=0,1,2,\ldots . \qquad (2)$$

Numerical modelling is required to predict the deformation shapes that can be achieved by a particular electrode pattern. In this paper two numerical models are developed and the results are compared. Firstly, a Rayleigh-Ritz model, using the Zernike polynomials directly to describe the deformation of the mirror is developed and presented. The second model is based on a more traditional finite element procedure with specially formulated hybrid axisymmetric elements with rotational degrees of freedom [2]. A prototype device is constructed for comparison to numerical results. Finally, an optimization routine is presented to predict the electrode pattern which best excites a particular polynomial.

Rayleigh–Ritz model formulation

The Rayleigh-Ritz method has previously been applied to a cantilever beam with attached piezoelectric ceramic patches by Hagood *et al.* [3], and has become popular for piezoelectric structures since then. The method is based on Hamilton's principle for coupled electromechanical systems. A more detailed formulation of the method specific to the piezoelectric unimorph under consideration may be found in [4]. Only a brief discussion of the salient features of the method is therefore presented here. The geometry under consideration is shown in Fig. 1.

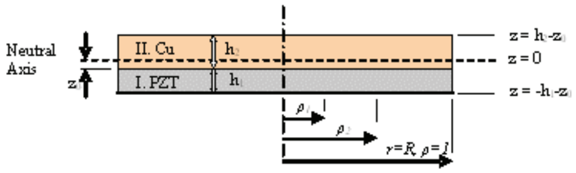

Fig. 1: *Unimorph mirror geometry and dimensions used in modelling.*

The method requires the selection of a set of assumed displacement and electrical potential distributions. Here, the Zernike polynomials are used *directly* for the assumed displacement distribution. The displacement may now be written as a superposition of assumed displacement functions with unknown coefficients,

$$w(r,t) = W(r)a(t) = R\{Z_1(\rho) \quad \cdots \quad Z_n(\rho)\}\begin{Bmatrix} a_1(t) \\ \vdots \\ a_n(t) \end{Bmatrix} \quad (3)$$

where, R is the radius of the disc, $Z_i(\rho)$ is the i^{th} Zernike polynomial, ρ is the non-dimensionalised radius and $a_i(t)$ is the amplitude coefficient for the i^{th} polynomial, which has to be determined. Only the first five non-constant Zernike polynomials are used in the modelling.

The method ultimately results in the coupled electromechanical equations:

$$\begin{aligned} Ka - \Theta v &= f \\ \Theta^T a + C_p v &= q \end{aligned} \quad (4)$$

where K is the mechanical stiffness matrix, Θ is the piezoelectric coupling matrix, C_p is the capacitance matrix, a is the vector of unknown Zernike polynomial coefficients, v is the vector of applied voltages, and f and q are the applied forces and charges. The integrations, required to form the matrices, can be performed analytically to obtain expressions for each term in the matrices. The matrix elements therefore contain the design dimensions (R, h_1, h_2, ρ_1, ρ_2) and material properties explicitly.

Elements with rotational dof's and polynomial extraction

To compare the numerical procedure based on the Rayleigh-Ritz method proposed in the previous section, a more conventional finite element analysis is conducted. For the purposes of this comparison, an axisymmetric finite element model using specially developed finite elements with rotational degrees of freedom is employed. These elements have been shown to be especially well suited to bending-dominated problems [2]. More detail regarding the elastostatic elements may be found in [2].

The displacement of the mirror surface can be extracted from the finite element analysis. A procedure is then required to determine which of the Zernike polynomials are excited. To this end, a least-squares fit of the surface displacements is employed, i.e. we minimize the function

$$\chi^2 = \sum_{i=1}^{N}\left[y_i - \sum_{k=0}^{M} a_k Z_k(r_i)\right]^2 \quad (5)$$

where y_i is the i^{th} of the N surface nodal displacements. The minimization is carried out using the procedure described in [1]. The output of this process is a vector of the coefficients a_k which scale the magnitude of the M non-constant Zernike polynomials.

Experimental details

In order to assess the practicality of the proposed numerical models, a physical prototype was constructed consisting of a 40 mm diameter, 0.3 mm thick, PZT4 piezoelectric ceramic disc bonded to a copper disc 44 mm in diameter and 0.3 mm thick. The slightly larger diameter of the copper provides a surface onto which is attached a grounding wire as shown in Fig. 2. The free electrode on the piezoelectric disc is segmented into three concentric rings. The electrode patterning was carried out using laser ablation with an excimer laser. The unimorph was driven by applying a harmonic voltage excitation to the segmented electrodes. Point deformations of the disc were measured using a Polytec laser vibrometer.

Fig. 2: *Prototype deformable mirror (rear view).*

Comparison of results

The results of the Rayleigh-Ritz and the finite element numerical models are compared in this section. For completeness, a commercial finite element code, Comsol Multiphysics, is also used in the comparison. For simplicity in the *numerical* comparison, both the piezoceramic and the copper discs are assumed to have the same diameter. A model with three annular electrodes is used for the comparison. A 40 mm diameter device comprising a 0.5 mm thick PZT4 disc bonded to a 0.5 mm thick copper disc is modelled. The first electrode extends from $\rho=0$ to ρ_1, the second from $\rho=\rho_1$ to ρ_2, and the third from $\rho=\rho_2$ to 1. The electrodes were positioned where they would be expected to best excite the third Zernike polynomial, i.e. $\rho_1=0.27$ and $\rho_2=0.72$.

These points are found by solving $\frac{\partial^2 Z_3}{\partial \rho^2}=0$.

This estimate neglects the influence of circumferential strains and a better estimate of the optimal electrode positions is presented in the next section.

The displacement of the mirror surface when 200 V is applied to the middle electrode (i.e. the electrode between the centre and outer electrodes) is illustrated in Fig. 3. Good agreement between the results from the three models is observed.

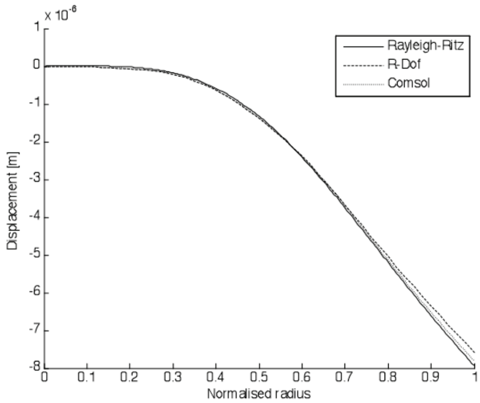

Fig. 3: *Comparison of mirror surface displacement prediction using different numerical models.*

As the numerical models produced essentially identical results only results from the axisymmetric finite element model using elements with rotational degrees of freedom are compared to an experimental measurement in Fig. 4. The measurement was performed on the device shown in Fig. 2 with 100V applied to the second electrode and the finite element model was adapted to this geometry and excitation. It is noted that the shape of the deformation is very similar but the model over predicts the deformation. It is believed that this is because the material properties of the piezoelectric disc used in the experiment are not known accurately and differ from those used in the model. A more thorough comparison of numerical model predictions and the experimental measurements may be found in [4].

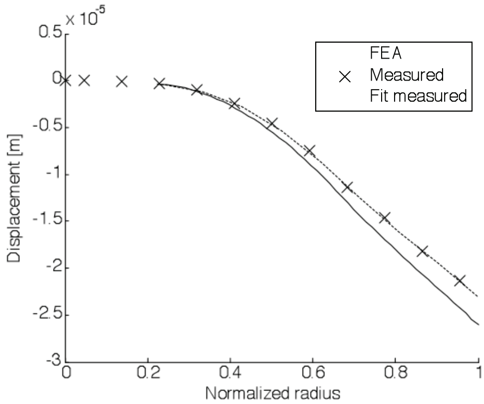

Fig. 4: *Comparison of mirror surface displacement prediction and measurement.*

Deformable mirror optimization

In this section it is briefly shown how the finite element model can be used to optimize the electrode configuration to best achieve a given objective.

Consider the problem of optimizing the layout of the electrodes to maximize the deformation of the third Zernike polynomial (a_3). To ensure proper scaling of the optimization problem, the objective function f is defined as:

$$f(x_i)=\left(\frac{a_3(x_i)}{a_3^{\,0}(x_i^{\,0})}\right)^2 \tag{6}$$

where $a_3(x_i)$ is the value of the third Zernike polynomial at design point x_i, and a_3^0 is value of a_3 at the starting point, x_i^0. The design variables (x_i) are defined as the 'prescribed' voltages at each individual node on the free electrode. The starting vector is chosen such that $x_i = V_{\max}$ for all nodes. The finite element mesh selected for evaluation resulted in 51 equally spaced nodes on the free electrode, each of which is an independent variable.

The optimization problem statement can therefore be written as:

$\min(-f)$, such that

$$-V_{\max} \le x_i \le V_{\max}, \quad i=1,2,\ldots,51$$

where $V_{\max}$=200V. The problem was solved iteratively using the gradient-based method of moving asymptotes (MMA) due to Svanberg [5].

The sensitivity of the objective function, f, to an individual voltage x_i can is computed analytically. A significant feature of the analytical sensitivity analysis is that $\partial a_3 / \partial x_i$ need only be computed once, and is constant throughout the procedure saving on computational effort.

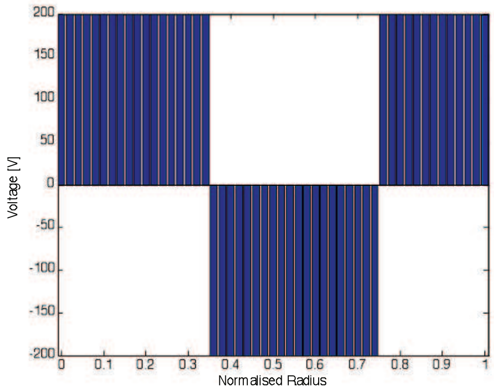

Fig. 5: *Optimal voltage distribution computed using finite element method.*

The optimal voltage distribution is depicted in Fig. 5. The result suggests that the optimal electrode configuration is ρ_1 =0.35 and ρ_2 = 0.75. It is important to emphasise that since each nodal voltage is a design variable, the optimization routine does not explicitly specify the number of electrodes. Rather, the optimal electrode configuration is implied by the optimal voltage distribution. The predicted deformation in the third polynomial, using this optimal electrode configuration, is approximately 10% greater than that predicted with the previous electrode configuration (ρ_1 =0.27 and ρ_2 = 0.72).

This optimization problem was solved analytically using the Rayleigh-Ritz model in [4] where it was assumed that three electrodes would be required and that the voltage applied to the second electrode would have opposite polarity to that applied to the first and third electrodes. The solution obtained was ρ_1 = 0.35 and ρ_2 = 0.7377.

The method presented here may be extended to consider various other optimization formulations. For example, we may wish to maximise a_3 while simultaneously minimizing the magnitude of the other Zernike polynomials. Furthermore, due to the flexibility of the finite element method, complicated geometries and boundary conditions may be considered.

Conclusions

Two numerical models of a piezoelectric unimorph are proposed, and produce very similar results. The Rayleigh-Ritz method produces a small model (stiffness matrix dimension equal to the number of polynomials used) that predicts the deformations of the piezoelectric mirror with remarkable accuracy. The FE method including rotational degrees of freedom is more efficient than the conventional FE method available in commercial software but retains the flexibility of the method for axisymmetric problems. Good agreement was obtained between experimental measurements and the FE method including rotational degrees of freedom. A mathematical optimization routine based on the FE method including rotational degrees of freedom was demonstrated by determining the optimal electrode layout to excite the third Zernike polynomial.

Acknowledgement

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