Methods for determining the effect of flatness deviations, eccentricity and pyramidal errors on angle measurements

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Abstract. In angle calibrations the uncertainty contributions of the autocollimator and index table are easily estimated as these uncertainties are well documented, but it is more difficult to determine the uncertainties associated with flatness deviations, eccentricity and pyramidal errors of the measuring faces. Deviations in the flatness of angle surfaces have been held responsible for the lack of agreement in angle comparisons. An investigation has been carried out using a small-angle generator and an autocollimator to determine the effect of flatness deviations of the measuring surfaces on angle measurements. The small-angle generator, together with a translation stage and tilt table, was also used to determine the effect of eccentricity and pyramidal errors. These methods were developed to calculate the related uncertainties associated with flatness deviations, eccentricity and pyramidal errors on face-to-face angle measurements. The results show that flatness and eccentricity deviations have less effect on angle measurements than do pyramidal errors.

1. Introduction

Polygons and angle blocks are the most important transfer standards in the field of angle metrology. Polygons are used by national metrology institutes (NMIs) as transfer standards to industry, where they are used in conjunction with autocollimators to calibrate index tables, rotary tables and other forms of angle-measuring equipment. Polygons and angle blocks are also the most common angle standards in comparisons of angle measurements between NMIs and accredited laboratories [1, 2].

Autocollimators, the main instruments used in the measurement of angle, are calibrated with small-angle generators, using laser interferometers [3, 4]. A vast amount of time is invested in their calibration. Index tables are usually calibrated using another index table and an autocollimator with a mirror. To measure the angle differences between the tables all the steps (i.e. every 30°) are measured. Research has shown that it is possible to calibrate both autocollimators and index tables very accurately.

When calibrating polygons and angle blocks, however, it is impossible to achieve comparable accuracies. According to a comparison of angle blocks carried out by the Western European Calibration Cooperation [2], "The additional investigations on the flatness deviation of the angle gauge faces have clearly shown that this has an important influence on the measured angle and its uncertainty". A correlation between the angle measurements of polygons and the flatness of the faces has also been demonstrated [3, 5].

We have developed methods to determine the influence of flatness deviations, eccentricity and pyramidal errors on the measurement of angle, and we report the results for a DA20 Rank Taylor Hobson autocollimator. The aim of each of these methods was to eliminate all the uncertainties except that under investigation.

2. Deviations from flatness

A small-angle generator with a resolution of 0.005 arc-sec was verified using a 2160 Moore index table and found to have a repeatability of better than 0.02 arc-sec. It is shown below that this repeatability is of greater significance than the absolute accuracy of the system. As a result of its excellent repeatability, the small-angle generator could be used to determine the effect that any deviation in flatness of the measuring faces may have on angle measurements.

The small-angle generator, with a "flat" mirror, was used to calibrate a DA20 autocollimator, after which the mirror on the angle generator was replaced with mirrors of different deviations of flatness. In total, five mirrors of 25 mm diameter were used, having different peak-to-valley (p-v)/root-mean-square (rms) flatness values, to simulate a polygon/angle block with different flatness

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deviations between the measuring faces. Each mirror used had a uniform flatness deviation, as shown by the plots in Figures 1 to 5. Table 1 gives the deviation in flatness of all five mirrors, mirror 1 being the flat mirror used for the initial calibration of the autocollimator. Mirror 5 represents the maximum flatness deviation and its p-v value exceeds the acceptable limit according to the National Physical Laboratory (UK) specification MOY/SCMI/87 inspection-grade polygon [6].

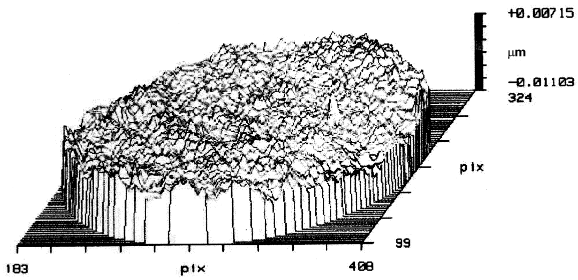

Figure 1. Mirror 1.

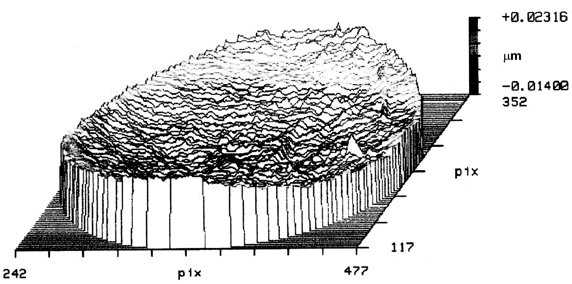

Figure 2. Mirror 2.

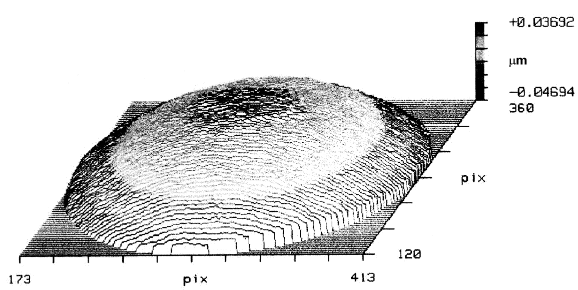

Figure 3. Mirror 3.

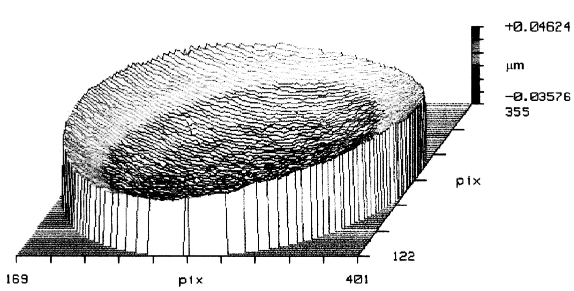

Figure 4. Mirror 4.

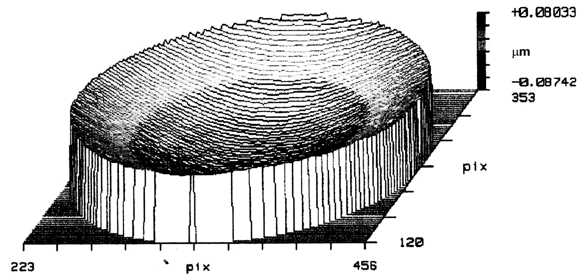

Figure 5. Mirror 5.

Figures 1 to 5. Flatness values of the five mirrors used in this experiment, measured using a Zygo flatness interferometer.

Table 1. Peak-to-valley and root-mean-square flatness values of each of the mirrors.

Mirror	p-v flatness/nm	rms flatness/nm
1	15	2
2	35	8
3	86	19
4	95	21
5	168	46

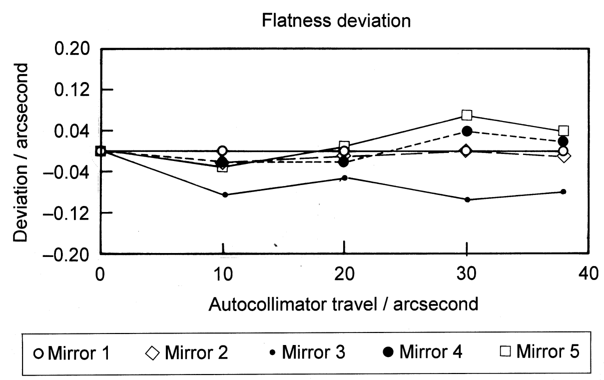

Figure 6. Results from the flat mirror, 1, on the small-angle generator, which was zeroed (normalized), compared with the results from the curved mirrors, 2 to 5.

The results from the flat mirror were zeroed and the results from the other mirrors compared with the zero line (Figure 6). Even at over 40 arc-sec, the total travel of the autocollimator, the results show smaller errors for different mirrors than those previously expected [3, 5]; all the results, even for mirror 5, are within 0.09 arc-sec.

A linear regression was fitted through the data points of each mirror:

$$u = bx + a, \tag{1}$$

where u is the “error”, or deviation from the result for the flat mirror; x is the travel/range of the autocollimator relative to the zero position; b is the gradient; and a is equal to zero, as the linear regression was fitted through zero. Gradient b is a function of the flatness deviation (rms) of the mirrors (measuring surfaces).

A linear regression between the gradient b of each mirror and its rms flatness deviation was calculated:

$$b = kz + c, \tag{2}$$

where z is the rms flatness deviation; the gradient k depends on the autocollimator under test; and c is again equal to zero.

Substitution of (2) into (1) gives

$$u = kzx. \quad (3)$$

Thus the possible error, i.e. the uncertainty, of the measurement depends on the rms flatness deviation z, the autocollimator characterization coefficient k, and the travel/range x of the autocollimator from the zero position.

A point of interest is that mirror 3 is the only convex mirror, all the other mirrors being concave. When compared with the flat mirror, the convex mirror gave negative results and the concave mirrors all gave positive results. Further work is required to establish if there is any correlation between the convex/concave nature and the negative/positive deviations with respect to the flat mirror.

3. Eccentricity measurements

A system has been developed to measure the influence of eccentricity errors of up to 0.5 mm.

Figure 7 shows the set-up: a translation stage with a flat mirror. The stage was moved laterally across the front of the autocollimator to simulate eccentricity of the polygon/angle block. The flat mirror was then replaced with curved mirrors to check for any influence from non-flat measuring faces. As the translation stage cannot be moved perfectly, i.e. without any yaw, a small-angle generator was also placed on top of the stage next to the mirror, to correct for any imperfections as it was moved through the 0.5 mm.

Figure 8 shows the effect of movement of the translation stage relative to the autocollimator read-out, the results having been corrected using the readings from the small-angle generator.

The results show that the error in the autocollimator reading is relatively small, being less than 0.12 arc-sec for all the mirrors, even for a large lateral movement of the mirror (eccentricity of 0.5 mm). If the eccentricity error is less than 0.1 mm, as prescribed in the protocol for EUROMET Project 371 [7], then the error in the autocollimator reading will be less than 0.04 arc-sec for all the mirrors. There is no significant difference in the results from the different mirrors, which proves that the use of non-flat faces does not affect the eccentricity errors in angle measurements.

4. Pyramidal errors

The pyramidal errors of a polygon/angle block are caused by non-alignment between the measuring faces in the y axis, due either to imperfections in the manufacture of the polygon or to misalignment in the experimental set-up.

Two methods were investigated: one similar to the previous method but using a tilt table and small-angle generator, and the other a polygon misaligned in the y axis.

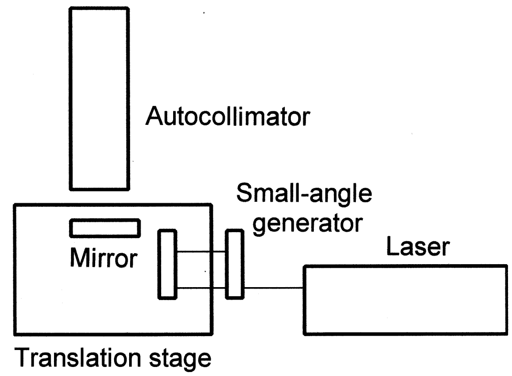

Figure 7. Experimental set-up for the eccentricity measurements using a translation stage equipped with a small-angle generator.

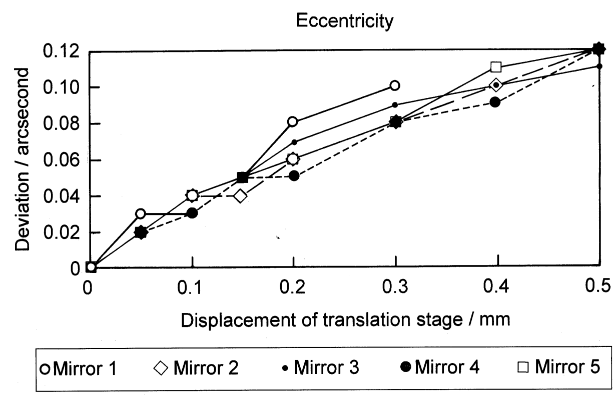

Figure 8. Eccentricity measurements of mirrors 1 to 5 using a translation stage.

Figure 9 shows the layout of the tilt table, with a small-angle generator, mirror and autocollimator. This layout was used to investigate if any difference was observed with curved mirrors. The small-angle generator was tilted around its apex, which in theory should not affect the laser readings. The readings from the flat mirror on the tilt table were compared with those from the polygon.

Figure 10 shows the results from the tilt table for mirrors 1 to 5. Although there is no significant difference in the results from the different mirrors, a substantial error in the x axis intercept is caused by misalignment/imperfection of the polygon in the y axis.

There is clearly a direct correlation between the misalignment/imperfection of the polygon, the y-axis reading, and the error in the x axis. The uncertainty can be calculated by fitting a line to the data of the flat mirror as follows:

$$u_{x1} = y_1 \times 0.09, \quad (4)$$

where y_1 is the y-axis reading and u_{x1} is the error in the x axis for the flat mirror;

$$u_{x2} = y_2 \times 0.11, \quad (5)$$

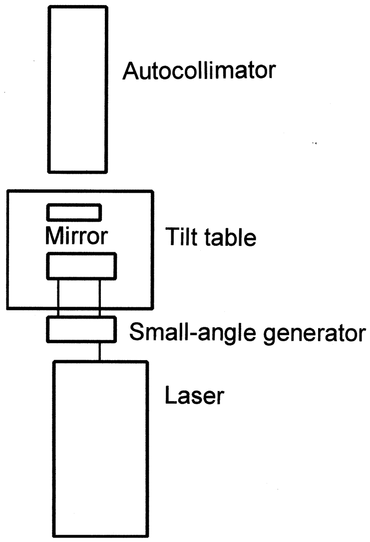

Figure 9. Experimental set-up for the pyramidal error measurements using a tilt table equipped with a small-angle generator.

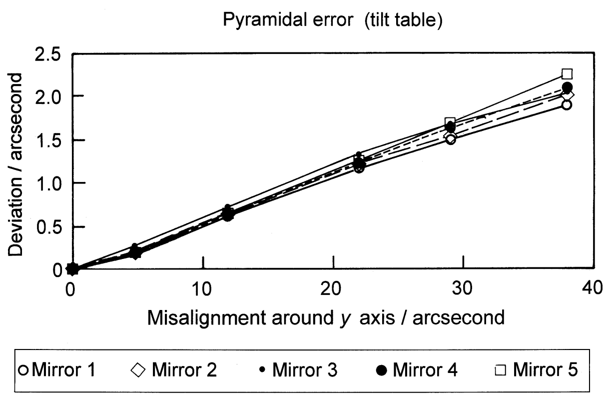

Figure 10. Pyramidal errors of mirrors 1 to 5 using a tilt table equipped with a small-angle generator.

where y_2 is the y-axis reading and u_{x2} is the error in the x axis for mirror 5.

Figure 11 shows the results from the polygon. The set-up consists of an autocollimator focused on a polygon on an index table. The polygon was set up to measure only the 0° and 180° faces, with these faces aligned to be 0° in the y axis. The difference in the x-axis readings from the index table was recorded. The polygon was then "misaligned" in the y axis and the x-axis reading recorded. This method was used for five different y-axis readings, as in the case of the tilt table, and a regression line was fitted through the points:

$$u_{x3} = y_3 \times 0.089, \tag{6}$$

where y_3 is the y-axis reading; and u_{x3} is the error in the x axis.

The results show excellent agreement with the previous method, leading to increased confidence in both methods. As the results from the tilt table do not show any significant differences for the different

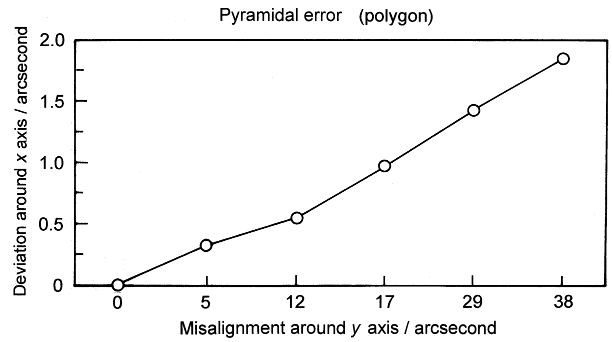

Figure 11. Pyramidal error using a polygon on an index table.

mirrors (see Figure 10), it is suggested that only the polygon/index table method need be used for the uncertainty calculation, as it is quicker and less complex.

The results indicate that the error in alignment of the polygon in the y axis should be minimized. Unfortunately, any imperfection in the manufacturing of the polygon will persist. The y-axis deviation of each face of the polygon can be measured and an uncertainty component calculated for each face-to-face measurement.

5. Conclusions

The results show that the flatness of the measuring face when using the autocollimator does not affect the angle measurements to the expected extent. The results are within ± 0.09 arc-sec, and increase as the flatness value of the mirror increases, with the exception of mirror 4 (the convex example), where the results decrease over the same range. The uncertainty for each measured face must be calculated according to (3).

The results also show that eccentricity does not have a large effect on the x-axis readings, the error being 0.04 arc-sec for an eccentricity error of 0.1 mm; this value can be included in the uncertainty budget. Using the same method, differences in flatness deviations between the mirrors were also shown to have no effect on the angle measurements.

Pyramidal errors proved to be the greatest contributor to the uncertainty. For the DA20 autocollimator, the pyramidal error is 0.09 arc-sec for 20 arc-sec error in the y axis. The y-axis deviation for each measurement, face-to-face, can be measured and calculated according to (6) and included in the uncertainty budget.

Note that if a closure method is not used, these uncertainties must be added to the autocollimator and index table uncertainties when compiling the comprehensive uncertainty budget for the calibration of polygon/angle blocks. This makes it possible to calculate a separate uncertainty for each face-to-face angle measurement.

It should be borne in mind that an autocollimator can yield different calibration data for different reflectivities and sizes of the measuring face (polygon). For this reason, wherever possible, the polygon being measured with the laser small-angle generator should have been used when calibrating the autocollimator.

It is important to note that these results are applicable only to this particular autocollimator. The methods will need to be repeated to determine the effect of flatness deviations, eccentricity and pyramidal errors on angle measurements on individual autocollimators.

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