Safety in Mines Research Advisory Committee

Draft Final Report

Reliable practical technique for in-situ rock stress measurements in deep gold mines

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Executive summary

The proposed Primary Output of this research project is the development of a set of equipment and method of analysis of data for a practical method of in situ stress measurement in a high stress environment typical of the deep level gold mines. The method involves:

- a borehole-based system, using percussion boreholes drilled with a conventional jackhammer;
- creating the enlargement, or stress/strain change by drilling parallel, overlapping holes;
- control of the direction of adjacent holes by means of a special borehole guide;
- measurement of deformation changes in the first borehole, as a result of the drilling of the third borehole;
- sequential deformation change measurements down the length of the borehole, alternately moving the deformation meter and advancing the third borehole;
- measurements carried out in three different borehole orientations at each site to provide sufficient data for the complete state of stress to be determined;
- calculation of the in situ stress field by back analysis of monitored deformations using an appropriate numerical stress analysis program;
- optimisation of results making use of the redundancy in number of data.

The theoretical feasibility of the method was proved in the GAP 220 project. Several specific aspects were identified as key issues:

- the development of a suitable drilling guide which could cope with the following:

    - variation in the straightness of the holes;
    - some variation in the diameter of the holes;
    - "wedging" due to some closure of the holes;
    - the impact and vibration action of the drill;
    - packing of the drill chippings.

- development of a suitable borehole deformation measurement device;

- development of the standard back analysis procedure.

The current project, GAP 314, has involved the development of these aspects as described below.

The back analysis system

Back analysis is a technique which has been developing for more than 20 years, and has been made feasible by the availability of computers and numerical stress analysis techniques. The principle of the method is that the in situ stresses and the deformation characteristics of the rock mass interact and thus, when an opening is created, the resulting deformations or stress changes, which are manifestations of this interaction, allow the in situ stresses and rock mass deformation properties to be back-calculated. A direct back analysis procedure/multiple stage has been implemented for the project. A fictitious stress boundary element method program was modified to apply this procedure, and the implementation of the method has been verified.

Calculation of the three dimensional stress tensor

The carrying out of an in situ stress measurement in one borehole set will provide the two dimensional stress tensor in the plane normal to the axis of the borehole set. To determine the complete three dimensional state of stress, measurements in three boreholes sets, with different orientations, at each measurement site are required. The three sets of two dimensional results are then be combined to determine the complete state of stress. A computer program has been prepared to carry out this calculation, and the program has been verified.

Mechanical drilling equipment

The development of the drilling guide proved to be the most challenging aspect of the research project, and was evolutionary. After initial trials in a concrete block had proved to be satisfactory, underground trials with the system were, conversely, completely unsuccessful. The drill jammed in the hole continuously. Trials reverted to the laboratory, with drilling being carried out in a block of norite. Various geomtries of the drilling guide were experimented with, with varying degrees of success.

It was ultimately found that a guide in the form of a tube was successful if a hole in the tube, through which the chips could pass, was present adjacent to the drill bit. Underground trials showed that the drill guide operated satisfactorily. It was also found necessary to develop a suitable guide system to prevent the deviation of the first hole as it passed through fractures or weaker zones.

Measurement of deformations

The deformation instrument developed consists of three pairs of strain-gauged cantilever elements, each pair measuring across a diameter. The cantilevers are pressed out into contact with the rock by means of an hydraulically operated cam action. Hydraulic "jacks" lock the instrument in place during measurements. Laboratory trials of the system showed that the agreement between measured and predicted deformations was very good.

Field trials of the system

Field trials of the complete system have been carried out in a gold mine at a depth of 2050m. The trials have demonstrated that overlapping percussion holes can be drilled very satisfactorily in quartzite at these depths, that the resulting deformations can be measured satisfactorily, and that approximately 6 measurements can be made in one shift. Insufficient numbers of measurements have been made at this stage to evaluate the magnitudes of the stresses. Further measurements are due to be carried out shortly, which will address this deficiency.

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1 INTRODUCTION

Knowledge of the in situ state of stress in a rock mass is essential for the proper planning and design of mine layouts to optimise stability and safety of mining operations. The in situ stresses are essential boundary conditions for all design and analysis methods making use of stress analyses. The use of incorrect boundary conditions will provide invalid design results which could be significant with regard to stability and safety.

The in situ state of stress in a rock mass can be determined by direct measurement, and there are numerous methods of measurement which are currently in use. Although some measurements of in situ stress were carried out prior to 1950, significant research into methods of in situ stress measurement began in the 1950's and has continued to the present time. In South Africa, research in the 1950's and early 1960's led to the development of borehole strain cells for the measurement of both two- and three-dimensional states of stress. This work was completed in mid-1960, and at that stage South African expertise was probably the best available in the world.

It is unfortunate that no significant research in the field of in situ stress measurements has been carried out in South Africa since that time, in spite of the fact that, in the past 30 years, mining has progressed to much greater depths, where stress conditions are even more critical. A consequence of this is that techniques that are appropriate for the very high stress conditions in our deep level mines were not developed.

In 1995 a SIMRAC Research Project (GAP 220) was awarded to Steffen, Robertson and Kirsten entitled "Reliable cost effective technique for in-situ ground stress measurements in deep gold mines". The purpose of this project was to identify a method of in situ stress measurement that would be suitable for deep level mines, and to carry out a conceptual feasibility study of the method. A report on this project was submitted to SIMRAC in July 1995.

The above is background to the present SIMRAC research project entitled "Reliable, practical technique for in situ rock stress measurements in deep gold mines" (GAP 314), which represents the implementation of the findings of Project GAP 220. A copy of the GAP 314 research proposal is contained in Appendix 1 for record purposes. The proposed Primary Output, as indicated in the proposal, is as follows:

- equipment and method of analysis of data for in situ stress measurement in a high stress environment typical of the deep level gold mines.

  This project has been carried out by Steffen, Robertson and Kirsten (SRK). The project team included personnel from CSIR Miningtek owing to their expertise with the carrying out of stress measurements, and their workshop and laboratory facilities. The latter were essential for the critical aspects of development of the mechanical and electronic equipment. SRK was responsible for the concepts, the development of the back analysis system, and the administration of the project.

2 Review of the GAP 220 project

It is considered appropriate to include a brief review of the work carried out under SIMRAC Project GAP 220, since the output from that project formed the basis of the current research. The activities carried out under GAP 220 included the following:

- a review of existing methods of in situ stress measurement as published in the literature;
- identification of those methods which are possibly more appropriate in South African gold mines;
- assessment of the applicablity of these more appropriate methods, and identification of the advantages and disadvantages of each of them;
- definition of the requirements, both theoretical and practical, for in situ stress measurements in deep level mines, and, based on these requirements,
- conceptual development of a reliable and cost-effective method of in situ stress measurement.

2.1 Summary from the review of available methods of in situ stress measurement

Most of the existing methods of in situ stress measurement involve the observation of a change in deformation or stress resulting from a change in the geometry of an opening in the rock, and the subsequent calculation of the field stresses from those measured changes. The most common "opening" in the rock mass is a borehole.

Of the available methods it was considered that only a limited number had potential application to the deep level gold mines:

- borehole overcoring methods, including the CSIR doorstopper strain cell, the CSIR triaxial strain cell, and the CSIRO HI cell;
- large diameter overcoring in a bored raise;
- hydrofracturing;
- back analysis from monitored deformations around excavations.

The practical applicability of each of the above four groups in deep level gold mines was summarised as follows:

Borehole overcoring methods

Borehole overcoring methods rely on the measurement of small changes in strains on the surfaces of the boreholes, or changes in diametral dimensions. The volume of rock involved in each measurement is very small, and the actual measurements made are therefore significantly influenced by the local behaviour of the rock at the measurement location. In fact the measurements can be affected by the grain size in the rock and strain gauges can be of similar size to grains. In addition, the condition of the rock at the measurement location can affect the results considerably. For example, the occurrence of minor micro-cracking beneath a strain gauge or diametral measurement point can affect the magnitude of the strain or deformation reading considerably. This is illustrated very clearly by the results reported by Chandler (1993) - microcracking in "perfect" granite led to large variations in the apparently good quality results obtained (Martin and Christiansson, 1991). In this good quality rock, variations of more than 50% in measured in situ stresses were obtained in a single hole.

Variation in results obtained by different overcoring methods, under laboratory conditions, are presented by Cai et al (1995). These results show a maximum error of about 30%, and a typical error of about 10%, even under the controlled laboratory test conditions. Under high stresses and less perfect rock conditions, large variations in apparently good measurements can be expected. To apply statistical techniques for analysis implies that a large number of results must be obtained. This has a significant disadvantage since overcoring measurements require a diamond drill, special bits, and associated services (water, compressed air and electricity usually) to be on standby for the duration of the measurement programme. An experienced driller and measurement technician are also required for the duration of the programme. Measurements can typically be carried out at the rate of one per shift for triaxials, and about two for doorstoppers. Time is also required after the site work to carry out laboratory core testing and to calculate and interpret the results. In addition to the above, there are many experimental difficulties which can develop, for example, breaking of the core during overcoring; instrumentation problems due to the harsh underground environment; the presence of water in the hole; glue creep and debonding. All of these can result in a failed measurement. It may be concluded that a successful overcoring programme is demanding on time and the provision of facilities.

Overcoring methods measure strain or deformation, and the in situ stress has to be calculated from these strain values. This requires that the deformation properties of the rock be determined, and the assumption that the properties determined are representative.

It may be concluded that, although overcoring methods are theoretically applicable in deep level mines, they have many disadvantages which make them non-ideal for this purpose. In summary, the disadvantages are their variability, the time and facilities required and the experimental unreliability.

Large diameter overcoring in a bored raise

Many raises are excavated by raise boring on the deep level gold mines. There is thus a source of measurement sites for application by this method. The method involves a much larger volume of rock than the borehole overcoring method, and thus overcomes the problem of small scale variability of rock material. In addition, a considerable amount of redundancy of measurement values can be introduced into the method, and this will allow dubious individual values to be eliminated, and statistical optimisation to be applied.

The method is dependent on the location of raise bores, and cannot be carried out at a specific location at which knowledge of in situ stress magnitudes is required.

The method requires physical access to the actual measurement location, and, for safety purposes, this may require special support to be installed in the raise. In addition, special large diameter overcoring equipment is required, with the associated services and personnel. Laboratory testing of the overcores, or of cores taken from the overcore, is required.

As with the borehole overcore methods, the in situ stresses are calculated from the measured strain relief values. The results are therefore dependent on the measured rock stress-strain relationships, and on the validity of the solution used for in situ stress calculation.

In conclusion, the method is likely to provide a more reliable stress measurement result than borehole overcoring. However, it is much less flexible, will be very time consuming and demanding on personnel, and may be difficult to carry out safely.

Hydrofracturing

Hydraulic fracturing is the best known method for determining in situ stress at great depth. This applies to the determination of crustal stresses in vertical boreholes. If the method has been applied in deep level gold mines, no results have been published.

The method requires special equipment, and associated services and personnel, to carry out a measurement. The borehole must be diamond drilled, or inspected with a television camera if percussion drilled. Since packers are inserted in the borehole to seal off the test sections, the straightness and wall quality of the borehole are important. In boreholes in which spalling is occurring, there may be a risk of not being able to insert (or recover) the packers, and it may also not be possible to seal off the borehole satisfactorily. In vertical boreholes in the oil industry in which hydrofracturing is usually carried out, drilling fluids are usually present which help to maintain the integrity of the borehole. Special techniques such as this may be necessary if the method was to be applied in the gold mines.

After hydrofracturing, the borehole has to be inspected again, using a television camera, or a special impression taken of its surface, to determine the orientation of the induced fracture.

Interpretation of hydrofracture records can require expert input if the shut-in pressure is not distinct. Interpretation of test results is not a straight-forward activity, and the experience of the interpreter has some effect on the in situ stress values ultimately determined. Different interpreters may derive somewhat different results from the same set of field data.

It is clear that the application of the hydrofracturing method in deep level gold mines would be very demanding on services. Perhaps the most severe restriction, however, is the requirement that the borehole be drilled in the direction of one of the principal stresses. In general in the gold mines this is not known and is one of the in situ stress parameters to be determined. The HTPF variation of the hydrofracture method (Cornet, 1986), which does not have this requirement, is not seen as a practical alternative owing to the number of boreholes required for a full in situ stress measurement.

It may be concluded that the hydrofracture method is not likely to provide a practical method for application in the deep level gold mines. However, the simplified approach (Vik and Tunbridge, 1986; de Witt, 1992), in which the magnitude of the minimum principal stress only is determined, is considered to have significant merit, since it provides a direct measurement of stress rather than a strain from which the stress is calculated. Such a test could be implemented in percussion boreholes with a very limited requirement for services. Although the test will yield only the magnitude of the minimum principal stress and not its orientation, it will provide an absolute stress magnitude which can be used to "calibrate" the results obtained from other in situ stress measurement methods. For this reason it is considered to be a practical addition to any other method of in situ stress measurement.

Back analysis from monitored deformations around excavations

The use of a back analysis technique, based on measured deformation or stress changes around an excavation, has the advantage of testing a large volume of rock. The larger the excavation involved the larger the volume of rock that will be affected. The benefits of such an approach compared with other methods of in situ stress measurement have been outlined by Wiles and Kaiser (1994a).

From a practical point of view, the calculation of in situ stresses requires linearly elastic rock deformation behaviour. This may be restrictive in very high stress environments, but, if required, it should be possible to engineer the enlargement such that the induced deformations being measured are essentially in the elastic regime.

The application of the method is restricted to the availablity of sites at which excavation enlargements are being made (unless enlargements are made specifically for the purposes of in situ stress measurement). This will have a large interference factor on mining operations. Sophisticated instrumentation is required, with associated special borehole drilling. The reliability and survivability of instrumentation in deep level gold mines is known to be a serious problem, particularly if it is required over a significant period of time.

It may be concluded that back analysis as a method of in situ stress determination has significant applicability in the deep level gold mining environment if suitable opportunities arise. However, it is inflexible in that it would not, under normal circumstances, be practical to implement the method only for the specific purpose of in situ stress measurement, since the interference factor will be significant.

Conclusions regarding the applicability of existing methods of in situ stress measurement

Of the many available methods of in situ stress measurement the majority were rejected immediately as not being practically applicable in deep level gold mines. Only a few of the methods were identified as being possibly applicable, and their advantages and disadvantages have been dealt with above. In summary, conclusions regading the applicability of these few methods were as follows:

- borehole overcoring methods: have been and can be used, but suffer from the disadvantages of poor reliability and considerable requirement for services. Applicability was considered to be poor.
- large diameter overcoring: greater reliability, but is inflexible in terms of location, and is demanding in terms of services. Applicability was considered to be poor.

- hydrofracturing: demanding in terms of services, specialised equipment and expert interpretation, and has a major drawback in that the borehole must be drilled in the direction of one of the in situ principal stresses. Applicability was considered to be very poor. However, applicability of a simplified method, in which only the magnitude of the minimum principal stress is determined, was considered to be significant. This, however, did not on its own satisfy the requirements of the SIMRAC Project.
- back analysis from monitored deformations around excavations: to implement excavation enlargements, with associated instrumentation, specifically for the purposes of in situ stress measurement was not considered to be practically applicable. Where appropriate opportunities arise to use the method, ie, when the excavation is going to be enlarged anyway and instrumentation can be easily installed, it was considered that it could be applicable. General applicability in the deep level gold mines was considered to be poor.

2.2 Requirements for a reliable practical technique for in situ ground stress measurements in deep gold mines

None of the existing methods of in situ stress measurement is considered to have practical applicability in deep gold mines. This is for a variety of reasons which have been outlined above. From experience of involvement in numerous in situ stress measurement programmes in mines and tunnels, using a range of the techniques described above, the following are considered to be the requirements for a reliable, practical technique for in situ stress measurement in deep gold mines:

- the technique must be undemanding on services provided by the mine. It would probably require the provision of compressed air, but should not require water or power. It must have as little impact as possible on mining production and exploration operations, and should require no input of time from these personnel. If possible, it should require no attendance by any mine personnel during the measurement programme;
- the technique should be simple with regard to all aspects - preparation, installation, instrumentation, and economical in terms of time requirements;
- the technique should allow many measurements to be made in a single mining shift. The preference is for a large number of lower accuracy measurements rather than one or a few apparently high accuracy results. This will allow the results to be treated statistically and therefore avoid localised effects;
- the technique should not be sensitive to high stress effects such as spalling and microcracking of the rock;
- the technique should be flexible. It should be possible to implement at very

short notice, require a minimum of preparation, be possible to apply in excavations of limited size, and be non-restrictive in terms of location. It should also be able to be implemented on a stop-start basis, not requiring a long continuous period for a measurement programme, ie it should be able to Table 3. fit in with the availablity of measurement sites, transport, personnel etc;

- the technique should preferably not require the retrieval of rock cores and laboratory testing to determine the deformation properties of the rock or rock mass.

Based on the above requirements, an in situ stress measurement technique which will be practically applicable in the deep gold mines was developed conceptually.

2.3 Conceptual development of a reliable practical method of in situ stress measurement

The method of in situ stress measurement, developed conceptually in SIMRAC Project GAP 220, is as illustrated in Figure 1:

- a borehole-based system, using percussion boreholes drilled using a conventional jackhammer;
- creating the enlargement, or stress/strain change, not by overcoring, but by drilling overlapping holes;
- control of the direction of adjacent holes, and prevention of sideways movement of drill chips, by means of a special drilling guide;
- measurement of deformation changes in the first borehole, as a result of the drilling of the third borehole, by means of a borehole deformation meter;
- sequential deformation change measurements down the length of the borehole, alternately moving the deformation meter and advancing the third borehole, thus obtaining numerous individual results in a single borehole set;
- measurements carried out in three different borehole orientations at each site to provide sufficient data for the complete state of stress to be determined;
- calculation of the in situ stress field by back analysis of monitored deformations using an appropriate numerical stress analysis program. Sufficient data would be obtained to allow the deformation properties to be determined as part of the back analysis procedure;
- optimisation of results making use of the redundancy in number of data.

It is considered that the technique defined by this concept will satisfy all the requirements identified in the sub-section above. Specifically, the use of jackhammers

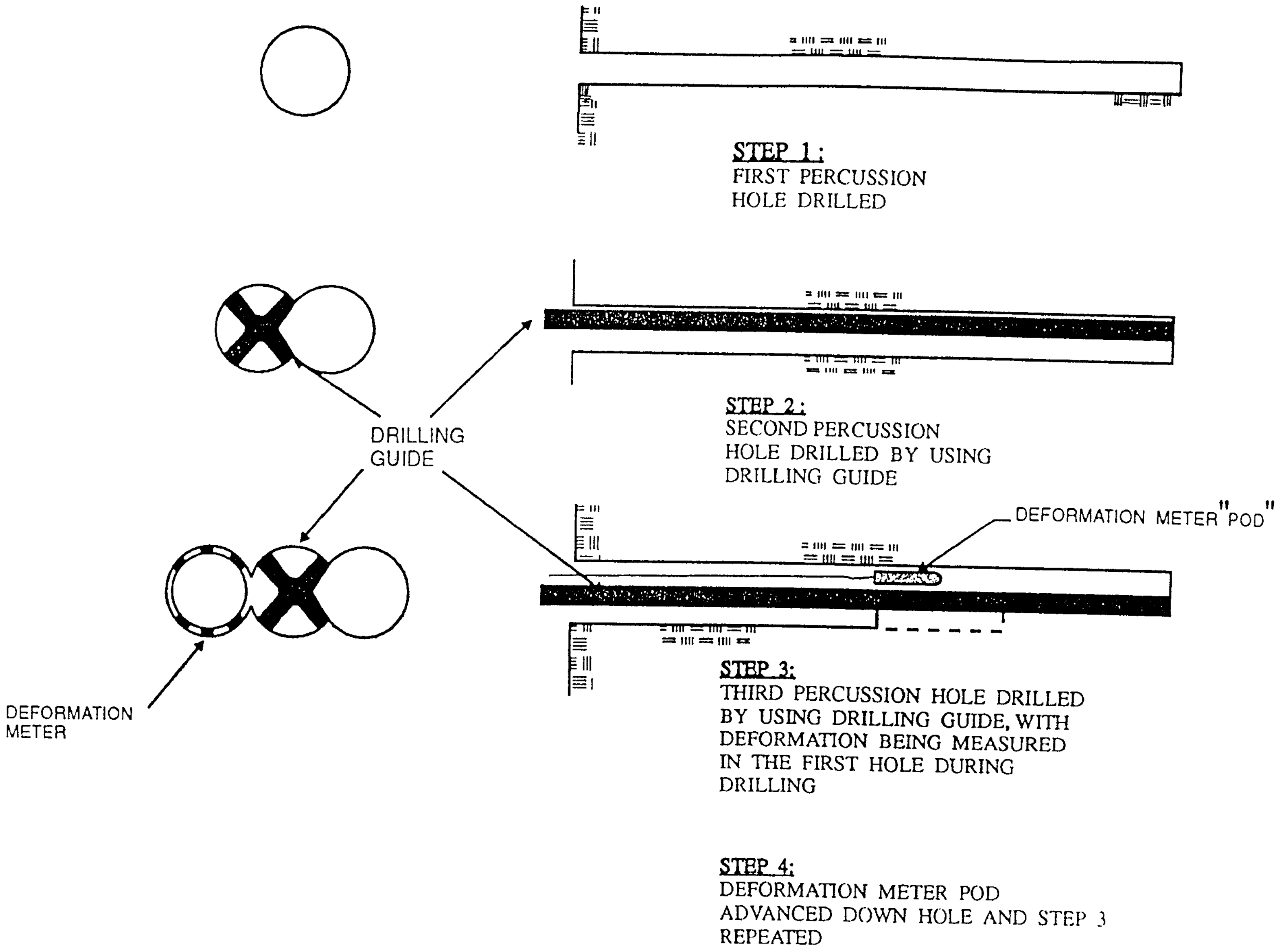


Figure 1: Conceptual method of in situ rock stress measurement

to drill percussion holes means that the operation will be quick and undemanding on the mine. The use of percussion drilling also means that many measurements can be made in one shift.

The use of a reusable deformation meter rather than disposable cells means that instrumentation problems should be reduced.

2.4 Stress analyses of the concept

Stress analyses of the proposed geometry of the method were carried out with the following aims:

- to prove that deformations induced around the first borehole by the drilling of the third borehole would be of sufficient magnitude to be easily measured;
- to determine the effect of failure around the boreholes due to overstress on the deformations;
- to determine the length over which the third borehole must be drilled to influence completely the measurement location in the first borehole.

The results showed that the net closure across the diameter of the first borehole as a result of the drilling of the third borehole is of the order of 0,04 mm. This order of magnitude can easily be measured with the required accuracy for back analysis purposes.

Analyses were carried out allowing rock failure to develop. In this "failure" situation, the deformation across the first borehole is increased by less than 5% compared with the elastic results. This is less than the likely measured variations in deformation owing to local variations in rock behaviour and experimental factors. The implication of the above is that the method is not sensitive to the occurrence of rock failure due to overstress in high field stress conditions. The extent of rock failure can also be restricted by appropriate choices of orientations of the borehole sets, that is, so that their axes are not normal to the probable orientation of the maximum principal stress.

Three dimensional analyses showed that, per measurement, the minimum length of percussion drilling required in the third borehole is about 200 mm. The implication of this is that about five measurements can be made per metre length of borehole.

The requirement for a large number of measurements in a single shift is therefore met on the basis of the theoretical analyses.

2.5 Conclusions from SIMRAC Project GAP 220

The method of in situ stress measurement, developed in concept, and based on the percussion drilling of a set of three overlapping boreholes, with back analysis of the in situ stresses from measured borehole deformations, overcomes the disadvantages of most of the available methods of in situ stress measurement. It should therefore be reliable, and is aimed at being specifically applicable under the conditions which occur in deep mines. It was recommended that the physical development of the system should proceed.

3 Development of a reliable practical technique for in situ rock stress measurements in deep gold mines

From the GAP 220 Project several specific aspects were identified as needing development to ensure that the method can be applied practically. These were:

- the development of a suitable drilling guide which can cope with the following:

  - variation in the straightness of the holes;
  - some variation in the diameter of the holes;
  - "wedging" due to some closure of the holes;
  - the impact and vibration action of the drill;
  - packing of the drill chippings.

  This was considered to be the most significant physical challenge;

- development of a suitable borehole deformation measurement device;

- development of the standard back analysis procedure;

Project GAP 314 involves the above developments and the laboratory and in situ testing of the system.

3.1 The back analysis system

Back analysis is a technique which has been developing for more than 20 years, and has been made feasible by the availability of computers and numerical stress analysis techniques. The principle of the method is that the in situ stresses and the deformation characteristics of the rock mass interact and thus, when an opening is created, the resulting defomations or stress changes, which are manifestations of this interaction, allow the in situ stresses and rock mass deformation properties to be back-calculated.

3.1.1 Theory of the back analysis method

When an excavation is made, or an excavation enlarged, in a rock mass under a stress field, displacements of the surfaces of the excavation occur in response to the action of the stresses. The relationship between the displacements and the stresses can be represented by a simple equation:

$$\{u\} = [F]\{\sigma\}$$

where $\{u\}$ is a matrix of displacements

$\{\sigma\}$ is a matrix of the in situ stresses

$[F]$ is the flexibility matrix, which is a function of the deformation properties of the rock, and the geometry and dimensions of the excavation

If the displacements can be measured, then the in situ stresses can be calculated (back analysed) by rearranging the equation as follows:

$$\{\sigma\} = ([F]^T[F])^{-1}[F]^T\{u_m\}$$

where $\{u_m\}$ is the matrix of the measured displacements

This approach is well suited to the application of numerical stress analysis methods, which have developed along with the development of digital computers, and are now widely used.

The method of implementation of the back analysis approach to the proposed in situ stress measurement technique is given in Appendix 2. The computer programs for back analysis are given in Appendix 3.

3.1.2 Theoretical verification of the back analysis method

Diametral convergences for the first borehole in the borehole set were **calculated** for two in situ stress fields:

- σ_{xx} = -37,5 MPa; σ_{yy} = -75,0 MPa; σ_{xy} = 0,0 MPa
- σ_{xx} = -37,5 MPa; σ_{yy} = -75,0 MPa; σ_{xy} = 10,0 MPa

A modulus of elasticity of 70 GPa and a Poisson's ratio of 0,2 were used in these calculations.

These calculated convergences were then assumed to be the **measured** convergences and the in situ stresses back analysed using the procedure described above. The results of the back analysis are given in Table 1. It can be seen that the procedure is exact within the limits of numerical rounding off errors.

Table 1. Verification of the back analysis program

	Correct	Calculated	Correct	Calculated
σ_{xx}	-37,5	-37,509	-37,5	-37,592
σ_{yy}	-75	-74,999	-75,0	-74,999
σ_{xy}	0	-0,001	-10,0	-10,001

The back analysis procedure requires the modulus of elasticity E and Poisson's ratio of the rock to be determined. These could be determined from laboratory tests. However, it is probable that the stress magnitudes determined from the measurements will be moderated so that the vertical stress corresponds with the overburden stress. In this case the back analysis can be carried out using assumed values for the deformation parameters. This is completely valid for the modulus of elasticity since stress magnitudes are directly proportion to the value of E, but is not the case for Poisson's ratio. Therefore, back analysis calculations were carried out for a range of Poisson's ratio values to determine the sensitivity of the back analysed stresses to incorrect assumptions (the correct value of Poisson's ratio in this case is 0,2). The results of these calculations are given in Table 2 and show that an incorrect assumption is unlikely to lead to an error exceeding approximately 10%. The values in brackets in the table are the percentage errors.

Table 2. Effect of the assumption of incorrect values of Poisson's ratio

	Exact	**0,1**	**0,15**	**0,2**	**0,25**	**0,3**	**0,35**	**0,4**
σ_{xx}	-37,5	-34,43 (8,2)	-35,84 (4,4)	-37,51 (0,0)	-39,36 (5,0)	-41,66 (11,1)	-44,47 (18,6)	-47,89 (27,7)
σ_{yy}	-75,0	-73,06 (1,9)	-73,83 (1,6)	-75,00 (0,0)	-76,60 (2,1)	-78,68 (4,9)	-81,32 (8,4)	-84,60 (12,8)
σ_{xy}	0	-0,00 (0,0)	-0,00 (0,0)	-0,00 (0,0)	-0,00 (0,0)	-0,00 (0,0)	-0,00 (0,0)	-0,00 (0,0)

3.1.3 Three dimensional stress tensor

As shown in the above description of the implementation of the back analysis method, the two dimensional stress tensor in the plane normal to the axis of the borehole will be determined. To determine the complete three dimensional state of stress, measurements in three sets of boreholes with different orientations at each measurement site will be required. The three sets of two dimensional results will then be combined to determine the complete state of stress. The computer program prepared for this is listed in Appendix 4, and is based on the publications of Panek (1966) and Smith (1982).

The inputs required by the program are the bearing and inclination of each borehole set, and the corresponding two dimensional stress tensor. There is redundancy in the amount of data and the program performs a least squares fit for the three dimensional stress tensor. The outputs from the program are:

- the six stress tensor components and their standard deviations;
- statistical parameters of the least squares fit;
- the three principal stresses, their bearing and inclination angles, and their standard deviations.

The program has been tested using theoretical data and has been shown to be correct. In this verification process, a three dimensional stress tensor was assumed. The two dimensional stress tensors for each of three borehole orientations (corresponding with the orientations of the stress measurement borehole sets) were calculated using elastic theory. These calculated two dimensional stresses and the borehole bearings and inclinations then constituted the input data for the program. A comparison of the assumed and calculated three dimensional stress tensors for two examples is summarised in Table 3.

Table 3. Verification of the three dimensional stress tensor program

		Example 1			Example 2	
	Assumed Stress Tensor	Calculated Stress Tensor	Standard Deviation	Assumed Stress Tensor	Calculated Stress Tensor	Standard Deviation
σ_x	-50	-52,2	10,7	-50	-50,0	0,0
σ_y	-45	-46,3	6,5	-25	-25,0	0,0
σ_z	-60	-59,9	0,2	-25	-25,0	0,0
σ_{xy}	-12,5	-14,2	8,6	-2	-2,0	0,0
σ_{yz}	-10,1	-10,2	0,5	-3	-3,0	0,0
σ_{zx}	-5,5	-5,5	0,6	-4	-4,0	0,0

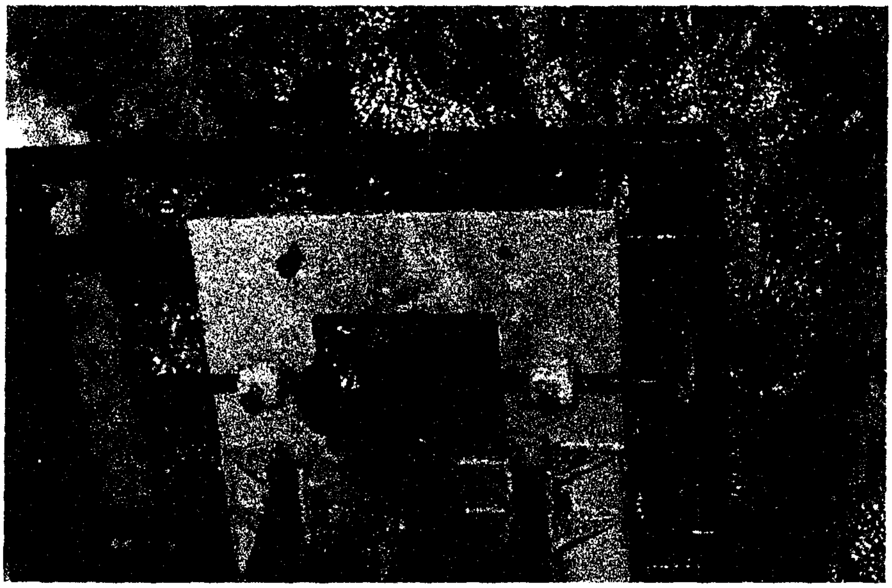
In the results in Table 3 it can be seen that that there is considerably more variation in Example 1 than in Example 2. This is due to the assumption, in Example 1, of borehole set orientations close to each other. The bearing/inclination assumptions were 60/3, 45/-5, 56/7. These similarly orientated holes do not produce data which is well conditioned. Nevertheless, the output results are acceptable. For Example 2 the bearing/inclination inputs were 100/-2, 25/-3 and 156/-10. These more widely distributed orientations produced almost exact results with negligible deviations.

3.2 Mechanical drilling equipment

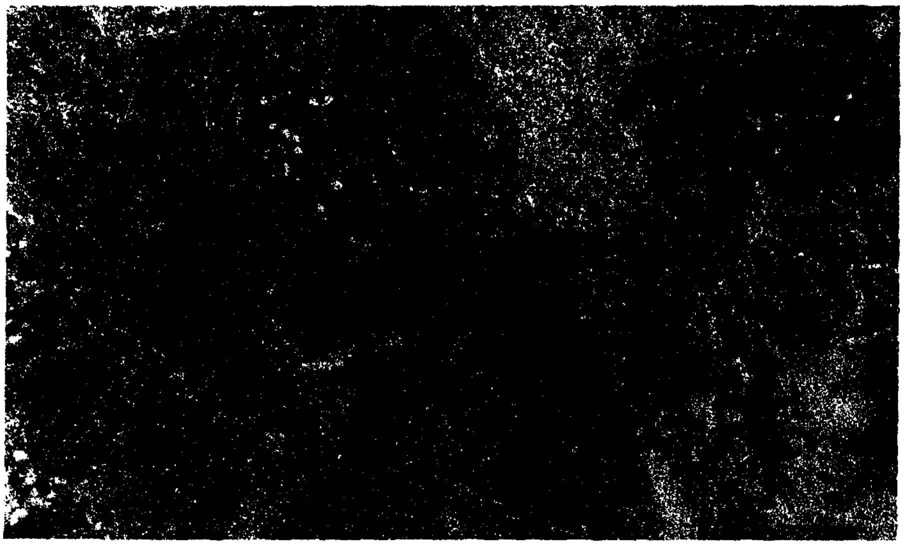
Drilling trials were initially carried out in a cast concrete block. After initial trials with the drill hand held, a drill cradle structure for the percussion drill was constructed, which allowed an air leg to be used. This considerably improved penetration rates and drilling accuracy. This structure is shown in Figure 2.

The development of the drilling guide, as expected, proved to be the most challenging aspect of the research project. After initial trials in the concrete block had proved to be satisfactory, underground trials with the system at Hartebeestfontein Gold Mine were, conversely, completely unsuccessful. It was found that the drill jammed in the hole continuously, and hole lengths greater than 1m were not possible to achieve. Trials therefore reverted to the laboratory, with drilling being carried out in a block of

PERCUSSION DRILLING SETUP



DETAIL OF DRILL AND GUIDE



APPEARANCE OF
THREE BOREHOLE SET

FIGURE 2: DRILL MOUNTING CRADLE AND HOLES PRODUCED

norite. Various geomtries of the drilling guide were experimented with, with varying degrees of success. It was also found that the stability of the system is critically dependent on the extent of overlap of the adjacent holes, and that the ideal overlap might be different for different sites.

It was ultimately found that a guide in the form of a tube was successful if a hole, through which the chips could pass, was present adjacent to the drill bit. Underground trials were carried out with this system at Oryx Mine. These showed that the drill guide operated satisfactorily, but an additional problem wa experienced. It was found that the first percussion hole deviated when it crossed a weaker zone or stress induced fracture. The resulting lack of straightness of the borehole prevented the drilling guide from passing down this first hole. It was therefore necessary to develop a suitable guide system to prevent the deviation of the first hole. The overall system proved to be very successful, and high quality holes were achieved, as shown in Figure 2.

3.3 Measurement of deformations

The deformation measurement system envisaged at proposal stage involved the scanning of the surface of the first borehole using a laser scanner. This scanning system was chosen since it would allow the deformation of a large number of points around the surface of the borehole to be measured. This availability of redundant data would provide security for the measurement - if a value at the exact required location is clearly invalid (such as due to spall fragments, joints, fractures, porous conditions etc) measurements at alternative adjacent locations would be available. A considerable amount of time and effort was spent on the development of this equipment. However, the required stability of the system could not be achieved, and it was necessary to develop a simpler, more conventional instrument for measurement of the borehole deformations.

The deformation instrument developed consists of three pairs of strain-gauged cantilever elements, each pair measuring across a diameter. The cantilevers are pressed out into contact with the rock by means of an hydraulically operated cam action. Hydraulic "jacks" lock the instrument in place during measurements. The instrument is shown in Figure 3.

3.4 Laboratory trials of the systems

Before the stress measurement system was subjected to underground conditions, its performance was tested in the laboratory under controlled conditions.

The system was tested in large blocks under known applied stress conditions.

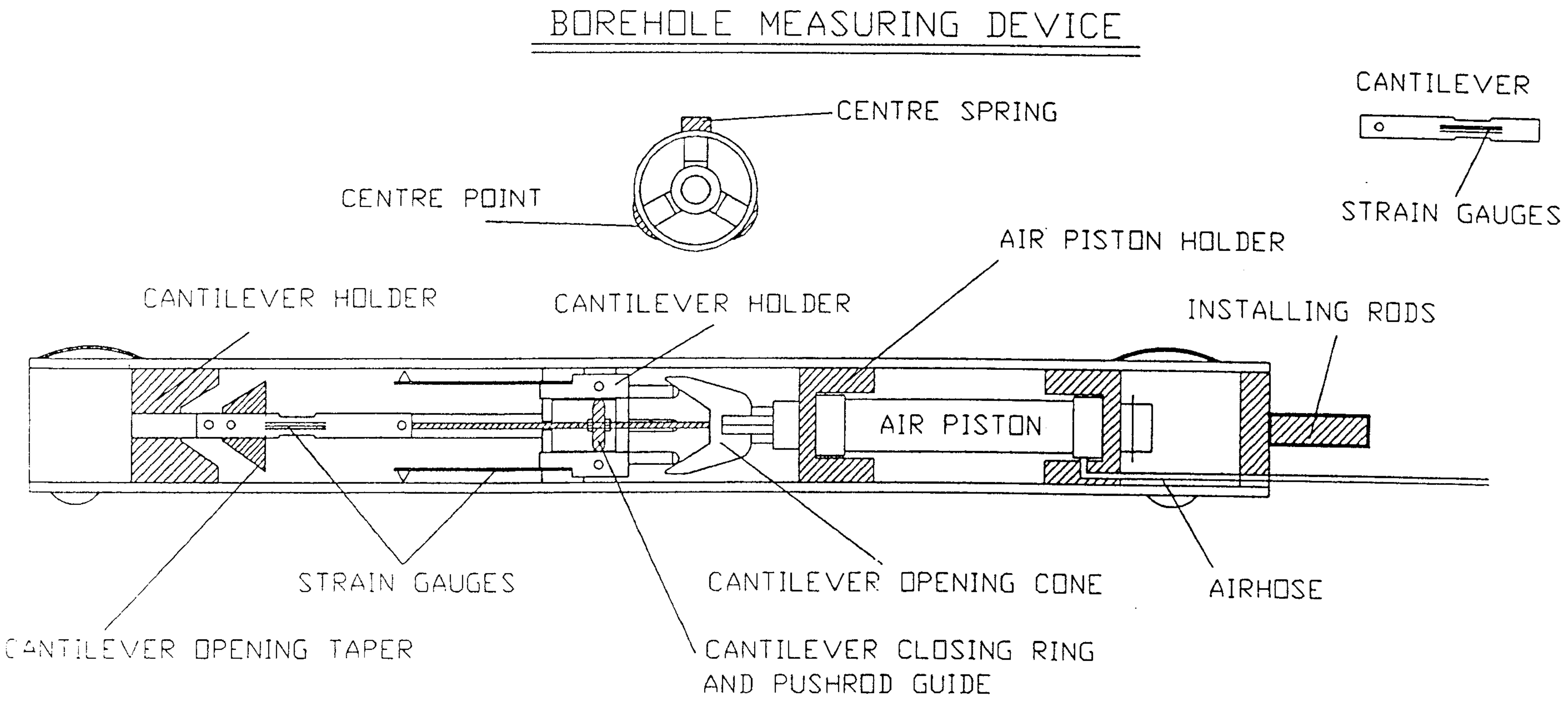


Figure 3: Layout of deformation meter

Boreholes were drilled into the blocks which were loaded in a standard laboratory compression testing machine. The initial plan was to use cast cement blocks for the verification purpose, and this plan was implemented. However, the cement blocks did not have the required strength, and developed cracks in the initial tests. In further tests, loads were therefore limited, and the corresponding deformations induced were small. This was considered to be unsatisfactory, and an aluminium block was used instead.

When the back analysis was applied to the measured deformations, it was found that the agreement between the aplied stresses and the back analysed stresses was very poor. This was very disappointing, and an explanation was sought. The explanation proved to be the small dimensions of the aluminium block, which resulted in the assumption of infinite boundaries (appropriate in the real application of the back analysis system) being incorrect. The holes in the block were substantially within the influence of the boundaries, as shown in Figure 4.

When the actual boundaries of the block were taken into account in the analyses, the agreement between measured and calculated borehole deformations proved to be excellent, as shown in Figure 5. The percentage values given in the tabular portion of this figure are the errors as a percentage of the average measured values. It can be seen that the agreement is excellent when the correct boundary conditions are taken into account, but the error is about 30% when infinite boundary conditions are assumed. It should be noted that little account should be taken of measurement 3 values since these were was not diametral change values.

It is concluded that the adequacy of the measurement system has been proved.

3.5 Field trials of the in situ stress measurement system

Initial drilling trials were carried out at Hartebeestfontein Mine as indicated above. This site was subsequently unavailable, and an alternative site at Oryx Mine was offered.

Figure 6 shows the location and geometry of the site. This location is 2050m below surface. The rock type in which the drilling was carried out is quartzite with a rock material strength of between 45 MPa and 145 MPa based on the results of uniaxial compressive strength tests. The borehole set was drilled at an upward inclined angle of 6° on a bearing of N55°E. Three measurements were carried out in the borehole set at depths down the hole of 1,45m, 2,9m and 3,2m during a single shift. Based on the experience gained from these activities, it has been estimated that a full set of measurements can be carried out in one borehole set in two mining shifts. This includes the setup of the drill, including the mounting of the drill support frame, the drilling of the overlapping holes, and the "leap frog" process of drilling and

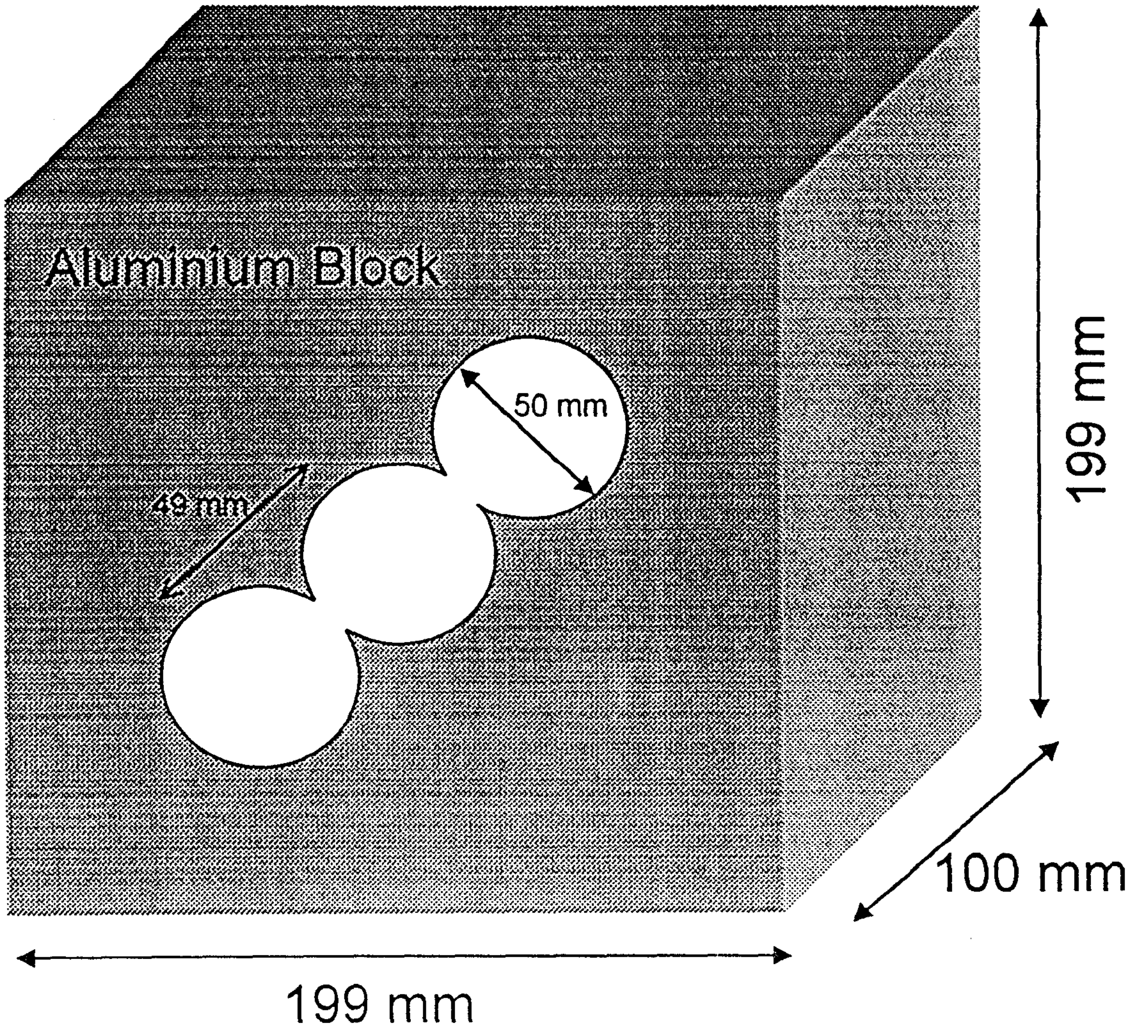
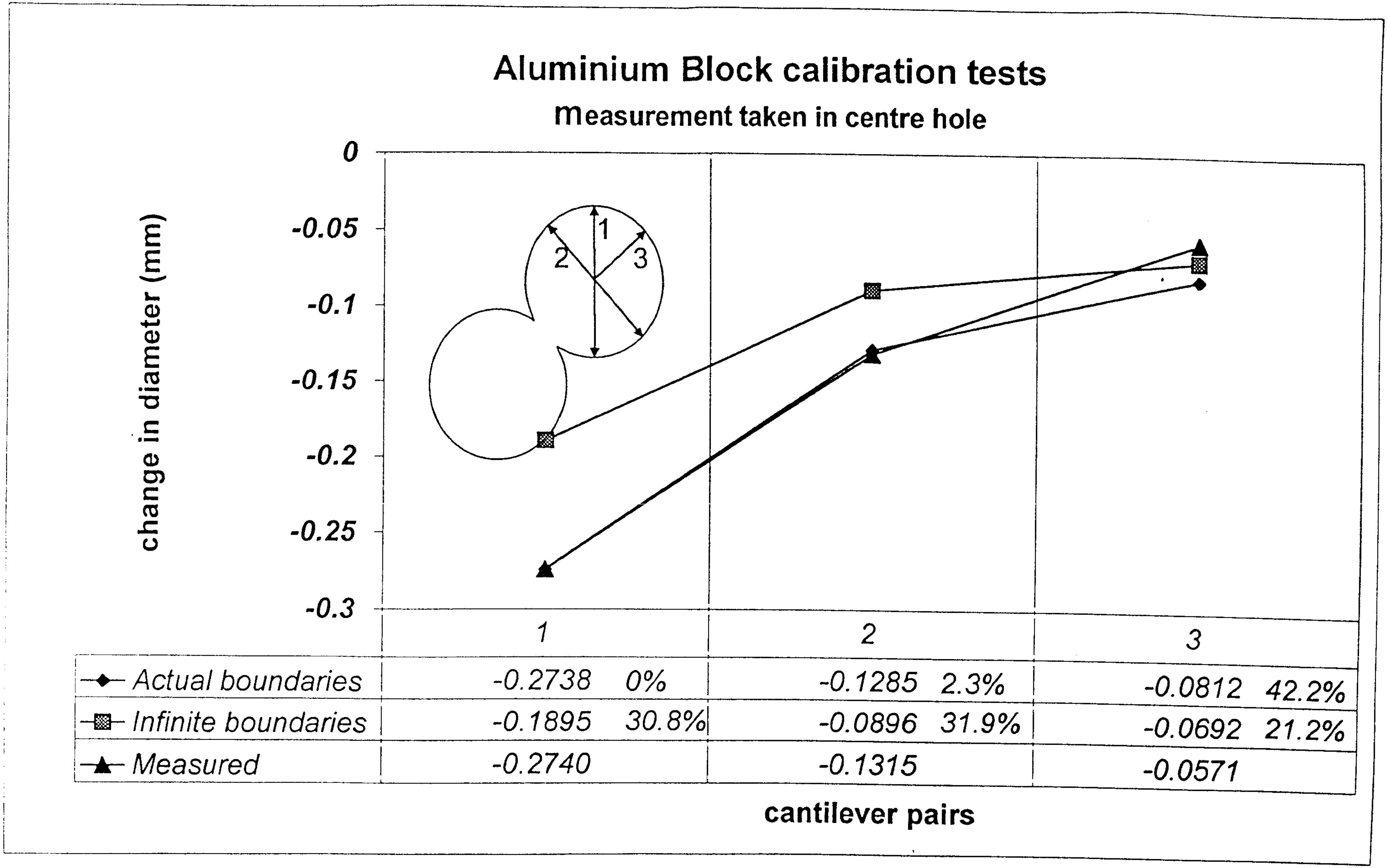


Figure 4: Laboratory test block



	1	2	3
◆ Actual boundaries	-0.2738 0%	-0.1285 2.3%	-0.0812 42.2%
▨ Infinite boundaries	-0.1895 30.8%	-0.0896 31.9%	-0.0692 21.2%
▲ Measured	-0.2740	-0.1315	-0.0571

Figure 5: Laboratory tests - measured and predicted results

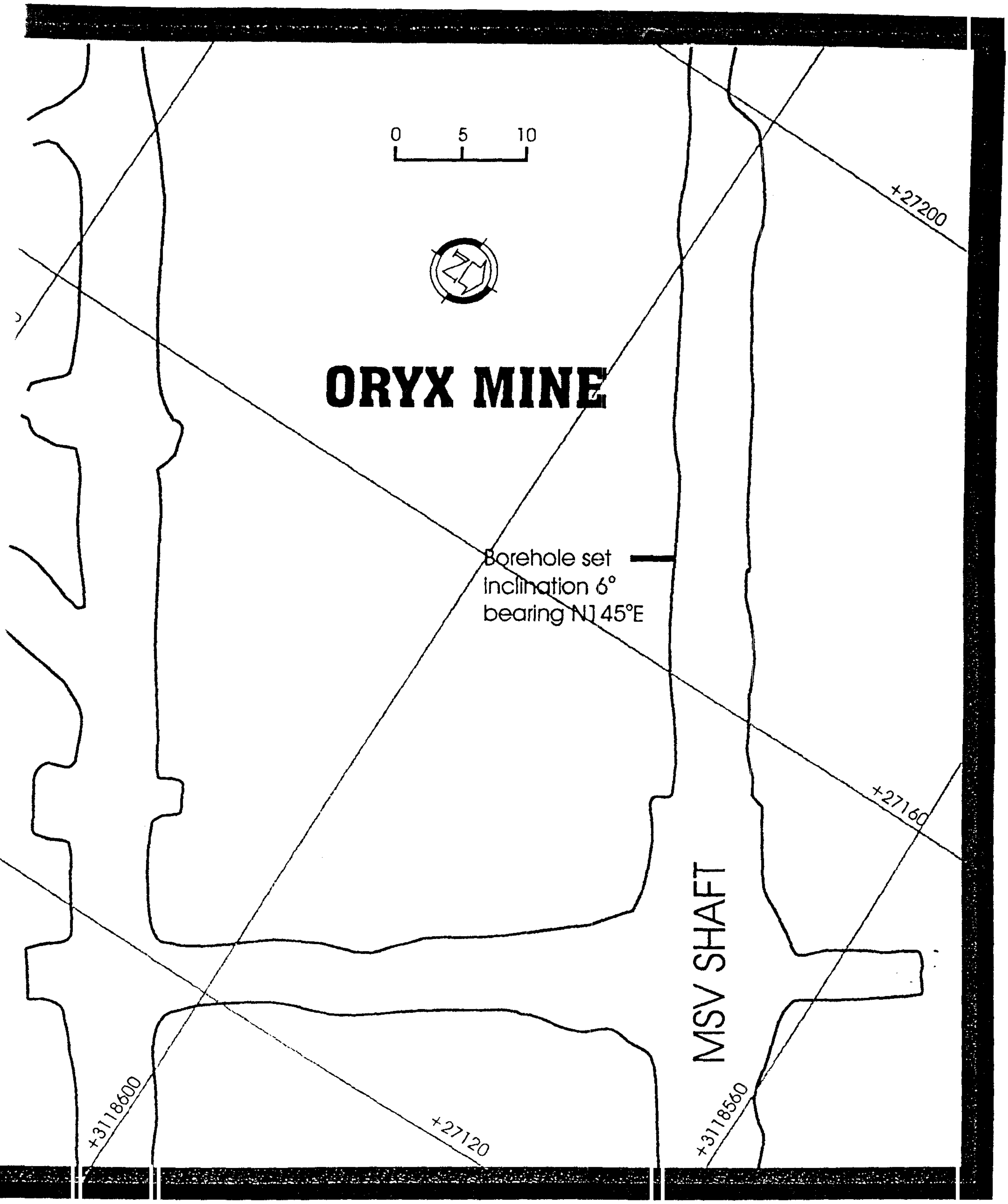


Figure 6: Location of stress measurement

measurement in the third hole.

Insufficient numbers of measurements have been made at this stage to evaluate the magnitudes of the stresses. Further measurements are due to be carried out shortly, which will involve three borehole sets and the use of a data logger to record deformation changes continuously as the drilling progresses. These additional measurements should address the deficiency in numbers of data.

4 Conclusions and recommendations

The following conclusions can be drawn from the research which has been carried out:

- a method of in situ stress measurement has been developed which has been shown to be practical for application in deep gold mines;
- a drilling guide which allows the satisfactory drilling of overlapping percussion boreholes was developed;
- a borehole deformation measurement meter was developed, and its adequacy proved in laboratory tests;
- a back analysis system, based on a boundary element numerical stress analysis program, was developed, and its application proved;
- based on the overall simplicity of the approach, and the fact that all the components of the system have been developed satisfactorily, the method should be reliable.
- application of the approach has shown that a full stress measurement in one borehole set can be carried out in two mining shifts, and that about 6 deformation measurements in one borehole set can be carried out in a single shift.

It is recommended that as much use as possible should now be made with the new method of stress measurement, to gain experience with the method, and to identify any further improvements that could be made to the equipment and the process.

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APPENDIX 1

SIMRAC RESEARCH PROPOSAL

RELIABLE, PRACTICAL TECHNIQUE FOR IN-SITU ROCK STRESS MEASUREMENTS IN DEEP GOLD MINES

DEPARTMENT OF MINERAL AND ENERGY AFFAIRS
PROPOSAL FOR A PROJECT TO BE FUNDED IN TERMS OF THE MINERALS ACT
-CONFIDENTIAL-

REVISED

DMEA REFERENCE NUMBER

GAP 314

(FOR OFFICE USE ONLY)

1. PROJECT SUMMARY

PROJECT TITLE: RELIABLE PRACTICAL TECHNIQUE FOR IN SITU ROCK STRESS MEASUREMENTS IN DEEP GOLD MINES.

PROJECT LEADER: TR STACEY

ORGANISATION: STEFFEN, ROBERTSON AND KIRSTEN
ADDRESS: PO BOX 55291, NORTHLANDS, 2116

TEL: (011) 441 1143 TELEFAX: (011) 880 8086 TELEX:

PRIMARY OUTPUT[1]: Equipment and method of analysis of data for in situ stress measurement in a high stress environment typical of the deep level gold mines.

HOW USED[2]?: The equipment and method could be used directly in the mines.

BY WHOM[3]? Rock mechanics practitioners and technicians.

CRITERIA FOR USE[4]? Some understanding of rock mechanics and use of electronic instruments.

POTENTIAL IMPACT[5]? Reliable field stress data for input to design analyses, making for better designed and safer working conditions.

FUNDING REQUIREMENTS (R000s)	YEAR 1	YEAR 2	YEAR 3
TOTAL PROJECT COST	383	284	
TOTAL SUPPORT REQUESTED FROM SIMRAC	383	284	

DURATION (YY/MM) 1996 TO 1997

SIMRAC SUB-COMMITTEE:

AU\PT	X	COAL		OTHER		GENERIC	

2. PROJECT DETAILS

2.1 Primary Output[1]

A prototype set of equipment and a report detailing the insitu rock stress measurement method and the method of analyses the results.

2.2 Other Outputs (deliverables)[6]

None

2.3 Enabling Outputs[7]

NO.	ENABLING OUTPUT	MILE-STONE DATE	MAN DAYS
1	Development of electronic measuring equipment	Month 6	35
2	Development of mechanical equipment	Month 6	25
3	Development of method of analysis of data	Month 6	63
4	Laboratory testing of electronic measuring equipment	Month 9	10
5	Laboratory testing of mechanical equipment	Month 9	15
6	Laboratory trials of system	Month 12	20
7	Field trials in underground mine	Month 15	30
8	Modifications as appropriate to equipment	Month 17	14
9	Further field trials	Month 20	40
10	Preparation of report and instruction manual	Month 24	35

2.4 Methodology[8]

NO. OF ENABLING OUTPUT	STEP NO.	METHODOLOGY TO BE USED TO ACCOMPLISH THE ENABLING OUTPUT (INDICATE STEPS/ ACTIVITIES)
1	i	Define measurement requirement, accuracy and identify suitable instruments
	ii	Design measurement `cell'
	iii	Manufacture prototype `cell'
2	i	Define mechanical requirements (mainly re drilling)
	ii	Design mechanical components
	iii	Manufacture prototype mechanical components
3	i	Develop theory for analysis of results
	ii	Write or modify computer program
	iii	Test method on theoretical results
4	i	Test measurement `cell' in sample rock in laboratory press
	ii	Modify prototype if necessary
5	i	Test operation of mechanical components in the laboratory or externally under easily controlled conditions
	ii	Modify prototype mechanical equipment if necessary
6	i	Carry out trial measurements with the system in the laboratory (stressed cement block)
	ii	Analysis results and compare with imposed stresses
7	i	Carry out trials in a `convenient' underground mine where access is easy and facilities convenient
	ii	Analyses results and compare with `reasonability'
8	i	Modify equipment as appropriate
9	i	Carry out trials in a deep level mine
	ii	Analyze results and compare with reasonability
	iii	Make modifications if appropriate
10	i	Prepare report and instruction manual

Key facilities and Procedures to be used in the Project

1 Input from the existing SIMRAC project GAP 220
2 Workshop and laboratory facilities, and experience, of CSIR EMATEK
3 Computer and analytical skills from Steffen, Robertson and Kirsten
4 Experience of having carried out a large number of insitu stress measurements using several different techniques.
5 The project will be carried ou as a joint Steffen, Robertson and Kirsten/ CSIR EMATEK project

3. FINANCIAL DETAILS[9]

3.1 Financial Summary

	R000s		
	YEAR 1	YEAR 2	YEAR 3
Project staff costs (from 3.2)	262	225	
Operating costs (from 3.3)	19	24	
Capital & plant costs (from 3.4)	55		
Sub-contracted work (from 3.5)			
Value added tax*	47	35	
Total cost of project	383	284	
Less funding from other sources (from 3.6)			
Support requested from SIMRAC	383	284	

*Only for VAT registered concerns

3.2 Project Staff Costs

	MAN DAYS		
NAME AND DESIGNATION	YEAR 1	YEAR 2	YEAR 3
T R Stacey Project Manager	15	25	
SJ Coetzer, Rock Mechanics Technologist	35	29	
Mining/ Numerical analysis Engineer	60	15	
Rock Mechanics Technician	35	30	
Electronics Specialist	13		
Tracer/ Draftsperson	10	20	
TOTAL (R000s)	262	225	

3.3 Operating Costs (Running)

	COST (R000s)		
ACTIVITY/ EQUIPMENT (Items above R10 000)	YEAR 1	YEAR 2	YEAR 3
Travelling	4	9	
Photography	1	2	
Photocopies, documentation etc	1	2	
Cement test blocks	7	2	
Other miscellaneous items	6	9	
TOTAL	19	24	

3.4 Capital and Plant Costs[10]

	COST (R000s)		
(i) ITEMS TO BE PURCHASED OR DEPRECIATED FOR MORE THAN R10 000 PER ITEM	YEAR 1	YEAR 2	YEAR 3
Displacement measuring transducer	18		
Data capture system	19		
Drilling equipment	10		
Other miscellaneous items	8		
TOTAL	55		

(ii) ITEMS TO BE MANUFACTURED WITH ASSEMBLED COST OF MORE THAN R10000 INCLUDING MATERIAL AND LABOUR	COST (R000s)		
	YEAR 1	YEAR 2	YEAR 3
Other miscellaneous items			
TOTAL			
TOTAL (i) and (ii)			

3.5 Sub-contracted Work

SUB-CONTRACTOR	ACTIVITY	COST (R000s)		
		YEAR 1	YEAR 2	YEAR 3
TOTAL				

3.6 Other Funding

ORGANISATION	NATURE OF SUPPORT/ COMMITMENT	AMOUNT (R000s)

4. MOTIVATION

(Provide a clear and quantified motivation or justification for the proposal, as well as the main conclusions of a literature survey and the findings of related local and international research. The motivation should include a synthesis of previous work in the project area, both locally and overseas, why the project is proposed, what the primary output will achieve and a cost benefit analysis, if applicable. Use continuation pages where necessary but in most cases it should be possible to clearly present the key and arguments in the space provided).

IN SITU STRESS MEASUREMENTS

Reliable values of the in situ stress are essential inputs to be able to obtain reliable output results from numerical design and layout analyses such as MINSIM-D and FLAC. The reliability of these methods of design is very important with regard to the safety of the underground operations.

A SIMRAC research project is currently in progress with the aim of identifying, conceptually, a reliable and cost effective method of in situ rock stress measurement applicable to the high stress conditions in deep level mines. The duration of this project is only for the 1995 year. Results obtained to date are believed to be very encouraging, and it is proposed that the approach should be investigated further to take the method from the conceptual stage to the practical implementation stage.

The proposed method is planned to have the following features:

- it will make use of percussion drilled boreholes. This will have the following benefits as far as mining operation are concerned:
    - it will require only compressed air, which is usually available in most mining areas
    - measurements, and their preparation, can be carried out quickly and without any significant interruption to production activities, and even without significant assistance from the mine itself
    - the costs of preparation, and the skills required for preparation, will be low
- multiple measurements can be made in a single hole, the aim being to obtain about 10 results in one shift.
- the large number of results obtained will allow data to be treated statistically, thus improving confidence in their reliability.
- reduction of results will be computer based, making use of numerical methods and techniques which were not available when previous in situ stress measurement methods were developed, but which are now commonly available.

5. CURRICULA VITAE OF PROJECT LEADER AND RESEARCH STAFF

5.1 Summary Information

Project leader

NAME & INITIALS: T R Stacey
AGE: 51
QUALIFICATIONS (eg. degree/ diploma, issuing institution and date):
See attached CV

SPECIAL AWARDS:

Principal Project Team Members

NAME & INITIALS: S J Coetzer
AGE: 47
QUALIFICATIONS (eg. degree/ diploma, issuing institution and date):
See attached CV

SPECIAL AWARDS:

NAME & INITIALS:
AGE:
QUALIFICATIONS (eg. degree/ diploma, issuing institution and date):

SPECIAL AWARDS:

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AGE:
QUALIFICATIONS (eg. degree/ diploma, issuing institution and date):

SPECIAL AWARDS:

NAME & INITIALS:
AGE:
QUALIFICATIONS (eg. degree/ diploma, issuing institution and date):

SPECIAL AWARDS:

Use a continuation sheet if necessary

5.2 Relevant Experience and Publications (one page for each individual listed in 5.1)

NAME: TR Stacey

Relevant Experience:

1 Hands on experience with CSIR doorstopper and triaxial stain cells in-situ stress measurements.

2 Developed computer programme to calculate 3D state of stress from 3 doorstopper measurements.

3 Involvement in stress measurement programmes, in a supervisory and evaluation position, with doorstopper, CSIRO HI cell, and borehole slotter methods.

4 Involved, at an expert consultant level, with hydrofracture stress measurements and large diameter overcore measurements (in a bored tunnel).

5 Familiarity with literature on all the above methods, as well as other lesser used methods such as borehole deepening, back analysis from measured deformations, flat jacks, and others.

6 Project leader on SIMRAC project GAP 220 `Reliable cost effective technique for insitu ground stress measurements in deep level gold mines'.

Relevant Publications:

1 Slotter stress measurements at Palabora Mine, Interfels News, No. 7, February 1993, pp 15 - 16.

2 Measurement of in situ stresses at Palabora Mine, Restricted Reports 179447/2 and 179447/5, Steffen, Robertson and Kirsten.

5.2 Relevant Experience and Publications (one page for each individual listed in 5.1)

NAME: SJ COETZER

Relevant Experience:

1 Hands on experience with CSIR doorstopper and triaxial stain cells, CSIRO H1 cell and borehole slotter, and in situ stress measurements.

2 Involvement in the development and upgrading of computer programs to calculate 3D state of stress from stress measurement projects.

3 Involvement in stress measurement projects and evaluation of results with doorstopper, CSIRO H1 cell, and borehole slotter methods.

4 Involved with hydrofracture stress measurement projects and evaluation of results.

5 Familiar with literature on all the above methods.

6 Project leader CSIR step projects and improvement of existing stress meaasurement techniques.

Relevant Publications:

1 Numerous stress measurement project reports for various clients.

2 Several presentations/ lectures on stress measurement at universities and various society meetings.

3 SJ COETZER Instruction Manual for the use of the CSIR Triaxial Rock Stress Measuring Equipment. CSIR Report EMA-H 8903, April 1989.

4 SJ COETZER Instruction Manual for the use of CSIR's Rock Stress Calculation Programme. CSIR Manual EMA-H 8905, September 1989.

5 SJ COETZER Rock stress and Mining: A Case History. CSIR Report and D Spencer EMAP-R-93001, January 1993.

6 SJ COETZER Rock Stress and Mining: A Case History. Proc. D Spencer and Symp. Rustenburg: Rock Engineering Problems Related to Hard W De Maar Rock Mining at Shallow Intermediates Depth, March 1993.

7 SJ COETZER Rock Stress and Mining: A Case History. CSIR report EMAP-R-93001, January 1993.

6. DECLARATION BY THE PROPOSING ORGANISATION

I, the undersigned, being duly authorized to sign this proposal, herewith declare that:

- The information given in this proposal is true and correct in every particular.
- This Organisation has the basic expertise and facilities required for satisfactory completion of the project and will adhere to the program of activities as set out in this proposal.
- The costs quoted are in accordance with the normal practice of this Organisation and can be substantiated by audit.

Signed on this 3rd day of August 1995 for

and on behalf of Steffen, Robertson and Kirsten

SIGNATURE

NAME T R STACEY

DESIGNATION DIRECTOR

:\PROJ\202778\stacey.1

APPENDIX 2

IMPLEMENTATION OF THE BACK ANALYSIS METHOD

IMPLEMENTATION OF THE BACK ANALYSIS METHOD

The direct back analysis procedure/multiple stage has been implemented for the project. In order to calculate the initial stress (in situ stress)

$$\{\sigma\} = [\sigma_{xx}, \sigma_{yy}, \sigma_{xy}]^T$$

from measurements taken during the ith excavation stage, the essential task is the computation of the flexibility matrix [F] for the ith excavation stage. The fictitious stress boundary element method program, published by Crouch and Starfield (1983), has been modified to allow the flexibility matrix to be calculated. The listing of the computer program which has been prepared is included in Appendix 3 for record purposes.

The steps in the flexibilty calculation process are as follows:

Step 1

For the two-borehole configuration geometry shown in Figure B1a, and referred to as Excavation A, the convergences across the three diameters d_1, d_2, d_3 (see Figure 2.1) are calculated for the unit applied stress field in the x direction

$$\{\sigma\} = [-1, 0, 0]^T$$

These convergences constitute the first column of the flexibility matrix $[F_A]$

$$[F_A] = \begin{bmatrix} u_{A,x,d} & * & * \\ u_{A,x,d} & * & * \\ u_{A,x,d} & * & * \end{bmatrix}$$

where	A	refers to excavation stage A
	x	refers to the unit stress field in the x direction
		d_1 to d_3 refer to the convergence locations
	*	are elements of the flexibility matrix still to be calculated

Step 2

Calculate the convergences along the diameters d_1 to d_3 under the unit applied stress field in the y direction

$$\{\sigma\} = [0, -1, 0]^T$$

These convergences constitute the second column of the flexibility matrix $[F_A]$

$$[F_A] = \begin{bmatrix} u_{A,x,d} & u_{A,y,d} & * \\ u_{A,x,d} & u_{A,y,d} & * \\ u_{A,x,d} & u_{A,y,d} & * \end{bmatrix}$$

Step 3

Repeat the procedure for the applied unit xy stress to determine the third column of the flexibility matrix $[F_A]$

Step 4

For the three-borehole configuration geometry shown in Figure B1b, referred to as excavation B, the three steps above are repeated to calculate the flexibility matrix $[F_B]$:

$$[F_B] = \begin{bmatrix} u_{B,x,d} & u_{B,y,d} & u_{B,xy,d} \\ u_{B,x,d} & u_{B,y,d} & u_{B,xy,d} \\ u_{B,x,d} & u_{B,y,d} & u_{B,xy,d} \end{bmatrix}$$

Step 5

Subtract matrix $[F_A]$ from matrix $[F_B]$ to obtain the flexibility matrix $[F]$ relevant to the convergence **increments** $\Delta u_{B.x.d}$ .... due to the drilling of the **third** borehole - that is, the change in configuration geometry from excavation A to excavation B.

$$[F] = \begin{bmatrix} \Delta u_{B,x,d_1} & \Delta u_{B,y,d_1} & \Delta u_{B,xy,d_1} \\ \Delta u_{B,x,d_2} & \Delta u_{B,y,d_2} & \Delta u_{B,xy,d_2} \\ \Delta u_{B,x,d_3} & \Delta u_{B,y,d_3} & \Delta u_{B,xy,d_3} \end{bmatrix}$$

Step 6

When the in situ stress measurement is carried out, the diametral changes Au_{d1m}, Au_{d2m} and Au_{d3m} resulting from the drilling of the third borehole are measured. Making use of the flexibility matrix calculated above, the in situ stresses can be calculated (back analysed). The relationship is as follows:

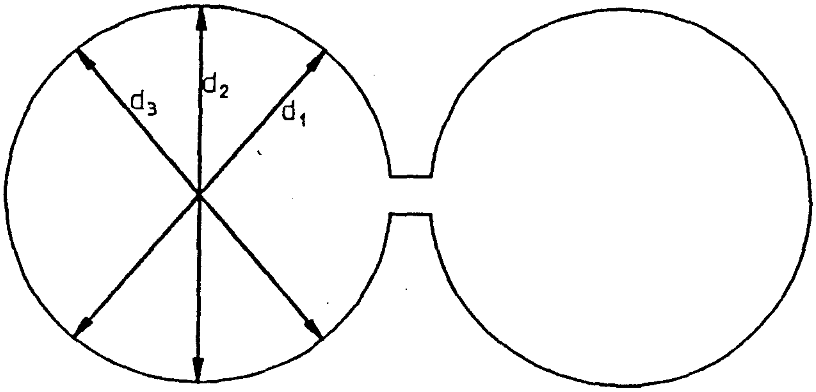

Figure 2.1a: Two borehole geometry

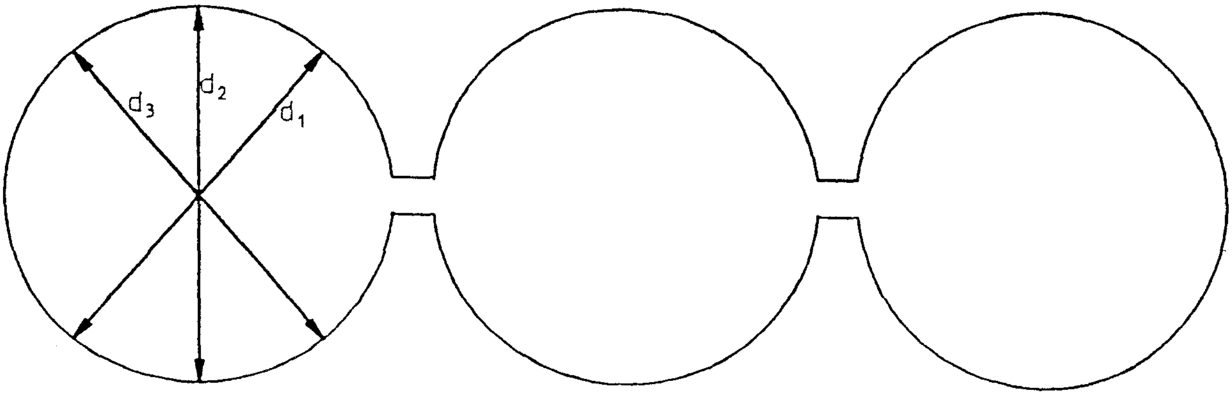

Figure 2.1b: Three borehole geometry

$$\begin{bmatrix} \Delta u_{B,x,d_1} & \Delta u_{B,y,d_1} & \Delta u_{B,xy,d_1} \\ \Delta u_{B,x,d_2} & \Delta u_{B,y,d_2} & \Delta u_{B,xy,d_2} \\ \Delta u_{B,x,d_3} & \Delta u_{B,y,d_3} & \Delta u_{B,xy,d_3} \end{bmatrix} \begin{matrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{matrix} = \begin{matrix} \Delta u_{d_1 m} \\ \Delta u_{d_2 m} \\ \Delta u_{d_3 m} \end{matrix}$$

Where the subscript *m* refers to measurements.

In matrix form:

$$[F]\{\sigma\} = \{\Delta u\}$$

The in situ stresses can then be determined by inversion of the flexibility matrix:

$$\{\sigma\} = [F]^{-1}\{\Delta u\}$$

This is a system of simultaneous linear equations which are easily solved.

APPENDIX 3

COMPUTER PROGRAM FOR BACK ANALYSIS

```
C     PROGRAM  BACKANAL.FOR
      INCLUDE 'C:\FORT\LIBCOMPS\FGRAPH.FI'
      INCLUDE 'C:\FORT\LIBCOMPS\FGRAPH.FD'
C     TO COMPILE FL BACKANAL.FOR  /link  GRAPHICS.LIB
C     THIS PROGRAM WAS WRITTEN FOR A MICROSOFT FORTRAN 77 COMPILER
C     MINOR CHANGES WAS MADE BY J.WESSELOO ON 16/1/98.  THE PROGRAM ASKS
C     THE USER FOR THE POSITIONS OF THE MEASURING POINTS WITH RESPECT TO
C     THE X-AXIS.  DEFAULT VALUES ARE 45,90 AND 135 DEG.  CHANGES MADE BY
C     J.WESSELOO IS MARKED WITH **** BEFORE AND AFTER CHANGES
C-----------------------------------------------------------------------
C-----------------------------------------------------------------------
C     THIS PROGRAM BACK-CALCULATES THE TWO NORMAL AND ONE SHEAR STRESS
C     TENSOR COMPONENTS, ON A PLANE NORMAL TO THE BOREHOLE SET AXIS FROM
C     CONVERGENCE MEASUREMENTS OF THREE BOREHOLE DIAMETERS.
C     THE BOREHOLE SET AXIS CAN HAVE ANY DIP AND DIP DIRECTION ANGLE.
C
C     THE BACK ANALYSIS ALGORITHM IMPLEMENTED IN THE PROGRAM IS THE
C     DIRECT BACK ANALYSIS PROCEDURE MULTIPLE STAGE ALGORITHM PRESENTED IN:
C     SHINICHI AKUTAGAWA. A BACK ANALYSIS PROGRAM SYSTEM FOR GEOMECHANICS
C     APPLICATIONS, PARAGRAPH 6.3. PhD THESIS, UNIVERSITY OF QUEENSLAND
C     NOVEMBER 1991.
C
C     THIS PROGRAM IS BASED ON THE FICTITIOUS STRESS BOUNDARY ELEMENT
C     METHOD AND FOLLOWS CLOSELY THE "TWOFS" PROGRAM DESCRIBED IN:
C     S.L CROUCH AND A.M.STARFIELD 1974. BOUNDARY ELEMENT METHODS IN
C     SOLIDS MECHANICS. LONDON GEORGE ALLEN & UNWIN.
C     MOST OF THE VARIABLES AND ARRAYS USED IN THIS PROGRAM ARE EXPLAINED
C     IN THE AFOREMENTIONED REFERENCE. WHEREVER NEW VARIABLES ARE INTRODUCED
C     THE ARE COMMENTED SEPARATELY. THE AUTHOR TRIED TO FOLLOW CROUCHES
C     ORIGINAL CODE AS CLOSELY AS POSSIBLE.
C
C     THIS PROGRAM REQUIRES TWO INPUT ASCII FILES TO RUN. THESE FILES
C     ARE NAMED 'CYCLOI2' AND 'CYCLOI3'. THE FORMAT OF THESE FILES
C     FOLLOWS CLOSELY WITH CROUCHES INPUT FILE FORMAT.
C     THE TWO FILES CONTAIN:
C     RECORD 1,  FORMAT 1
C      1) TITLE
C     RECORD 2,  FORMAT 3
C      2) NUMBER OF BOUNDARY SEGMENTS NECESSARY TO DISCRETIZE THE BOREHOLE
C      CONFIGURATIONS. BECAUSE OF THE GEOMETRY OF THE BOREHOLES  THE
C      NUMBER OF BOUNDARY ELEMENTS COINSIDES WITH THE NUMBER OF BOUNDARY
C      SEGMENTS. VARIABLE NUMBS.
C      3) SYMMETRY LINES, VARIABLE KSYM. DESPITE THE LACK OF ANY SYMMETRY
C      LINES (i.e KSYM = 1) THE PROGRAM CONTINUES TO TAKE INTO ACCOUNT
C      VARIABLE KSYM IN AN EFFORT NOT TO MODIFY THE ORIGINAL PROGRAM.
C     RECORD 3,  FORMAT 4
C      4) LINES OF SYMMETRY. VARIABLES XSYM, YSYM. NO LINES OF SYMMETRY
C      EXIST. HOWEVER THE VARIABLES ARE RETAINED FOR CONFORMITY.
C     RECORDS NUMBS, FORMAT 14
C      5) BOUNDARY ELEMENT NUMBER. VARIABLE NUM
C      6) ELEMENT COORDINATES. VARIABLES XBEG,YBEG,XEND,YEND. EACH ELEMENT
C      ELEMENT COINCIDES WITH A STRAIGHT LINE BOUNDARY SEGMENT.
C      7) TYPE OF PRESCRIBED STRESS OR DISPLACEMENT 'RESULTANT' BOUNDARY
C      CONDITIONS FOR EACH BOUNDARY ELEMENT. VARIABLE KODE. THE BOREHOLES
C      HAVE ZERO SURFACE TRACTIONS. KODE IS THEREFORE 1. THE VARIABLE IS
C      RETAINED.
C      8) PRESCRIBED 'RESULTANT' BOUNDARY CONDITION VALUES FOR EACH ELEMENT.
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C       VARIABLES BVS (SHEAR) AND BVN (NORMAL).
C       FOR ZERO SURFACE TRACTIONS THE SHEAR AND NORMAL STRESSES ON EACH
C       BOUNDARY ELEMENT ARE ZERO. THE VARIABLES BNS,BVN ARE RETAINED AND
C       SET TO ZERO.
C
C       INPUT ARGUMENTS
C       THE  PROGRAM REQUIRES POISSON'S RATIO, YOUNG'S MODULUS AND
C       THREE CONVERGENCE MEASUREMENTS ALONG THREE DIAMETERS OF THE FIRST
C       BOREHOLE. THE THREE DIAMETERS OF THE FIRST BOREHOLE JOIN MIDPOINTS
C       OF BOUNDARY ELEMENTS. THE BOUNDARY ELEMENTS NUMBERS DIFFER FOR THE
C       TWO CONFIGURATIONS. THE CONVERGENCE OF ONE DIAMETER
C       IS THE SUM OF THE NORMAL DISPLACEMENTS OF THE MIDPOINTS OF THE TWO
C       DIAMETRICALLY OPPOSED BOUNDARY ELEMENTS.
C-----------------------------------------------------------------------
C-----------------------------------------------------------------------
C      COMMON BLOCKS USED
       COMMON/S1/PI, PR, PR1, PR2, PR3, CON , COND
       COMMON/S2/SXXS,SXXN,SYYS,SYYN,SXYS,SXYN,UXS,UXN,UYS,UYN
C      COMMON/S3/C(100,100), B(100), P(100)  ! USED IN SUBROUTINE SOLVE
          ! IN THE ORIGINAL  'TWOFS' PROGRAM.
C
C      VARIABLE DECLARATIONS
C
       DIMENSION C(200,200), B(200), P(200)  ! EXPLAINED IN 'TWOFS'; ALSO
C         EQUATIONS (4.7.3) IN CROUCH.
C         ARRAY C CONTAINS INFLUENCE COEFFICIENTS
C         MATRIX B CONTAINS KNOWN BOUNDARY VALUES OF STRESS OR DISPLACEMENT
C         MATRIX P CONTAINS THE FICTITIOUS NORMAL AND SHEAR STRESS
C         COMPONENTS FOR EACH BOUNDARY ELEMENT. INITIALLY UNKNOWN
C
       DIMENSION XM(200),YM(200),A(200),COSBET(200),SINBET(200),KOD(200)
C         MATRICES XM, YM CONTAIN MIDPOINT COORDINATES FOR EACH BOUNDARY
C         ELEMENT.
C         MATRIX A CONTAINS ELEMENT HALF LENGTH.
C         MATRICES COSBET, SINBET DESCRIBE THE ORIENTATION OF EACH BOUNDARY
C         ELEMENT.
C         MATRIX KOD CONTAINS THE 'TYPE' OF BOUNDARY CONDITIONS.
C
C      THE FOLLOWING VARAIBLES ARE NOT CONTAINED IN 'TWOFS'
C
       DIMENSION BDN(3,200)  ! CONTAINS NORMAL DISPLACEMENTS AT ELEMENT
C         MIDPOINTS FOR EACH OF THE THREE RUNS.
C
       DIMENSION STR(3)    ! CONTAINS THE APPLIED STRESS FIELD PXX,PYY,PXY
          ! FOR THE THREE RUNS. FOR EACH RUN, ONE COMPONENT IS UNIT STRESS
          ! AND THE OTHER TWO ARE ZERO.
          ! THEREFORE FOR RUN ONE   Pxx=UNIT STRESS, Pyy=Pxy=ZERO;
          ! THEREFORE FOR RUN TWO   Pyy=UNIT STRESS, Pxx=Pxy=ZERO;
          ! THEREFORE FOR RUN THREE Pxy=UNIT STRESS, Pyy=Pxx=ZERO;
          ! IT IS USED IN THE COMPUTATION OF THE FLEXIBILITY MATRIX FLEX(3,3).
       DIMENSION  FLEX(3,3) ! FLEXIBILITY MATRIX
C
       DIMENSION  CON_MES(3) !  CONVERGENCE MEASUREMENTS, INPUT
C
       DIMENSION  TASIS(3)   ! MATRIX OF BACK CALCULATED IN SITU STRESS
C         COMPONENTS IN THE PLANE NORMAL TO THE AXES OR THE THREE BOREHOLES.
       DIMENSION TITLE(20)
```

```
C
      REAL*8  STR, CON_MES, FLEX, TASIS, C, B, P, BDN
C
      INTEGER FSN, DS  ! FSN Variable to open input files
C
      CHARACTER*8 FILENAME, FNAME
C
C
C     VARIABLE INITIALIZATIONS
C
      WRITE (*, 700)
700   FORMAT ('  ENTER THE ANGLES BETWEEN THE X-AXIS AND THE SAMPLING PO
     +INTS',/,/)
      WRITE (*, 710)
C****
710   FORMAT (/,/,'  ENTER 1ST ANGLE (45 DEG)->  ', \)
      READ (*,'(F6.0)') A1
      IF (A1.EQ.0) A1=45       ! DEFAULT VALUE

      WRITE (*, 720)
720   FORMAT (/,/,'  ENTER 2ST ANGLE (90 DEG)->  ', \)
      READ (*,'(F6.0)') A2
      IF (A2.EQ.0) A2=90          ! DEFAULT VALUE

      WRITE (*, 730)
730   FORMAT (/,/,'  ENTER 3ST ANGLE (135 DEG)-> ', \)
      READ (*,'(F6.0)') A3
      IF (A3.EQ.0) A3=135         ! DEFAULT VALUE

      INDEX_1=NINT((A1-45)/9)+1   ! SERIAL NUMBERS OF BOUNDARY ELEMENTS WITH
      INDEX_2=INDEX_1+20        ! DIAMETRICALLY OPPOSED MIDPOINTS IN THE FIRST

      INDEX_3=NINT((A2-45)/9)+1  ! BOREHOLE. CORRESPOND TO 45,90,135 DEGREES
      INDEX_4=INDEX_3+20         ! ANGLES FROM X AXIS, COUNTERCLOCKWISE.

      INDEX_5=NINT((A3-45)/9)+1
      INDEX_6=INDEX_5+20
C****
C        THESE LOCATIONS CORRESPOND WITH THE LOCATIONS OF PHYSICAL
C        CONVERGENCE MEASUREMENTS IN THE ACTUAL STRESS MEASUREMENT PROCESS.

      DATA STR/ 0. ,0., 0./    ! INITIALIZE IN SITU STRESS FIELD
C-----------------------------------------------------------------------
C-----------------------------------------------------------------------
      FILENAME = 'CYCLOI2' ! FIRST FILE FOR DATA INPUT. TWO BOREHOLES.
C------------------------------------------------------------------------
C------------------------------------------------------------------------
C     Input POISSON'S  and YOUNG'S  moduli
C
      CALL clearscreen($GCLEARSCREEN)

609   WRITE (*, 610)
610   FORMAT (/,/,'  ENTER POISSON''S RATIO ->  ', \)
      READ (*, *) PR
      IF (PR .LT. 0.0  .OR. PR .GT. 0.5) GOTO 609 ! CHECK VALIDITY

      WRITE (*, 620)
```

```
620   FORMAT (/,/,'  ENTER YOUNG''S MODULUS IN MPa ->   ', \)
      READ (*,*) E

      WRITE (*,'(///,5X,A,2X, F6.4,/,/)') ' POISSON''S RATIO IS ', PR
      WRITE (*,'(5X,A,2X, E10.5,A3,/,/)') 'YOUNG''S MODULUS IS ',
     + E,'MPa'
      PAUSE   ' CHECK INPUT VALUES, PRESS ENTER TO CONTINUE'
C
C----------------------------------------------------------------------
C     Input convergence measurements
      CALL clearscreen($GCLEARSCREEN)

      WRITE (*, '(/,/,/, 10X,A,/)' ) 'INPUT CONVERGENCE MEASUREMENTS
     + IN mm'
      WRITE (*, 630)
630   FORMAT (/,/,'  ENTER FIRST VALUE IN mm  ->    '  \)
      READ (*,'(E16.9)') CON_MES(1)
      WRITE (*, 640)
640   FORMAT (/,/,'  ENTER SECOND VALUE IN mm ->  '  \)
      READ (*,'(E16.9)') CON_MES(2)
      WRITE (*, 650)
650   FORMAT (/,/,'  ENTER THIRD VALUE IN mm  ->  '  \)
      READ (*,'(E16.9)') CON_MES(3)
      WRITE (*, *)  '             CONVERGENCE MEASUREMENTS '
      WRITE (*, '(/,15X,E16.9)') (CON_MES(I), I=1,3)
      WRITE (*, *)
      PAUSE   ' CHECK INPUT VALUES, PRESS ENTER TO CONTINUE'
C----------------------------------------------------------------------
C     REQUEST USER FOR OUTPUT FILE NAME
C
      CALL clearscreen($GCLEARSCREEN)
      WRITE (*, '(A35,\)') '   ENTER FILE NAME FOR OUTPUT -> '
      READ (*, '(A)' )  FNAME
      OPEN (4, FILE = FNAME, ACCESS='SEQUENTIAL', STATUS='NEW')
C
C----------------------------------------------------------------------
      DO FSN = 8,9              ! FSN FILE SPECIFICATION NUMBER
      OPEN (UNIT=FSN,  FILE=FILENAME)
      WRITE  (4,  '(A12,\)') 'THIS IS FILE',  FILENAME
      READ (FSN, 1) (TITLE(I), I=1,20)
      WRITE (4, 2) (TITLE(I), I=1,20)
      READ (FSN, 3) NUMBS, KSYM
      READ (FSN, 4)  XSYM, YSYM
C
C     READ (3, 5) PXX, PYY, PXY   ! INITIAL STRESS FIELD IN 'TWOFS'
C     IN 'BACKANAL' THE INITIAL STRESS FIELD IS ASSUMED FOR THE CALCULATION
C     OF THE FLEXIBILITY MATRIX.
C
      WRITE (4, 6) NUMBS
      GO TO (80, 85, 90, 95), KSYM
80    WRITE (4, 7)
      GO TO 100
85    WRITE (4, 8) XSYM
      GO TO 100
90    WRITE (4, 9) YSYM
      GO TO 100
95    WRITE (4, 10) XSYM, YSYM
```

```
C
100   CONTINUE
      WRITE (4, 11) PR, E
C     WRITE (4, 12) PXX, PYY, PXY
C
      PI=4.*ATAN(1.)
      CON =1./(4.*PI*(1.-PR))
      COND = (1.+PR)/E
      PR1 =1.-2.*PR
      PR2 =2.*(1.-PR)
      PR3 = 3.-4.*PR
C
C     DEFINE LOCATION, SIZES, ORIENTATIONS AND BOUNDARY CONDITIONS OF
C     BOUNDARY ELEMENTS
C
      NUMBE =0
      DO 110 N=1, NUMBS   ! NUMBER OF BOUNDARY LINE SEGMENTS
      READ (FSN, 14) NUM, XBEG, YBEG, XEND, YEND, KODE, BVS, BVN
      write (4, 14) NUM, XBEG, YBEG, XEND, YEND, KODE, BVS, BVN
      XD=(XEND-XBEG)/NUM
      YD=(YEND-YBEG)/NUM
      SW= SQRT(XD*XD + YD*YD)  ! Element length
C
      DO 110 NE=1, NUM ! Number of elements in each straight line b. seg
      NUMBE= NUMBE +1
      M=NUMBE  ! Start from first element
      XM(M) =XBEG +0.5 *(2.*NE-1.)*XD
      YM(M) =YBEG +0.5 *(2.*NE-1.)*YD  ! Element midpoint coordinates
      A(M)=0.5*SW                      ! element half length
      SINBET(M)=YD/SW
      COSBET(M)=XD/SW
      KOD(M)=KODE  ! TYPE OF BOUNDARY CONDITIONS
      MN=2*M
      MS=MN-1
      B(MS)=BVS    ! Stores total shear disp. or stress for the element
110   B(MN)=BVN    ! Stores total normal disp. or stress for the element
C
C****
      IF (A1.LT.(45-4.5)) INDEX_1 = (NUMBE+1)-NINT((45-A1)/9)
C****
      WRITE (4,13)
      DO 115 M=1, NUMBE
      SIZE=2.*A(M)
      ANGLE =180.*ATAN2(SINBET(M),COSBET(M))/PI  !beta angle for element
      WRITE (4, 15) M,KOD(M), XM(M),YM(M), SIZE, ANGLE,B(2*M-1), B(2*M)
115   CONTINUE
C
C     ADJUST STRESS BOUNDARY VALUES TO ACCOUNT FOR INITIAL STRESSES
C
      DO  1000 ST=1,3
          STR(ST) = 1.  ! IN EACH RUN ONLY ONE IN SITU SRESS COMPONENT
                        ! HAS THE VALUE OF UNITY. THE TWO OTHERS ARE ZERO
          DS = ST-1
             IF (DS .GE. 1.) STR(DS) = 0.
          PXX= STR(1)   ! IN SITU STRESS FIELD
          PYY= STR(2)
          PXY= STR(3)
```

```
C
      WRITE (4, 12) PXX, PYY, PXY
C
      DO 150 N=1, NUMBE
      NN=2*N
      NS=NN-1
      COSB=COSBET(N)
      SINB=SINBET(N)
      SIGS=(PYY-PXX)*SINB*COSB+PXY*(COSB*COSB-SINB*SINB)
      SIGN=PXX*SINB*SINB-2.*PXY*SINB*COSB+PYY*COSB*COSB
      GO TO (120, 150, 130,140), KOD(N)
120   B(NS)=0.-SIGS            ! induced = resultant - initial
      B(NN)=0.-SIGN
      GO TO 150
130   B(NN)=B(NN) -SIGN
      GO TO 150
140   B(NS)=B(NS)-SIGS
150   CONTINUE
C
C     COMPUTE INFLUENCE COEFFICIENTS AND SET UP SYSTEM OF ALGEBRAIC
C     EQUATIONS .
C
      DO 300 I=1, NUMBE
      IN=2*I
      IS=IN-1
      XI=XM(I)  ! element i midpoint coordinates
      YI=YM(I)
      COSBI=COSBET(I)
      SINBI=SINBET(I)
      KODE=KOD(I)
C
      DO 300 J=1, NUMBE
      JN=2*J
      JS=JN-1
      CALL INITL
      XJ=XM(J)
      YJ=YM(J)
      COSBJ=COSBET(J)
      SINBJ=SINBET(J)
      AJ= A(J)                  ! element half length
      CALL  COEFF(XI,YI,XJ,YJ,AJ,COSBJ,SINBJ,+1)
      GO TO (240, 210, 220, 230),  KSYM
C
210   XJ=2.*XSYM-XM(J)
      CALL  COEFF(XI,YI,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
      GO TO 240
C
220   YJ=2.*YSYM-YM(J)
      CALL  COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,SINBJ,-1)
      GO TO 240
C
230   XJ=2.*XSYM-XM(J)
      CALL  COEFF(XI,YI,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
      XJ=XM(J)
      YJ=2.*YSYM-YM(J)
      CALL  COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,SINBJ,-1)
      XJ=2.*XSYM-XM(J)
```

```
      CALL  COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,-SINBJ,+1)
C
240   CONTINUE
      GO TO (250, 260, 270, 280),  KODE
C
250   C(IS, JS)=(SYYS-SXXS)*SINBI*COSBI+SXYS*(COSBI*COSBI-SINBI*SINBI)
      C(IS, JN)=(SYYN-SXXN)*SINBI*COSBI+SXYN*(COSBI*COSBI-SINBI*SINBI)
      C(IN, JS)=SXXS*SINBI*SINBI-2.*SXYS*SINBI*COSBI+SYYS*COSBI*COSBI
      C(IN, JN)=SXXN*SINBI*SINBI-2.*SXYN*SINBI*COSBI+SYYN*COSBI*COSBI
      GO TO 300
C
260   C(IS, JS) = UXS*COSBI+ UYS*SINBI
      C(IS, JN) = UXN*COSBI+ UYN*SINBI
      C(IN, JS)= -UXS*SINBI +UYS*COSBI
      C(IN, JN)= -UXN*SINBI+ UYN*COSBI
      GO TO 300
C
270   C(IS, JS) = UXS*COSBI +UYS*SINBI
      C(IS, JN) = UXN*COSBI +UYN*SINBI
      C(IN, JS) = SXXS*SINBI*SINBI-2.*SXYS*SINBI*COSBI+SYYS*COSBI*COSBI
      C(IN, JN) = SXXN*SINBI*SINBI-2.*SXYN*SINBI*COSBI+SYYN*COSBI*COSBI
      GO TO 300
C
280   C(IS, JS)=(SYYS-SXXS)*SINBI*COSBI+SXYS*(COSBI*COSBI-SINBI*SINBI)
      C(IS, JN)=(SYYN-SXXN)*SINBI*COSBI+SXYN*(COSBI*COSBI-SINBI*SINBI)
      C(IN, JS)=-UXS*SINBI +UYS*COSBI
      C(IN, JN)=-UXN*SINBI +UYN*COSBI
C
300   CONTINUE
C
C     SOLVE SYSTEM OF EQUATIONS
C
      N=2*NUMBE
      CALL SOLVE(N, C,B,P)
C
C      COMPUTE BOUNDARY DISPLACEMENTS AND STRESSES
C
      WRITE (4, 16)
      DO 600 I=1, NUMBE
      XI =XM(I)
      YI =YM(I)
      COSBI=COSBET(I)
      SINBI=SINBET(I)
C
      UX= 0.
      UY= 0.
      SIGXX = PXX
      SIGYY = PYY
      SIGXY = PXY
C
      DO 570 J= 1, NUMBE
      JN = 2*J
      JS = JN-1
      CALL  INITL
      XJ = XM(J)
      YJ = YM(J)
      AJ = A(J)
```

```
        COSBJ = COSBET(J)
        SINBJ = SINBET(J)
        CALL COEFF(XI,YI,XJ,YJ,AJ,COSBJ,SINBJ,+1)
        GO TO (540, 510 , 520, 530),   KSYM
C
510     XJ=2.*XSYM-XM(J)
        CALL COEFF(XI,YI,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
        GO TO 540
C
520     YJ=2.*YSYM-YM(J)
        CALL COEFF(XI,YI,  XJ,YJ,  AJ,  -COSBJ,SINBJ, -1)
        GO TO 540
C
530     XJ=2.*XSYM-XM(J)
        CALL COEFF(XI,YI,  XJ,YJ,  AJ,  COSBJ,-SINBJ, -1)
        XJ=XM(J)
        YJ=2.*YSYM-YM(J)
        CALL COEFF(XI,YI,  XJ,YJ,  AJ,  -COSBJ,SINBJ, -1)
        XJ=2.*XSYM-XM(J)
        CALL COEFF(XI,YI,  XJ,YJ,  AJ,  -COSBJ,-SINBJ, +1)
C
540     CONTINUE
C
        UX = UX + UXS*P(JS) + UXN*P(JN)
        UY = UY + UYS*P(JS) + UYN*P(JN)
        SIGXX = SIGXX + SXXS*P(JS) + SXXN*P(JN)
        SIGYY = SIGYY + SYYS*P(JS) + SYYN*P(JN)
        SIGXY = SIGXY + SXYS*P(JS) + SXYN*P(JN)
C
570     CONTINUE
C
        US = UX*COSBI + UY*SINBI
        UN =-UX*SINBI + UY*COSBI
        BDN(ST,I)  =  UN
        SIGS= (SIGYY-SIGXX)*SINBI*COSBI+SIGXY*(COSBI*COSBI-SINBI*SINBI)
        SIGN= SIGXX*SINBI*SINBI-2.*SIGXY*SINBI*COSBI+SIGYY*COSBI*COSBI
        SIGT= SIGXX*COSBI*COSBI+2.*SIGXY*SINBI*COSBI+SIGYY*SINBI*SINBI
C
        WRITE (4, 17) I,UX,UY,US,UN,SIGXX,SIGYY,SIGXY,SIGS,SIGN,SIGT
C
600     CONTINUE
C
C       CALCULATION OF FLEXIBILITY MATRIX  3-2
        FLEX(1,ST) = -FLEX(1,ST) + ( BDN(ST, INDEX_1) + BDN(ST,INDEX_2) )
        FLEX(2,ST) = -FLEX(2,ST) + ( BDN(ST, INDEX_3) + BDN(ST,INDEX_4) )
        FLEX(3,ST) = -FLEX(3,ST) + ( BDN(ST, INDEX_5) + BDN(ST,INDEX_6) )

1000    CONTINUE  !  END OF THE THREE RUNS FOR THIS INPUT FILE
                  !  TWO BOREHOLES
C-----------------------------------------------------------------------------
C       PREPARE FOR THE THREE RUNS OF THE SECOND FILE
C       THREE  BOREHOLES
          FILENAME = 'CYCLOI3' ! SPECIFY FILE FOR INPUT
          STR(3)= 0.       ! initialize PXY to 0 for second file run

        END DO !END FLEXIBILITY MATRIX CALCULATION
C-----------------------------------------------------------------------------
```

```
C
C     PRINT FLEXIBILITY MATRIX
      CALL clearscreen($GCLEARSCREEN)
      WRITE (*,'(15X,A,/)') 'FLEXIBILITY MATRIX'
      WRITE (*,660) ((FLEX(FF, GG), GG=1,3), FF=1,3)
660   FORMAT (3(5X, 3(E10.4,4X), /))
      PAUSE  '   PRESS ENTER TO CONTINUE  '
C-----------------------------------------------------------------------
C    {Dqi} = [Fi] {sigma o} EQUATION (6.3.25) PAGE 175 AKUTAGAWA
      CALL SOLVE_2(3, FLEX, CON_MES, TASIS)
C-----------------------------------------------------------------------
      CALL clearscreen($GCLEARSCREEN)
C-----WRITE TO SCREEN
      WRITE (*,'(5X,A,2X, F6.4,/,/)') ' POISSON''S RATIO IS ', PR
      WRITE (*,'(5X,A,2X, E10.5,A3,/,/)') 'YOUNG''S MODULUS IS ',
     + E,'MPa'
      WRITE (*,'(5X,A,E14.8,A3,/,/)') 'SXX=', TASIS(1), 'MPa'
      WRITE (*,'(5X,A,E14.8,A3,/,/)') 'SYY=', TASIS(2), 'MPa'
      WRITE (*,'(5X,A,E14.8,A3,/,/)') 'SXY=', TASIS(3), 'MPa'
      PAUSE ' '
C-------------------------------------------------------------------------
C     FORMAT STATEMENTS
C
      FORMAT (20A4)
      FORMAT (1H ,/,25X,20A4,/)
      FORMAT (2I4)
      FORMAT ( 2F12.4)
  5      FORMAT (3E11.4)
      FORMAT (/,109H NUMBER OF STRAIGHT-LINE SEGMENTS (EACH CONTAINING A
     +T LEAST ONE BOUNDARY ELEMENT) USED TO DEFINE BOUNDARIES =,I3,/)
      FORMAT (/,32H NO SYMMETRY CONDITIONS IMPOSED.)
      FORMAT (/,18H THE LINE X = XS =,F12.4,23H IS A LINE OF SYMMETRY.)
      FORMAT (/,18H THE LINE Y = YS =,F12.4,23H IS A LINE OF SYMMETRY.)
0     FORMAT (/,19H THE LINES X = XS =,F12.4,13H AND Y = YS =,F12.4,
     +  23H ARE LINES OF SYMMETRY.)
1     FORMAT (/,18H POISSON'S RATIO = F6.2,//,18H YOUNG'S MODULUS =,E11.
     +4)
2     FORMAT (/,31H XX-COMPONENT OF FIELD STRESS =,E11.4,//,31H YY-COMPO
     +NENT OF FIELD STRESS =,E11.4,//,31H XY-COMPONENT OF FIELD STRESS =
     +,E11.4)
3     FORMAT (1H ,/,27H      BOUNDARY ELEMENT DATA.,//,96H  ELEMENT     K
     +ODE  X (CENTER)   Y (CENTER)     LENGTH       ANGLE  US OR SIGMA-S
     +  UN OR SIGMA-N,/)
4     FORMAT (I4,4F12.6,I4,2E11.4)
5     FORMAT (2I9,3F12.4,F12.2, 2E15.4)
6     FORMAT (1H ,/,66H      DISPLACEMENTS AND STRESSES AT MIDPOINTS OF B
     +OUNDARY ELEMENTS.,//,40H    ELEMENT       UX        UY       US  ,
     + 60H        UN      SIGXX      SIGYY      SIGXY   SIGMA-S     SIGM-N,
     + 10H   SIGMA-T,/)
7     FORMAT (I10,4E10.4, 6F10.5)
8     FORMAT (1H ,/,64H      DISPLACEMENTS AND STRESSES AT SPECIFIED POIN
     +TS IN THE BODY.,//,93H    POINT     X CO-ORD     Y CO-ORD          U
     +X           UY        SIGXX        SIGYY        SIGXY,/)
C-----------------------------------------------------------------------
      END          ! Main program end
C-----------------------------------------------------------------------
```

```
C
      SUBROUTINE INITL
C
      COMMON/S2/SXXS,SXXN, SYYS,SYYN, SXYS,SXYN, UXS,UXN,  UYS,UYN
C
      SXXS = 0.
      SXXN = 0.
      SYYS = 0.
      SYYN = 0.
      SXYS = 0.
      SXYN = 0.
C
      UXS   = 0.
      UXN   = 0.
      UYS   = 0.
      UYN   = 0.
C
      RETURN
      END
C-----------------------------------------------------------------------
C
C
      SUBROUTINE   COEFF(X,Y,CX,CY,A,COSB,SINB,MSYM)
C
      COMMON/S1/PI, PR, PR1, PR2, PR3, CON, COND
      COMMON/S2/SXXS,SXXN, SYYS,SYYN, SXYS,SXYN, UXS,UXN, UYS,UYN
C
      COS2B = COSB*COSB - SINB*SINB
      SIN2B = 2.*SINB*COSB
C
      XB = (X-CX)*COSB + (Y-CY)*SINB
      YB = -(X-CX)*SINB + (Y-CY)*COSB
C
      R1S = (XB-A)*(XB-A)+YB*YB
      R2S = (XB+A)*(XB+A)+YB*YB
      FL1 = 0.5*ALOG(R1S)
      FL2 = 0.5*ALOG(R2S)
      FB2 = CON*(FL1-FL2)
      IF (YB .NE. 0.) GO TO 10
      FB3 = 0.
      IF (ABS(XB) .LT. A)  FB3 = CON*PI
      GO TO 20
10    FB3 = -CON* (ATAN((XB+A)/YB) - ATAN((XB-A)/YB))
20    FB1 = YB*FB3 + CON*( (XB-A)*FL1 - (XB+A)*FL2 )
      FB4 = CON*(YB/R1S-YB/R2S)
      FB5 = CON*( (XB-A)/R1S - (XB+A)/R2S )
C
      UXPS = COND*(PR3*COSB*FB1 + YB*(SINB*FB2 + COSB*FB3))
      UXPN = COND*(-PR3*SINB*FB1 - YB*(COSB*FB2 -SINB*FB3))
      UYPS = COND*(PR3*SINB*FB1 - YB*(COSB*FB2 - SINB*FB3))
      UYPN = COND*(PR3*COSB*FB1 - YB*(SINB*FB2 + COSB*FB3))
C
      SXXPS = FB2+PR2*(COS2B*FB2 - SIN2B*FB3)+YB*(COS2B*FB4+SIN2B*FB5)
      SXXPN = FB3-PR1*(SIN2B*FB2 + COS2B*FB3)+YB*(SIN2B*FB4-COS2B*FB5)
      SYYPS = FB2-PR2*(COS2B*FB2 - SIN2B*FB3)-YB*(COS2B*FB4+SIN2B*FB5)
      SYYPN = FB3+PR1*(SIN2B*FB2 + COS2B*FB3)-YB*(SIN2B*FB4-COS2B*FB5)
      SXYPS = PR2*(SIN2B*FB2 + COS2B*FB3)+YB*(SIN2B*FB4-COS2B*FB5)
```

```
      SXYPN = PR1*(COS2B*FB2 - SIN2B*FB3)-YB*(COS2B*FB4+SIN2B*FB5)
C
      UXS = UXS +MSYM*UXPS
      UXN = UXN + UXPN
      UYS = UYS + MSYM*UYPS
      UYN = UYN + UYPN
C
      SXXS = SXXS + MSYM *SXXPS
      SXXN = SXXN + SXXPN
      SYYS = SYYS + MSYM *SYYPS
      SYYN = SYYN + SYYPN
      SXYS = SXYS + MSYM *SXYPS
      SXYN = SXYN + SXYPN
C
      RETURN
      END
C     END OF SUBROUTINE COEF
C-----------------------------------------------------------------------
C
      SUBROUTINE SOLVE(N, A, B, X)
C
      DIMENSION  A(200,200), B(200), X(200)
C
      REAL*8  A,B,X
      NB = N-1
      DO 20 J=1,NB
      L = J+1
      DO 20 JJ =L,N
      XM = A(JJ,J)/A(J,J)
      DO 10 I=J,N
10    A(JJ,I) = A(JJ,I)-A(J,I)*XM
20    B(JJ) = B(JJ) - B(J)*XM
C
      X(N) = B(N)/A(N,N)
      DO 40 J=1,NB
      JJ = N-J
      L = JJ+1
      SUM = 0.
      DO 30 I=L,N
30    SUM = SUM +A(JJ,I)*X(I)
40    X(JJ) = (B(JJ) -SUM)/A(JJ,JJ)
C
      RETURN
      END
C     END  OF SUBROUTINE SOLVE
C-----------------------------------------------------------------------
C
      SUBROUTINE SOLVE_2(N, A, B, X)
C     THIS SUBROUTINE DIFFERS FROM THE PREVIOUS ONE ONLY IN THE
C     ARRAY SIZES
C
      DIMENSION  A(3,3), B(3), X(3)
C
      REAL*8  A,B,X
      NB = N-1
      DO 20 J=1,NB
      L = J+1
```

```
      DO 20 JJ =L,N
      XM = A(JJ,J)/A(J,J)
      DO 10 I=J,N
10    A(JJ,I) = A(JJ,I)-A(J,I)*XM
20    B(JJ) = B(JJ) - B(J)*XM
C
      X(N) = B(N)/A(N,N)
      DO 40 J=1,NB
      JJ = N-J
      L = JJ+1
      SUM = 0.
      DO 30 I=L,N
30    SUM = SUM +A(JJ,I)*X(I)
40    X(JJ) = (B(JJ) -SUM)/A(JJ,JJ)
C
      RETURN
      END
C     END OF SUBROUTINE  SOLVE_2
C----------------------------------------------------------------------------
C----------------------------------------------------------------------------
```

APPENDIX 4

COMPUTER PROGRAM TO CALCULATE THE THREE DIMENSIONAL STRESS TENSOR

```
   PROGRAM MODIF.FOR
-------------------------------------------------------------------------
-------------------------------------------------------------------------
   INCLUDE 'C:\FORT\LIBCOMPS\FGRAPH.FI'
   INCLUDE 'C:\FORT\LIBCOMPS\FGRAPH.FD'
   TO COMPILE      FL MODIF.FOR /link GRAPHICS.LIB
   THIS PROGRAM HAS BEEN WRITTEN FOR A FORTRAN 77 COMPILER.
-------------------------------------------------------------------------
-------------------------------------------------------------------------
   THIS PROGRAM COMPLEMENTS THE BACK ANALYSIS PROGRAM USED TO
   CALCULATE THE IN SITU STRESS TENSOR COMPONENTS ON A PLANE NORMAL
   TO THE BOREHOLE SET AXES.

   REFERENCES
   1. LOUIS A. PANEK. CALCULATION OF THE AVERAGE GROUND STRESS
   COMPONENTS FROM MEASUREMENTS OF THE DIAMETRAL DEFORMATION OF A
   DRILL HOLE. REPORT OF INVESTIGATIONS 6732. UNITED STATES DEPARTMENT
   OF THE INTERIOR. BUREAU OF MINES 1966.
   2. WILLIAM K. SMITH. TWO BASIC COMPUTER PROGRAMS FOR THE DETERMINATION
   OF IN-SITU STRESSES USING THE CSIRO HOLLOW INCLUSION STRESS CELL
   AND THE USBM BOREHOLE DEFORMATION GAGE. OPEN-FILE REPORT 82-489, 1982.
   UNITED STATES DEPARTMENT OF THE INTERIOR GEOLOGICAL SURVEY.

   COORDINATE SYSTEMS.
   THE COORDINATE SYSTEMS USED IN THIS PROGRAM ARE EXPLAINED IN SMITH
   PAGES 4-6 AND FIGURE 1 IN PARTICULAR. THE BOREHOLE SET AXIS IS DEFINED
   BY THE BEARING AND INCLINATION ANGLES OF THE BOREHOLE SET, IN TERMS
   OF THE MINE COORDINATE SYSTEM XYZ AND CORRESPONDS TO THE h2 AXIS IN
   FIGURE 1.
   THE OUTPUT OF THE BACK ANALYSIS PROGRAM IS THE THREE IN SITU STRESS
   TENSOR COMPONENTS ON THE PLANE NORMAL TO THE h2 AXIS IN FIGURE 1.
   THESE STRESS COMPONENTS ARE SIGMA-1 SIGMA-3 AND TAU-13; FORMULAE (13),
   (15) AND (16) ON PAGE 8 IN PANEK AND FIGURE 1 IN SMITH.
   THE OUTPUT OF THE BACK ANALYSIS PROGRAM IS SXX, SYY AND SXY. HOWEVER
   XYZ IS NOT THE MINE COORDINATE SYSTEM (Y NORTH, Z VERTICAL, X EAST)
   OF THIS PROGRAM BUT THE COORDINATE SYSTEM 123. THE CONNECTIONS BETWEEN
   THE BACK ANALYSIS PROGRAM OUTPUT AND THIS PROGRAM STRESS COMPONENTS ARE:
           BACK ANALYSIS PROGRAM           THIS PROGRAM
                  SXX                          SIGMA-1
                  SYY                          SIGMA-3
                  SXY                          TAU-13  (SHEAR)
-------------------------------------------------------------------------
   THIS PROGRAM PERFORMS A LEAST SQUARE FIT FOR THE STRESS TENSOR BASED
   ON DATA OF THREE BOREHOLE SETS.

   PROGRAM INPUT

   THE USER IS PROMPTED FOR THE FOLLOWING DATA INPUT:
   1. BEARING ANGLE OF THE BOREHOLE SET.
   2. INCLINATION ANGLE OF THE BOREHOLE SET.
   3. SIGMA-1
   4. SIGMA-3
   5. TAU-13   (SHEAR)

   PROGRAM OUTPUT
   1. THE SIX STRESS TENSOR COMPONENTS AND THEIR STANDARD DEVIATIONS.
   2. STATISTICAL PARAMETERS OF THE LEAST SQUARES FIT.
   3. THE THREE PRINCIPAL STRESS VECTORS, THEIR BEARING AND INCLINATION
      ANGLES IN TERMS OF THE MINE COORDINATE SYSTEM AND THEIR STANDARD
      DEVIATIONS.

-------------------------------------------------------------------------
-------------------------------------------------------------------------
```

```
  PARAMETERS USED
  PARAMETER (R1=57.29578) ! CONVERTS DEGREES TO RADIANS

  VARIABLE DECLARATIONS

  ARRAYS, MATRICES

  REAL*8 BE(3), INCLI(3)  ! BEARING AND INCLINATION ANGLE MATRICES
      ! OF THE THREE BOREHOLE SETS
  REAL*8 J(9,6)    ! ARRAY OF J FACTORS
  REAL*8 BAI(9) !BAI(1)=STRESS1,BAI(2)=STRESS3,BAI(3)=SHEAR13;
      ! STRESS VALUES. INPUT FROM THE BACK ANALYSIS PROGRAM
  REAl*8 DCOS(3,9) ! DIRECTION COSINES OF BOREHOLE SETS
  REAL*8 A(6,12) ! A(i,j) = Sum(J(n,i)*J(n,j)) n=1,9
      ! WHERE A[6,12] APPEARS IN EQUATION  [A]x[B]=[G]
      ! IN THE EQUATION ONLY THE FIRST 6 COLUMNS OF A[6,12] APPEAR;
      ! A[6,12] HAS THE UNIT DIAGONAL MATRIX IN THE SIX  AUGMENTATION
      !COLUMNS IN ORDER TO FIND IT'S INVERSE C[6,6]
  REAL*8 G(6)     ! G(j) = Sum(BAI(n)*J(n,j))  n=1,9
  REAL*8 B(6)     ! MATRIX OF UNKNOWN STRESS TENSOR COMPONENTS
  REAL*8 V(6)     ! MATRIX OF STANDARD DEVIATIONS OF STRESS COMPONENTS
      ! STORES ALSO STANDARD DEVIATIONS OF PRINCIPAL STRESSES
  REAL*8 S(3,3)  ! STRESS TENSOR IN ARRAY FORM
  REAL*8 Z(3,3)  ! Z(N,1) PRINCIPAL STRESS, Z(N,2) BEARING ANGLE,
      ! Z(N,3) INCLINATION. N=1,3 LABEL.
  REAL*8 Y(3,3)  ! Y[3,3] = S[3,3] - Z(N)*[I] FOR CALCULATION OF
      ! DIRECTION COSINES OF STRESS TENSOR COMPONENTS
  REAL*8 P(3,3)  ! ARRAY OF DIRECTION COSINES OF PRINCIPAL STRESS
      ! VECTORS
  REAL*8 PR_DCOS(3,9) ! ARRAY OF DIRECTION COSINES OF BOREHOLE SETS
      ! WITH RESPECT TO THE PRINCIPAL STRESS DIRECTIONS

  NON-ARRAY VARIABLES
  REAL*8 S1,S2,S3,R2,R,S4 ! STATISTICAL PARAMETERS
  REAL*8 I1,I2,I3 ! INVARIANTS OF THE STRESS TENSOR
  REAL*8 I4, Q1,Q2,Q3,Q4,Q5,Q6,Q7,X1,X2,X3,N2,N3,N4
  REAL*8 B5, I5 ! TEMPORARY VARIABLES STORING BEARING AND INCLINATION
      ! ANGLES OF PRINCIPAL STRESSES
  REAL*8 Z1,Z3 ! TEMPORARY  VARIABLES SORTING PRINCIPAL STRESSES
  REAL*8 CHECK, FF ! FOR CHECKING PRINCIPAL STRESSES
  CHECK (FF) = -FF**3 + I1*FF**2 -I2*FF + I3  ! STATEMENT FUNCTION

  INITIALIZATIONS
  DATA J /54*0.0/ !  INITIALIZE J(9,6)
  DATA A /36*0.0, 1.0, 6*0.0, 1.0, 6*0.0, 1.0, 6*0.0, 1.0,
 + 6*0.0, 1.0, 6*0.0, 1.0 / ! INITIALIZE AUGMENTED MATRIX A
-----------------------------------------------------------------------
---------------------------------------------------------------------------
  COORDINATE SYSTEMS AND CONVENTIONS FOR BEARING AND INCLINATION
  ANGLES
  CALL   clearscreen($GCLEARSCREEN)
  WRITE (*,'(//,A)')   '                    COORDINATE SYSTEMS AND CON
 +VENTIONS '
  WRITE (*,'(/,A)')      '  Two right-handed coordinate systems are
 +used in the analysis of stress determi-nation. The first of these
 + is a local, or borehole coordinate system, h1, h2,  h3, as define
 +d by Panek (1966) and Smith (1982). The h2 axis coincides with the
 + axis of the borehole. The h1 axis is horizontal and extends to th
 +e right as the observer looks into the borehole. The h3 axis lies
 +in a vertical plane and gene-rally points upward. The h3 axis is t
 +ruly vertical only if the borehole is hori-zontal; it is horizonta
 +l for a vertical drill hole.'

  WRITE (*,'(//,A)')      '  The second coordinate system is the glob
```

```
+al, or geographic system, x,y,z, which gives a common base to whic
+h the computations are referred. The system used is  x is positive
+ east, y is positive north, and z is positive up.'

 WRITE (*,'(//,A)')    '  Bearings are given in degrees clockwise f
+rom north, and inclinations are in de-grees from the horizontal, p
+ositive upwards.'
 PAUSE ' PRESS ENTER TO CONTINUE'




----------------------------------------------------------------------
---------------------------------------------------------------------------

  DATA INPUT
  DO  N = 1,3
    CALL   clearscreen($GCLEARSCREEN)
    WRITE (*, '(///,10X, A, I1)')  'BOREHOLE SET ', N
    WRITE (*, '( /,10X, A \)') 'ENTER BEARING (Degrees) -->   '
    READ (*,*)  BE(N)
    WRITE (*, '( /,10X, A \)') 'ENTER  INCLINATION ANGLE (Degrees)
+   -->  '
    READ (*, *) INCLI(N)


    WRITE (*, '(/,10X,A,\)')  'ENTER STRESS ONE   (SXX) (MPa) -->        '
    READ (*,*) BAI(3*N-2)
    WRITE (*, '(/,10X,A,\)')  'ENTER STRESS THREE (SYY) (MPa) -->        '
    READ (*,*) BAI(3*N-1)
    WRITE (*, '(/,10X,A,\)')  'ENTER SHEAR STRESS (TXY) (MPa) -->        '
    READ (*,*) BAI(3*N)
  ENDDO
---------------------------------------------------------------------------
-----------------------------------------------------------------------
  ECHO PRINT THE INPUT DATA

  CALL   clearscreen($GCLEARSCREEN)
  WRITE (*, '(/,16X,A40,/)')  '                 INPUT DATA
+         '
  WRITE (*, '(/,16X,3A13,/)') 'BOREHOLE 1', 'BOREHOLE 2',
 + 'BOREHOLE 3'
  WRITE (*, '(A16, 3F13.4)') '   BEARING     |', (BE(LL), LL=
 +1,3)
  WRITE (*, '(A16, 3F13.4)') '   INCLINATION |', (INCLI(LL),
 + LL = 1,3)
  WRITE (*, '(A16, 3F13.4)') '   STRESS 1    |', (BAI(LL),
 + LL=1,9,3)
  WRITE (*, '(A16, 3F13.4)') '   STRESS 3    |', (BAI(LL),
 + LL=2,9,3)
  WRITE (*, '(A16, 3F13.4),//)') '   SHEAR  13   |', (BAI(LL),
 + LL=3,9,3)
  PAUSE     '   '
------------------------------------------------------------------------
    WRITE (* , '(6(F8.3, 1X))')  ((J(L,JJ), JJ=1,6), L=1,9)

   DO N=1,3
     BE(N) = BE(N)/R1         ! CONVERT TO RADIANS BEARING
     INCLI(N) = INCLI(N)/R1  ! AND INCLINATION ANGLES
     CALCULATE DIRECTION COSINES
     DCOS(N,1) = COS(BE(N))                    ! L1
     DCOS(N,2) = SIN(BE(N))*COS(INCLI(N))      ! L2
     DCOS(N,3) = -SIN(BE(N))*SIN(INCLI(N))     ! L3
     DCOS(N,4) = -SIN(BE(N))                   ! M1
```

```
      DCOS(N,5) = COS(BE(N))*COS(INCLI(N))        ! M2
      DCOS(N,6) = -COS(BE(N))*SIN(INCLI(N))       ! M3
      DCOS(N,7) = 0.0                             ! N1
      DCOS(N,8) = SIN(INCLI(N))                   ! N2
      DCOS(N,9) = COS(INCLI(N))                   ! N3


      CALL  JCALC(DCOS, N, J) ! CALCULATE J ARRAY FACTORS
      BE(N) AND INCLIN(N) ARE ALTERED; THEY ARE IN RADIANS
    ENDDO


    CALL   clearscreen($GCLEARSCREEN)
    WRITE (*, *)  '  RETURN FROM SUB  JCALC, J ARRAY FACTORS'
    WRITE (* , '( 6(F10.6, 1X))')  ((J(L,JJ), JJ=1,6), L=1,9)
    PAUSE   '   '
------------------------------------------------------------------------
------------------------------------------------------------------------
    PREPARE SYSTEM OF EQUATIONS [A]x[B] = [G]

    CALL   clearscreen($GCLEARSCREEN)
    CALL SYST_EQ(BAI, J, A, G)
    CALL   clearscreen($GCLEARSCREEN)
    WRITE  (*,*)  '  RETURNED FROM SYST_EQ '
    WRITE (*, *)  '  A(6,12) ARRAY'
    WRITE (* , '( 12(F10.4,1X))')  ((A(L,JJ), JJ=1,12), L=1,6)
    WRITE (*, *)  '  G(6) ARRAY'
    WRITE (* , '( 6(F8.3, 1X))')  (G(JJ), JJ=1,6)
    PAUSE 'CALL ELIM NOW  '

    INVERT AUGMENTED MATRIX A(6,12)
    CALL  ELIM ( A, 6, 12, 6)
    RETURNING FROM SUBROUTINE ELIM MATRIX A CONTAINS ITS INVERSE
    MATRIX C[6X6] IN IT'S AUGMENTATION COLUMNS. HOWEVER THE
    ORIGINAL FIRST SIX COLUMNS OF A HAVE BEEN CORRUPTED.
    CALL   clearscreen($GCLEARSCREEN)
    PAUSE '  RETURNED FROM ELIM NOW AUGMENTED A(6,12) ARRAY'
    WRITE (* , '(12(F10.4, 1X))')  ((A(L,JJ), JJ=1,12), L=1,6)
    WRITE (*,*)  '  C(6,6) ARRAY IS THE AUGMENTED COLUMNS OF A(6,12)'
    WRITE (* , '(6(F10.4, 1X))')  ((A(L,JJ), JJ=7,12), L=1,6)
    PAUSE ' '
------------------------------------------------------------------------
--------------------------------------------------------------------------
    CALCULATE STRESS TENSOR COMPONENTS B(6) = [C][G]

    CALL  STR_CALC(A,G,B)
------------------------------------------------------------------------
--------------------------------------------------------------------------
    COMPUTE STATISTICAL PARAMETERS
    CALL   clearscreen($GCLEARSCREEN)
    CALL  STATS( BAI, A, B, G ,V)
--------------------------------------------------------------------------
------------------------------------------------------------------------
    PRINT RESULTS
    CALL   clearscreen($GCLEARSCREEN)
    WRITE (*, *) '       STRESS COMPONENTS        STANDARD DEVIATIONS '
    WRITE (*,'(/A,F9.3,5X,F9.3)')  '    NORMAL N-S = ', B(2), V(2)
    WRITE (*,'(/A,F9.3,5X,F9.3)')  '    NORMAL E-W = ', B(1), V(1)
    WRITE (*,'(/A,F9.3,5X,F9.3)')  '    NORMAL VER = ', B(3), V(3)
    WRITE (*,'(/A,F9.3,5X,F9.3)')  '    SHEAR  HOR = ', B(4), V(4)
    WRITE (*,'(/A,F9.3,5X,F9.3)')  '    SHEAR  N-V = ', B(5), V(5)
    WRITE (*,'(/A,F9.3,5X,F9.3)')  '    SHEAR  E-V = ', B(6), V(6)
    PAUSE  '  '
--------------------------------------------------------------------------
```

```
---------------------------------------------------------------------------
      CALCULATION OF PRINCIPAL STRESSES AND ORIENTATIONS
---------------------------------------------------------------------------
      S(1,1) = B(1)    ! Sxx
      S(2,1) = B(4)    ! Txy
      S(3,1) = B(6)    ! Tzx
      S(1,2) = B(4)    ! Txy
      S(2,2) = B(2)    ! Syy
      S(3,2) = B(5)    ! Tyz
      S(1,3) = B(6)    ! Tzx
      S(2,3) = B(5)    ! Tyz
      S(3,3) = B(3)    ! Szz

      CALL   clearscreen($GCLEARSCREEN)

      WRITE (*, *)  '   THIS IS S(3,3) ARRAY'
      WRITE (*, '(3F9.3)')  ((S(K,N), N=1,3), K=1,3)

      COMPUTE INVARIANTS OF THE STRESS TENSOR
      I1 = S(1,1) + S(2,2) + S(3,3)
      WRITE (*,'(A,F15.6)')  '  I1=', I1
      I2 = S(1,1)*S(3,3) + S(2,2)*S(3,3) + S(1,1)*S(2,2)
      WRITE (*,'(A,F15.6)')  '  I2=', I2
      I4 = S(2,3)**2 + S(1,2)**2 + S(1,3)**2
      WRITE (*,'(A,F15.6)')  '  I4=', I4
      I2 = I2 - I4
      WRITE (*,'(A,F15.6)')  '  I2=', I2
      I3 = S(1,1)*S(2,2)*S(3,3) +2*S(1,2)*S(2,3)*S(1,3) -
     +S(3,1)*S(2,2)*S(3,1)-S(3,2)*S(3,2)*S(1,1)-S(3,3)*S(2,1)*S(2,1)
      WRITE (*,'(A,F15.6)')  '  I3=', I3

      SOLVE CUBIC EQUATION BY TRIGONOMETRIC SUBSTITUTION
      Q1 = -I1
      Q2 =  I2
      Q3 = -I3
      Q4 = (3*Q2-Q1*Q1)/3
      WRITE (*,'(A,F15.6)')  '  Q4=', Q4
      Q5 = (2*Q1**3 - 9*Q1*Q2 + 27*Q3)/27
      WRITE (*,'(A,F15.6)')  '  Q5=', Q5
      Q6 = -Q5/2/SQRT(-Q4**3/27)
      WRITE (*,'(A, F12.6)')      ' Q6 =',  Q6
      Q7 = ACOS(Q6)/3
      WRITE (*, '(A, F12.6)')      ' Q7 =',  Q7
      X1 = 2*SQRT(-Q4/3)*COS(Q7)
      X2 = 2*SQRT(-Q4/3)*COS(Q7 + 120/R1)
      X3 = 2*SQRT(-Q4/3)*COS(Q7 + 240/R1)
      Z(1,1) = X1 - Q1/3
      Z(2,1) = X2 - Q1/3
      Z(3,1) = X3 - Q1/3

      WRITE (* , '(//,A, F14.6)')  '  I1=', I1
      WRITE (* , '(A, F14.6)')  '  I2=', I2
      WRITE (* , '(A, F14.6)')  '  I3=', I3
      WRITE (* , *)  '  ARRAY OF PRINCIPAL STRESSES'
      WRITE (*, '(F12.6)')  (Z(N,1), N=1,3)
      PAUSE ' '
      WRITE (*,*) ' THIS IS A CHECK'
      WRITE (*, '(A, F12.6)')      ' FIRST INVARIANT =',  Z(1,1)+Z(2,1)+
     +  Z(3,1)
      WRITE (* , '(A, F14.6)')  '  FIRST  CHECK =',  CHECK(Z(1,1))
      WRITE (* , '(A, F14.6)')  '  SECOND CHECK =',  CHECK(Z(2,1))
      WRITE (* , '(A, F14.6)')  '  THIRD  CHECK =',  CHECK(Z(3,1))
      PAUSE '  '
      END OF CUBIC SOLUTION
```

```
    SORT PRINCIPAL STRESSES
    Z1 = MAX (Z(1,1),Z(2,1),Z(3,1))
    Z3 = MIN (Z(1,1),Z(2,1),Z(3,1))
    Z(1,1) = Z1
    Z(2,1) = I1 -Z1-Z3
    Z(3,1) = Z3
    WRITE (* , *)  '  ARRAY OF PRINCIPAL STRESSES'
    WRITE (*, '(F12.6)')  (Z(N,1), N=1,3)
    PAUSE ' '
    END OF SORT
 -----------------------------------------------------------------------
 -----------------------------------------------------------------------
    COMPUTE DIRECTION COSINES FOR PRINCIPAL STRESSES
 -----------------------------------------------------------------------
    CALL   clearscreen($GCLEARSCREEN)
    DO 410  N=1,3
      Y = S  !
      WRITE (*, '(3F9.3)')  ((Y(K,NN), NN=1,3), K=1,3)
        DO 420 I=1,3
          Y(I,I) = Y(I,I)  -  Z(N,1)  ! [Y] = [S] - EIGENVALUE*[I]
        CONTINUE
      WRITE (*, *)  '   THIS IS Y(3,3) ARRAY'
      WRITE (*, '(3F9.3)')  ((Y(K,NN), NN=1,3), K=1,3)
      PAUSE ' '
      N3 = (Y(1,1)*Y(2,2)-Y(2,1)**2)/(Y(1,2)*Y(2,3)-Y(2,2)*Y(1,3))
      WRITE (*,'(A,F15.6)') '   N3=', N3
      N2 = -(Y(1,1) + Y(1,3)*N3)/Y(1,2)
      WRITE (*,'(A,F15.6)') '   N2=', N2
      N4 = SQRT(1+N2*N2 +N3*N3)
      WRITE (*,'(A,F15.6)') '   N4=', N4
      P(1,N) = 1/N4    ! N IS THE PRINCIPAL STRESS VECTOR LABEL
      WRITE (*,'(A,F15.6)') '   P(1,N)=', P(1,N)
      P(2,N) = N2/N4   ! DIRECTION COSINES
      WRITE (*,'(A,F15.6)') '   P(2,N)=', P(2,N)
      P(3,N) = N3/N4
      WRITE (*,'(A,F15.6)') '   P(3,N)=', P(3,N)
      CALL  DIRCOS(P,N,I5, B5)
      Z(N,2) = B5
      Z(N,3) = I5
      WRITE (*, '(A, F12.4)') '  BEARING =',      Z(N, 2)
      WRITE (*, '(A, F12.4)') ' INCLINATION = ',  Z(N,3)
      PAUSE  ' '
0   CONTINUE
    WRITE (*, *) '  PRINCIPAL STRESS        BEARING      INCLINATION'
    WRITE (*, '(F12.4,6X, 2F15.4)')  ((Z(N, K), K=1,3), N=1,3)
    PAUSE  ' '
-----------------------------------------------------------------------
-----------------------------------------------------------------------
    CALCULATION OF STANDARD DEVIATIONS OF PRINCIPAL STRESSES
-----------------------------------------------------------------------
    RE-COMPUTE DIRECTION COSINES OF THE THREE BOREHOLE SETS
    WITH RESPECT TO PRINCIPAL AXES.

    CALL  DC_IN_PS (DCOS, P, PR_DCOS)

    CALL   clearscreen($GCLEARSCREEN)
    WRITE (*, *)    '    DIRECTION COSINES OF BOREHOLE SETS'
    WRITE (*, *)    '    IN XYZ COORDINATE SYSTEM'
    WRITE (*, '(3F10.6)')  ((DCOS(N,JJ), N=1,3), JJ=1,9)
      PAUSE  '  '
    WRITE (*, *)    '    DIRECTION COSINES OF BOREHOLE SETS'
    WRITE (*, *)    '    IN 123 COORDINATE SYSTEM'
    WRITE (*, '(3F10.6)')  ((PR_DCOS(N,JJ), N=1,3), JJ=1,9)
```

```
     PAUSE  '  '
-----------------------------------------------------------------------------
-----------------------------------------------------------------------------
    CALCULATE J ARRAY FACTORS AGAIN
    DO N=1,3
      CALL  JCALC(PR_DCOS, N, J) ! CALCULATE J ARRAY FACTORS
    ENDDO
-----------------------------------------------------------------------------
-----------------------------------------------------------------------------
    PREPARE SYSTEM OF EQUATIONS  [A]x[B] =[G]
    CLEAN UP A, G, B AND V  ARRAYS
    A = 0.0
    G = 0.0
    B = 0.0
    V = 0.0
    CREATE UNIT MATRIX IN AUGMENTATION COLUMNS OF ARRAY A FOR
    MATRIX INVERSION
    DO W=1,6
       A(W, W+6) = 1.0
    ENDDO

    CALL SYST_EQ(BAI, J, A, G)
    WRITE (*,'(12(F12.4,1X))')  ((A(R,F), F=1,12),R=1,6)
    PAUSE '  A ARRAY'

    INVERT AUGMENTED MATRIX A[6,12]
    CALL  ELIM ( A, 6, 12, 6)
    CALL    clearscreen($GCLEARSCREEN)
    PAUSE '  RETURNED FROM ELIM NOW AUGMENTED A(6,12) ARRAY'
    WRITE (* , '(12(F10.5, 1X))')  ((A(L,JJ), JJ=1,12), L=1,6)
    WRITE (*,*)  '  C(6,6) ARRAY IS THE AUGMENTED COLUMNS OF A(6,12)'
    WRITE (* , '(6(F12.5, 1X))')  ((A(L,JJ), JJ=7,12), L=1,6)
    PAUSE ' '

    CALCULATE STRESS TENSOR COMPONENTS B(6) = [C][G]
    CALL  STR_CALC(A,G,B)
    WRITE (*,'(F12.4)')  (B(F), F=1,6)

    COMPUTE STATISTICAL PARAMETERS
    CALL   clearscreen($GCLEARSCREEN)
    CALL  STATS( BAI, A, B, G ,V)



-----------------------------------------------------------------------------
-----------------------------------------------------------------------------


    STOP '  '
    END    ! OF MAIN
-------------------------------------------------------------------------

-------------------------------------------------------------------------
    SUBROUTINE JCALC(DIRC, L, K)

    THIS SUBROUTINE CALCULATES THE J ARRAY FACTORS FOR THE
    LEAST SQUARES FIT

    INPUT ARGUMENTS

    REAL *8 DIRC(3,9) ! IS THE MATRIX OF DIRECTION COSINES OF THE
    ! BOREHOLES; DCOS IS THE MATRIX IN THE MAIN PROGRAM
    OUTPUT ARGUMENTS
```

```
      REAL*8 K(9,6)  ! IS THE J FACTOR  ARRAY IN THE MAIN PROGRAM

      LOCAL VARIABLES

      INTEGER  L ! L IS THE BOREHOLE SET COUNTER L=1,3
      C IS THE ROW OF THE ARRAY
      INTEGER C !  ROW COUNTER
      DATA C /1/
      SAVE C       NOT NECESSARY.  STATIC BY DEFAULT

      SUBROUTINE BODY

      CALCULATE J ARRAY FACTORS
      EQUATIONS USED  ARE DERIVED FROM PANEK 13-15-16 PAGE 8
      STRESS1=SX*L1^2+SY*M1^2+SZ*N1^2+2*TXY*L1*M1+2TYZ*M1*N1+
      2*TZX*N1*L1 EQ13
      K(C,1) = DIRC(L,1)*DIRC(L,1)    ! COS(B)*COS(B)  B:bearing
      K(C,2) = DIRC(L,4)*DIRC(L,4)    ! SIN(B)*SIN(B)  I:inclination
      K(C,3) = DIRC(L,7)*DIRC(L,7)    ! 0.0
      K(C,4) = 2*DIRC(L,1)*DIRC(L,4)  ! -2*COS(B)*SIN(B)
      K(C,5) = 2*DIRC(L,4)*DIRC(L,7)  ! 0.0
      K(C,6) = 2*DIRC(L,7)*DIRC(L,1)  ! 0.0
      STRESS3=SX*L3^2+SY*M3^2+SZ*N3^2+2*TXY*L3*M3+2TYZ*M3*N3+
      2*TZX*N3*L3 EQ15
      C=C+1
      K(C,1) = DIRC(L,3)*DIRC(L,3)    ! [SIN(B)*SIN(I)]^2
      K(C,2) = DIRC(L,6)*DIRC(L,6)    ! [COS(B)*SIN(I)]^2
      K(C,3) = DIRC(L,9)*DIRC(L,9)    ! COS(I)^2
      K(C,4) = 2*DIRC(L,3)*DIRC(L,6)  ! 2*[-SIN(B)*SIN(I)]*[-COS(B)*SIN(I)]
      K(C,5) = 2*DIRC(L,6)*DIRC(L,9)  ! 2*[-COS(B)*SIN(I)]*COS(I)
      K(C,6) = 2*DIRC(L,9)*DIRC(L,3)  ! 2*COS(I)*[-SIN(B)*SIN(I)]
 3    SHEAR13= SX*L1*L3 + SY*M1*M3 + SZ*N1*N3 + TXY*(L1*M3+L3*M1) +
      TYZ*(M1*N3+M3*N1) + TZX*(N1*L3 + N3*L1)  EQ 15
      C = C+1
      K(C,1) = DIRC(L,1)*DIRC(L,3)    ! COS(B)*[-SIN(B)*SIN(I)]
      K(C,2) = DIRC(L,4)*DIRC(L,6)    ! -SIN(B)*[-COS(B)*SIN(I)]
      K(C,3) = DIRC(L,7)*DIRC(L,9)    !  0.
      K(C,4) = DIRC(L,1)*DIRC(L,6) + DIRC(L,3)*DIRC(L,4)
      !    COS(B)*[-COS(B)*SIN(I)]+ [-SIN(B)*SIN(I)]*[-SIN(B)]
      K(C,5) = DIRC(L,4)*DIRC(L,9) + DIRC(L,6)*DIRC(L,7)
      !    -SIN(B)*COS(I)+ 0.
      K(C,6) = DIRC(L,7)*DIRC(L,3) + DIRC(L,9)*DIRC(L,1)
      ! 0. + COS(I)*COS(B)
      C=C+1

      IF (C .EQ. 10) C=1

      WRITE (*, *)   '   TO EXIT SUB  JCALC'
      WRITE (*, '(A,I2)')   '      C =', C
      PAUSE ' '
       WRITE (* , '( 6(F8.3, 1X))')  ((K(LL,JJ), JJ=1,6), LL=1,9)
       PAUSE

      RETURN
      END           ! OF SUBROUTINE  JCALC
---------------------------------------------------------------------

---------------------------------------------------------------------
      SUBROUTINE SYST_EQ(SYBAI, SYJ, SYA, SYG)

      PREPARES SYSTEM OF EQUATIONS [A]x[B] = [G]

      INPUT ARGUMENTS
      REAL*8 SYBAI(9) ! MATRIX OF BACK ANALYSIS INPUT
```

```
     REAL*8 SYJ(9,6) ! ARRAY OF J COMPONENTS

     OUTPUT ARGUMENTS
     REAL*8 SYA(6,12) ! A[6,12] ARRAY
     REAL*8 SYG(6)    ! G[6] MATRIX
     FUNCTION BODY

     CALL   clearscreen($GCLEARSCREEN)
     WRITE (*,*)   '   INSIDE SYST_EQ'
     WRITE (*, *)  '  A(6,12) ARRAY   INITIALLY'
     WRITE (* , '( 12(F6.3))')  ((SYA(L,JJ), JJ=1,12), L=1,6)
     WRITE (*, *)  '  G(6) ARRAY      INITIALLY'
     WRITE (* , '(6(F8.3, 1X))')  (SYG(JJ), JJ=1,6)
     PAUSE  '   '
     COMPUTE [A] AND [G] ARRAYS
     DO 130 K=1,9
       DO 120 JJ=1,6
           SYG(JJ) = SYG(JJ) + SYBAI(K)*SYJ(K,JJ)
             DO 110 I=1,6
                SYA(I,JJ)=SYA(I,JJ)+SYJ(K,I)*SYJ(K,JJ)
0            CONTINUE
0      CONTINUE
0    CONTINUE
     RETURN
     END
     END OF SUBROUTINE  SYST_EQ

---------------------------------------------------------------------------
---------------------------------------------------------------------------
     INCLUDE 'ELIMROUT.FOR'
     FILE INCLUDING THE MATRIX INVERSION SUBROUTINE
---------------------------------------------------------------------------
--------------------------------------------------------------------------
     SUBROUTINE STR_CALC(STA, STG, STB)

     CALCULATE STRESS TENSOR COMPONENTS B(6) = [C][G]
     C ARRAY DOES NOT APPEAR EXPLICITLY. IT IS CONTAINED IN A(6,12)

     INPUT ARGUMENTS
     REAL*8  STA(6,12) ! A[6,12] ARRAY
     REAL*8  STG(6)    ! G[6] MATRIX

     OUTPUT ARGUMENTS
     REAL*8  STB(6)    ! MATRIX OF STRESS COMPONENTS

     FUNCTION BODY

     CALL   clearscreen($GCLEARSCREEN)
     WRITE (*, *)  '  STRESS CALCULATION FOLLOWS INSIDE STR_CALC'
     DO 210 I=1,6
        DO 220 JJ=1,6
           STB(I) = STB(I) + STA(I,JJ+6)*STG(JJ)
0       CONTINUE
        WRITE (*, '(A,I1,A, F9.3)')  '  B(',I, ')=', STB(I)
0    CONTINUE
        PAUSE  '  '
     RETURN   '
     END
     END OF SUBROUTINE STR_CALC
--------------------------------------------------------------------------
--------------------------------------------------------------------------
     SUBROUTINE STATS(SSBAI, SSA, SSB, SSG , SSV)

     COMPUTE STATISTICAL PARAMETERS OF LEAST SQUARES FIT.
```

```
     STANDARD DEVIATIONS OF STRESS TENSOR COMPONENTS

     INPUT ARGUMENTS

     REAL*8  SSA(6,12) ! A[6,12] ARRAY
     REAL*8  SSBAI(9)  ! BACK ANALYSIS INPUT MATRIX
     REAL*8  SSB(6)    ! STRESS COMPONENT MATRIX
     REAL*8  SSG(6)    ! G MATRIX

     OUTPUT ARGUMENTS

     REAL*8  SSV(6)    ! STANDARD DEVIATIONS OF STRESS COMPONENTS

     LOCAL VARIABLES
     REAL*8 S1, S2, S3, S4, R2, R
     AUTOMATIC S1, S2, S3, S4, R2, R ! THE ROUTINE IS CALLED TWICE
     ! THEREFORE THE VARIABLES SHOULD NOT BE STATIC; THE DEFAULT

     SUBROUTINE BODY

     CALL   clearscreen($GCLEARSCREEN)
     DO 310 K=1,9
        S1 = S1 + SSBAI(K) * SSBAI(K)
0    CONTINUE
     S2 = SSB(1)*SSG(1) + SSB(2)*SSG(2) +SSB(3)*SSG(3) +
    + SSB(4)*SSG(4) + SSB(5)*SSG(5) + SSB(6)*SSG(6)
     S3 = S1 - S2
     R2 = 1 - S3/S1
     R = SQRT(R2)
     S4 = SQRT( S3/(9-6) )
     WRITE (*,'(//,A,/)') '             STATISTICAL PARAMETERS'
     WRITE (*,*) '   1) INPUT FROM BACK ANALYSIS              '
     WRITE (*,'(A,F18.5)') '      STRESSES SUM OF SQUARES            =
    +' ,  S1
     WRITE (*,'(A,F18.5)') '   2) [B].[G]                            =
    +' ,  S2
     WRITE (*,'(A,F18.5)') '   3) RESIDUAL SUM OF SQUARES            =
    +' ,  S3
     WRITE (*,'(A,F18.5)') '   4) STANDARD DEVIATION OF FITTED DATA  =
    +' ,  S4
     WRITE (*,'(A,F18.5)') '   5) GOODNESS OF FIT (R**2)             =
    +' ,  R2
     WRITE (*,'(A,F18.5)') '   6) CORRELATION COEFFICIENT            =
    +' ,  R
     COMPUTE STANDARD DEVIATIONS OF STRESS COMPONENTS
     WRITE (*, '(/////)')
     DO 320 I=1,6
     SSV(I) = S4*SQRT( SSA(I,I+6) )
     THE INVERSE OF A(6,12) C(6,6) IS IN THE AUGMENTATION COLUMNS OF A
     WRITE (*,'(A, 2I2,A, F9.3,5X,A,I1,A,F9.3)')   '  C(',I,I,')=',
    + SSA(I,I+6), 'V(',I,')=', SSV(I)
20   CONTINUE
     PAUSE ' '
     RETURN
     END
     END OF SUBROUTINE STATS
---------------------------------------------------------------------------
---------------------------------------------------------------------------
     SUBROUTINE DIRCOS(SP,SN,I5, B5)

     THIS SUBROUTINE CALCULATES THE BEARING AND INCLINATION ANGLES OF
     THE PRINCIPAL STRESS VECTORS

     PARAMETER (R1=57.29578)  ! CONVERT TO RADIANS
```

```
     INPUT ARGUMENTS

     INTEGER  SN
     REAL*8  SP
     DIMENSION  SP(3,3) ! ARRAY OF DIRECTION COSINES, INPUT ARGUMENT
     SP(1,SN) SP(2,SN) SP(3,SN): SN IS THE STRESS VECTOR LABEL i.e.
     SP(1,3) IS THE DIRECTION COSINE WITH THE X AXIS OF THE THIRD
     PRINCIPAL STRESS VECTOR.

     OUTPUT ARGUMENTS

     REAL*8 I5,B5 ! B5 BEARING, I5 INCLINATION ANGLES

     FUNCTION BODY

     I5 = 90 -ACOS(SP(3,SN))*R1
     IF  (SP(1,SN) .NE. 0.0)   GOTO 3040
     IF  (SP(2,SN) .GT. 0.0)   GOTO 3000
     IF  (SP(2,SN) .LT. 0.0)   GOTO 3020
     WRITE (*, *) '   IF THE TWO FIRST DIRECION COSINES ARE ZERO '
     WRITE (*, *) '   THEN TAN(BEARING OF PRINCIPAL STRESS)= 0/0. '
     WRITE (*, *) '   SEE PAGE 22 IN PANEK FORMULA (58) '
     GOTO 3150
 00  B5 = 0
     GO TO 3150
 20  B5 = 180
     GO TO 3150
 40  B5 = ATAN(SP(1,SN)/SP(2,SN))*R1
     IF (SP(2,SN) .LT. 0.0)   GOTO 3090
     IF (SP(1,SN) .GT. 0.0)   GOTO 3100
     B5 = 360 + B5
     GOTO 3100
 90  B5 = 180 + B5
     INCLINATION OF PRINCIPAL STRESSES ARE NEGATIVE BY CONVENTION
     AND BEARINGS ARE ADJUSTED ACCORDINGLY IF NECESSARY
 00  IF (I5 .LT. 0.0)  GOTO 3130
     I5 = -I5
     B5 = 180 +B5
 30  IF (B5 .LT. 360)  GOTO 3150
     B5 = B5 -360
 50  CONTINUE
     RETURN
     END
     END OF ROUTINE DIRCOS
--------------------------------------------------------------------------------
-----------------------------------------------------------------------------------
     SUBROUTINE DC_IN_PS (D_C, PC, PR_D)

     RECOMPUTE DIRECTION COSINES OF THREE BOREHOLE SETS WITH RESPECT
     TO PRINCIPAL AXES TO COMPUTE STANDARD DEVIATIONS OF PRINCIPAL
     STRESSES.

     INPUT ARGUMENTS

     REAL*8 D_C(3,9) ! ARRAY OF DIRECTION COSINES OF THREE BOREHOLES
     ! WITH RESPECT TO XYZ COORDINATE SYSTEM; DCOS IN MAIN
     REAL*8 PC(3,3) ! ARRAY OF DIRECTION COSINES OF PRINCIPAL STRESSES
     ! WITH RESPECT TO XYZ COORDINATE SYSTEM; P IN MAIN

     OUTPUT ARGUMENTS

     REAL*8 PR_D(3,9) ! ARRAY OF DIRECTION COSINES OF THREE
     ! BOREHOLES  WITH RESPECT TO 123 COORDINATE SYSTEM; PR_DCOS IN MAIN
```

```
      LOCAL VARIABLES
      INTEGER L ! L=1,3 COUNTER

      SUBROUTINE BODY

      DO L=1,3
        ! L1
        PR_D(L,1)= D_C(L,1)*PC(1,1)+ D_C(L,4)*PC(2,1)+ D_C(L,7)*PC(3,1)
        ! M1
        PR_D(L,4)= D_C(L,1)*PC(1,2)+ D_C(L,4)*PC(2,2)+ D_C(L,7)*PC(3,2)
        ! N1
        PR_D(L,7)= D_C(L,1)*PC(1,3)+ D_C(L,4)*PC(2,3)+ D_C(L,7)*PC(3,3)

        ! L2
        PR_D(L,2)= D_C(L,2)*PC(1,1)+ D_C(L,5)*PC(2,1)+ D_C(L,8)*PC(3,1)
        ! M2
        PR_D(L,5)= D_C(L,2)*PC(1,2)+ D_C(L,5)*PC(2,2)+ D_C(L,8)*PC(3,2)
        ! N2
        PR_D(L,8)= D_C(L,2)*PC(1,3)+ D_C(L,5)*PC(2,3)+ D_C(L,8)*PC(3,3)

        ! L3
        PR_D(L,3)= D_C(L,3)*PC(1,1)+ D_C(L,6)*PC(2,1)+ D_C(L,9)*PC(3,1)
        ! M3
        PR_D(L,6)= D_C(L,3)*PC(1,2)+ D_C(L,6)*PC(2,2)+ D_C(L,9)*PC(3,2)
        ! N3
        PR_D(L,9)= D_C(L,3)*PC(1,3)+ D_C(L,6)*PC(2,3)+ D_C(L,9)*PC(3,3)
      ENDDO
      RETURN
      END
      END OF SUBROUTINE  DC_IN_PS
-----------------------------------------------------------------------
--------------------------------------------------------------------------
```

```
      SUBROUTINE ELIM(AB, N, NP, NDIM)
      DIMENSION AB(6,12)     !NP
      REAL*8 AB
      INTEGER N,NP,NDIM
       WRITE (*, *)   '   ENTERED SUBROUTINE ELIM NOW'
       WRITE (* ,23) ((AB(I,J), J=1, NP ), I=1, NDIM)
23     FORMAT ((12(F8.2), /))
       PAUSE  ' MATRIX INVERSION STARTS'

 THIS SUBROUTINE SOLVES A SET OF LINEAR EQUATIONS.
 THE GAUSS ELIMINATION METHOD IS USED, WITH PARTIAL PIVOTING.
 MULTIPLE RIGHT HAND SIDES ARE PERMITTED, THEY SHOULD BE SUPPLIED
 AS COLUMNS THAT AUGMENT THE COEFFICIENT MATRIX.
 PARAMETERS ARE -
    AB   COEFFICIENT MATRIX AUGMENTED WITH R.H.S VECTORS
    N    NUMBER OF EQUATIONS
    NP   TOTAL NUMBER OF COLUMNS IN THE AUGMENTED MATRIX.
    NDIM FIRST DIMENSION OF MATRIX AB IN THE CALLING PROGRAM.
 THE SOLUTION VECTOR(S) ARE RETURNED IN THE AUGMENTATION
 COLUMNS OF AB.

 BEGIN THE REDUCTION
```

```
    NM1 = N-1
    DO 35 I = 1,NM1
FIND THE ROW NUMBER OF THE PIVOT ROW. WE WILL THEN
INTERCHANGE ROWS TO PUT THE PIVOT ELEMENT ON THE DIAGONAL.
    IPVT = I
    IP1 =I+1
    DO 10 J = IP1 , N
      IF (ABS(AB(IPVT,I)) .LT. ABS(AB(J,I))) IPVT = J
    CONTINUE
CHECK TO BE SURE THE PIVOT ELEMENT IS NOT TOO SMALL, IF SO
PRINT A MESSAGE AND RETURN.
    IF ((ABS(AB(IPVT,I)) .LT. 1.E-5)) GO TO 99
NOW INTERCHANGE, EXCEPT IF THE PIVOT ELEMENT IS ALREADY ON
THE DIAGONAL. DON'T NEED TO.
    IF (IPVT .EQ. I)  GO TO 25
    DO 20 JCOL = I, NP
    SAVE = AB(I ,JCOL)
    AB(I,JCOL) = AB(IPVT, JCOL)
    AB(IPVT,JCOL) = SAVE
    CONTINUE
NOW REDUCE ALL ELEMENTS BELOW THE DIAGONAL IN THE I-TH ROW. CHECK
FIRST TO SEE IF A ZERO ALREADY PRESENT. IF SO,
CAN SKIP REDUCTION FOR THAT ROW.
      DO 32 JROW = IP1,N
        IF (AB(JROW,I) .EQ. 0) GO TO 32
        RATIO = AB(JROW, I)/AB(I,I)
        DO 30 KCOL = IP1, NP
          AB(JROW,KCOL) = AB(JROW,KCOL) - RATIO*AB(I,KCOL)
        CONTINUE
      CONTINUE
    CONTINUE
WE STILL NEED TO CHECK A(N,N) FOR SIZE.
    IF (ABS(AB(N,N)) .LT. 1.E-5) GO TO 99
NOW WE BACK SUBSTITUTE
    NP1 = N+1
    DO 50 KCOL = NP1, NP
      AB(N, KCOL) = AB(N,KCOL)/AB(N,N)
      DO 45 J=2,N
        NVBL = NP1 - J
        L = NVBL + 1
        VALUE = AB(NVBL, KCOL)
        DO 40 K = L,N
          VALUE = VALUE - AB(NVBL,K)*AB(K,KCOL)
        CONTINUE
        AB(NVBL, KCOL) = VALUE/AB(NVBL, NVBL)
      CONTINUE
    CONTINUE
    RETURN
MESSAGE FOR A NEAR SINGULAR MATRIX
    WRITE (*, 100)
0   FORMAT(1H0, 'SOLUTION NOT FEASIBLE. A NEAR ZERO PIVOT WAS ENCOUN
   +TERED.')
    RETURN
    END
```