Materials Science and Engineering A309-310 (2001) 227–228

MATERIALS SCIENCE & ENGINEERING A

www.elsevier.com/locate/msea

The time constant of logarithmic creep and relaxation

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Abstract

Under certain conditions, the plastic extension of a sample subjected to a constant stress is to a good approximation proportional to the logarithm of the time. Similarly, if a sample is plastically strained and unloaded, there are changes in its length and hardness which vary logarithmically with time. For dimensional reasons, a logarithmic variation must involve a time constant τ characteristic of the process, so that the deformation is proportional to $\ell n(t/\tau)$.

Two distinct mechanisms of logarithmic creep have been proposed, the work-hardening of a set of barriers to dislocation motion, all having the same activation energy, or the progressive exhaustion of the weaker barriers in a set which has a distribution of activation energies, these energies remain constant during the process of creep. It has been suggested that the experimentally observed value of τ can be used to decide which of the two mechanisms is operative. It is shown here that the work-hardening mechanism expresses τ in terms of parameters which are not easy to estimate, while, if the exhaustion mechanism operates, the observed value of τ is determined by the experimental conditions rather than by the parameters of the dislocation mechanism.

Keywords: Dislocation mechanism; Exhaustion mechanism; Logarithmic creep

1. Introduction

When a material is loaded under a constant stress at a constant temperature, it undergoes a time-dependent creep strain $\epsilon(t)$. If the stress and the temperature are low, ϵ usually increases linearly with the logarithm of the time. This is called logarithmic creep. [1,2]. Similarly, if a sample is plastically extended under an increasing load, and the elongation is suddenly held constant, the stress in the sample decreases as the logarithm of the time [3]. There are analogous cases of dielectric and magnetic relaxation (e.g. [4]). For a thermally activated process with a single activation energy one expects a decay which is exponential in time, not logarithmic. A suitable spectrum of activation energies will lead to logarithmic recovery, but the assumption that such a spectrum is frequently present is very artificial.

There are two models of logarithmic creep and similar effects. The first is, in the case of logarithmic creep, called the work-hardening model. The second is called the exhaustion model. It is important to know which model is appropriate in a particular case.

Mott [5] and Cottrell [6] suggested that a distinction could be made along the following lines. For dimensional reasons, a logarithmic dependence on time must involve the function $\ell n(t/\tau)$, where τ might be expected to be the order of the period of a lattice vibration. We argue below that the experimentally measured value of τ represents, in the work-hardening model, a rather complicated combination of physical parameters, and, in the exhaustion model, the duration of the transient deformation which occurs before logarithmic creep is established.

2. The work-hardening model

In the work-hardening model, all the sites at which deformation can be initiated are assumed to have the same activation energy U. However, U is not constant in time. It depends on the effective stress σ_{eff} acting on each site. The effective stress is the applied stress σ minus the frictional stress of the material. As a result of work-hardening, this frictional stress increases linearly with the small plastic strain ϵ. Thus

$$\sigma_{\text{eff}} = \sigma - k\varepsilon \tag{1}$$

It is then reasonable to assume that, for small changes in σ_{eff}, U will be a linearly decreasing function of σ_{eff}, so that

$$U(t) = U_0 + \lambda\varepsilon(t) \tag{2}$$

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0921-5093/01/$ – see front matter
PII: S0921-5093(00)01692-0

The cases of stress relaxation and elastic after effects differ slightly from that of creep under a constant applied stress. Here [4], the driving stress σ is an internal stress, which decreases linearly with the relaxation strain. Eq. (2) again applies. In all cases, the rate of creep is given by

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp\left(-\frac{U}{kT}\right) \quad (3)$$

The value of $\dot{\varepsilon}_0$ depends on the volume density of the barriers, and on the strain which is produced each time a barrier is overcome. The value of λ in Eq. (2) is not easy to estimate, even if it is assumed that the rate of work hardening during logarithmic creep is the same as that in rapid deformation.

From Eqs. (2) and (3) it follows that

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \dot{\varepsilon}_0 \exp\left(-\frac{U_0 + \lambda\varepsilon}{kT}\right) \quad (4)$$

The solution is

$$\varepsilon = \left(\frac{kT}{\lambda}\right) \ell n \left(1 + \frac{t}{\tau}\right) \quad (5)$$

where

$$\tau = \left(\frac{kT}{\lambda\dot{\varepsilon}_0}\right) \exp\left(\frac{U_0}{kT}\right) \quad (6)$$

Experiment determines kT/λ and τ, and so gives the values of λ and of $\dot{\varepsilon}_0 \exp(-U_0/kT)$. The latter is proportional to the frequency of lattice vibrations, but involves several unknown physical quantities.

3. The exhaustion model

The usual outline of the exhaustion model is as follows. There is a distribution of barriers such that all values of the activation energy are equally likely. Each activation process produces the same increment of strain. The dislocation segments attack each barrier with a frequency ν equal to the vibration frequency of a dislocation segment about 100 atoms long, say about $10^{10}\,\mathrm{s}^{-1}$. Then at time t those barriers for which the activation energy satisfies

$$\exp\left\{-\nu t \exp\left[\frac{-U(t)}{kT}\right]\right\} = \frac{1}{2} \quad (7)$$

are equally likely to be present or to have been overcome. Those with U less than $U(t) - 2kT$ are almost certain to have been overcome, these with U greater than $U(t) + 2kT$ are almost certain not to have been overcome.

It follows from Eq. (7) that

$$U(t) = kT\ell n \frac{\nu t}{\ell n 2} \quad (8)$$

and from the physical assumptions that ϵ is a linear function of $U(t)$. Hence

$$\varepsilon = \varepsilon_0 + AkT\ell n \frac{\nu t}{\ell n 2} \quad (9)$$

However, while Eq. (5) gives $\epsilon = 0$ at $t = 0$, Eq. (9) gives a divergence at $t = 0$. This is because it has been assumed that all values of U, even negative values, are equally likely, so that there is an infinite instantaneous extension when the stress is applied. It is therefore necessary to measure the time from some epoch τ at which ϵ is defined to be zero. Then Eq. (9) is replaced by

$$\varepsilon(t) = \varepsilon_i + AkT\ell n \left[\nu \frac{t + \tau}{\ell n 2}\right] \quad (10)$$

subject to the condition

$$\varepsilon(0) = 0 = \varepsilon_i + AkT\ell n \left(\frac{\nu t}{\ell n 2}\right) \quad (11)$$

The creep strain is given by

$$\varepsilon(t) = AkT\ell n \left(1 + \frac{t}{\tau}\right) \quad (12)$$

The experimental characteristic time is then τ, which is of the order of the duration of the rapid extension on loading, a few seconds. The observation [1] that τ is of this order is therefore compatible with the exhaustion model. It could also be compatible with the work-hardening model. The latter would only be excluded if the measured τ was found to depend very little on temperature.

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