

# Computationally Efficient Analysis and Optimization of Stiffened Thin-Walled Panels in Shear

Awie Viljoen\* and A. G. Visser†

*Council for Scientific and Industrial Research, Pretoria 0001, South Africa*  
and

Albert A. Groenwold‡

*University of Pretoria, Pretoria 0002, South Africa*

The computationally efficient analysis and optimum design of the buckling of stiffened, thin-walled shear panels in aircraft structures is discussed. Namely, the postbuckling behavior of these panels is assessed using the iterative procedure developed by Grisham. This procedure requires only linear finite element analyses, whereas convergence is typically achieved in as few as five iterations. An algorithm developed by (A. F. Grisham, “A Method for Including Post-Buckling of Plate Elements in the Internal Loads Analysis of Any Complex Structure Idealized Using Finite Element Analysis Methods,” AIAA Paper 78-515, April 1978) using connect format, is compared with empirical methods of analysis frequently used in aircraft structures and also with a refined, nonlinear quasi-static finite element analysis. It is shown that the procedure proposed by Grisham overcomes some of the conservatism inherent in conventional methods of analysis. In addition, the method is notably less expensive than a complete nonlinear finite element analysis, which makes it attractive for use during initial design iterations, even though global collapse of a structure cannot be predicted. As an illustration of the optimal design of buckled, stiffened thin-walled structures, the Grisham algorithm is combined with a microgenetic algorithm. Important reductions in weight are obtained within relatively few function evaluations.

## Nomenclature

$A$	=	area
$E$	=	Young's modulus
$G$	=	shear modulus
$k$	=	diagonal tension factor
$L_x$	=	length of web along the $x$ axis
$L_y$	=	length of web along the $y$ axis
$N$	=	internal loads of structure
$t$	=	thickness of web
$\alpha$	=	diagonal tension angle
$\gamma_{xy}$	=	web shear distortion
$\gamma_{xyc}$	=	web shear distortion component due to compressive buckling
$\gamma_{xyDT}$	=	Postbuckled shear distortion component of web
$\epsilon_x, \epsilon_y$	=	web normal strain
$\epsilon_{xc}, \epsilon_{yc}$	=	web compressive buckling strain
$\epsilon_{xDT}, \epsilon_{yDT}$	=	web diagonal tension strain
$\mu$	=	poisson's ratio
$\sigma_x, \sigma_y$	=	web normal stress
$\sigma_{xc}, \sigma_{yc}$	=	web compressive buckling stress
$\sigma_{xcr}, \sigma_{ycr}$	=	web modified critical normal buckling stress
$\sigma_{xDT}, \sigma_{yDT}$	=	web diagonal tension stress
$\tau_{cr}$	=	web critical buckling shear stress
$\tau_{xy}$	=	web shear stress
$\tau_{xycr}$	=	web modified critical buckling shear stress

## I. Introduction

IN many aircraft structures, thin sheet structural components or panels are designed to buckle under shear load. On buckling, the internal loads and stresses in neighboring panels and the surrounding structure can change significantly. Hence, detailed analysis of the effects of buckling are important.

In the early days of aircraft design, postbuckling effects were taken into account through the theory of pure diagonal tension (PDT) proposed by Wagner.<sup>1,2</sup> However, in practice PDT proved to be very conservative. Wagner's approach was gradually modified to eventually become the more general approach of incomplete diagonal tension (IDT). The IDT approach was developed by NACA in the 1950s, after conducting an extensive testing program to generate empirical relations. This approach, also known as the NACA method, later became an accepted design approach used by many aircraft manufacturers, even though the theory is still considered conservative.

One of the factors neglected in the NACA approach is the interaction of stresses in each panel element on the element allowable, namely, the combination of compression and shear buckling, diagonal tension, and postbuckled skin softening in shear. As an alternative to IDT, nonlinear finite element codes can be used to assess buckling, even though design-by-rule failure criteria are more difficult to assess.<sup>3</sup> In addition, nonlinear finite element analyses are computationally very expensive during initial design iterations.

In 1978, Grisham<sup>4</sup> proposed an iterative procedure for the analysis of postbuckling behavior of thin-walled shear panels as used in aircraft structures. This procedure requires only linear finite element analyses, whereas convergence is typically achieved in as few as five iterations. Grisham's algorithm is attractive in optimization and during initial design iterations because the computational effort required is relatively low. Some of the salient features of the algorithm are as follows:

- 1) Convergence is usually achieved rapidly. Typically, as few as five iterations are required to obtain convergence to within 2% variation between successive values of diagonal tension  $\sigma_{xDT}$  and  $\sigma_{yDT}$ .
- 2) Provision is made for compressive buckling in both the length and width directions of panels, as well as shear buckling. The latter causes the development of diagonal tension, accompanied by

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\*Engineer, Division of Manufacturing and Materials Technology, Pretoria 0001, South Africa.

†Technical Specialist, Division of Manufacturing and Materials Technology.

‡Professor, Department of Mechanical Engineering, University of Pretoria.