

A homogeneity test for spatial line patterns

Renate Thiede ^{a,*}, Inger Fabris-Rotelli ^a, Pravesh Debba ^b, Christopher W. Cleghorn ^c

^a Department of Statistics, University of Pretoria, Corner of Roper and Lynnwood Roads, Pretoria, South Africa

^b Council of Scientific and Industrial Research, CSIR Main Site, Meiring Naude Road, Pretoria, South Africa

^c School of Computer Science and Applied Mathematics, University of the Witwatersrand, Corner of Bertha and Jorissen Streets, Braamfontein, Johannesburg, South Africa

ARTICLE INFO

Keywords:

Line patterns
Homogeneity
Road networks
Hypothesis tests

ABSTRACT

Spatial linear networks exhibit a variety of patterns. Like spatial point patterns, they can be homogeneous or heterogeneous. Although there exist a wide variety of tests for the homogeneity of point patterns, no statistical tests currently exist to quantify the homogeneity of spatial linear networks. This research provides two statistical methodologies to test for homogeneity in spatial linear networks, using point pattern methods. The first methodology approximates spatial linear networks by point patterns, obtained by taking the midpoint of each line. The second methodology projects each line of a spatial linear network into a space defined by the distance from the origin and orientation of the line, thus representing lines as three dimensional points. In both methodologies, existing tests for homogeneity of point patterns are then applied to the point pattern representations of the linear networks. The methodologies are applied to test for homogeneity of formal and informal road networks in South Africa. This research is in line with UN Sustainable Development Goals 3, 9 and 10.

Video to this article can be found online at <https://doi.org/10.1016/j.sctalk.2024.100296>.

* Corresponding author.

E-mail address: renate.thiede@up.ac.za (R. Thiede).

Figures and tables



Fig. 1. Satellite imagery of formal and informal roads in Mamelodi township, South Africa. The formal roads in the upper part of the image have a more planned structure with larger gaps between roads. The informal roads in the lower part of the image have a less planned, more organic structure and smaller spaces between roads.

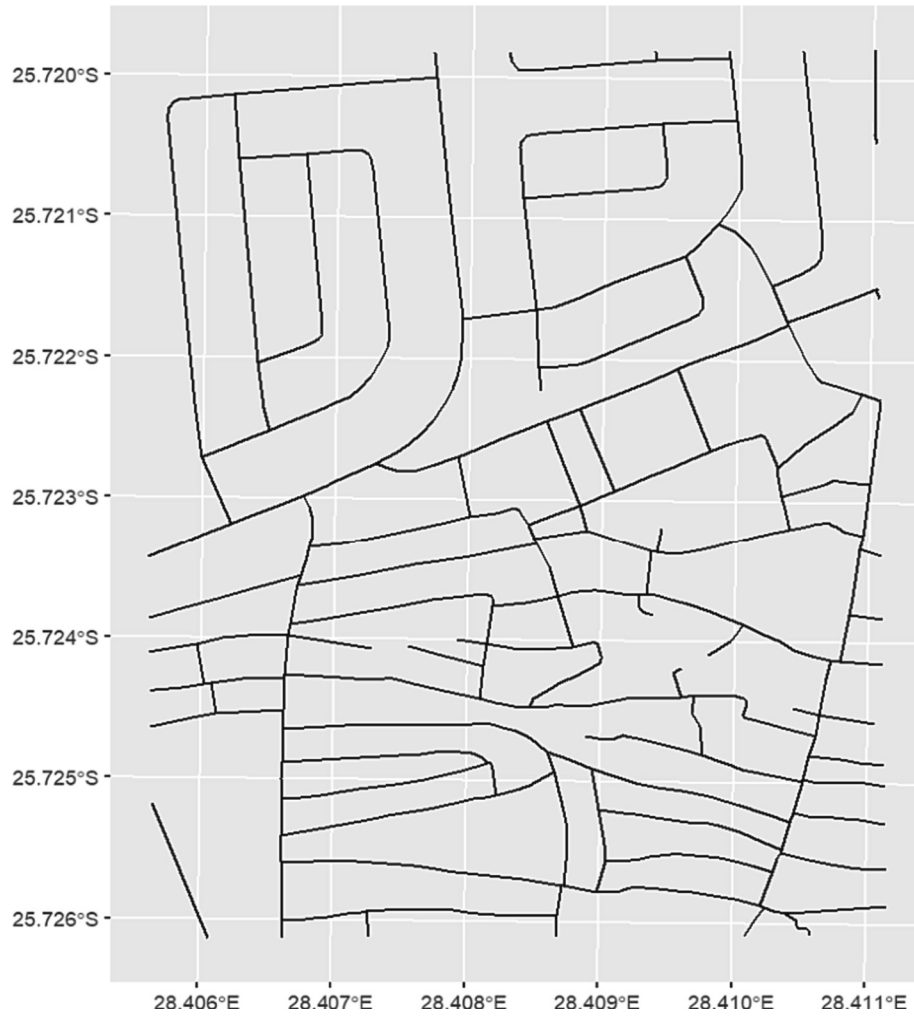


Fig. 2. The formal and informal roads of Fig. 1 represented as a line pattern.

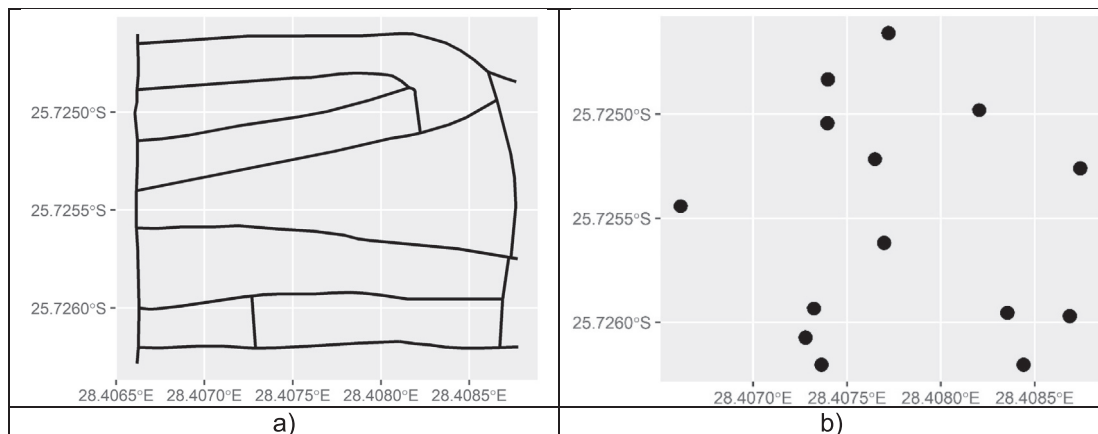


Fig. 3. A road network as a spatial line pattern (a) with its midpoint representation in b).

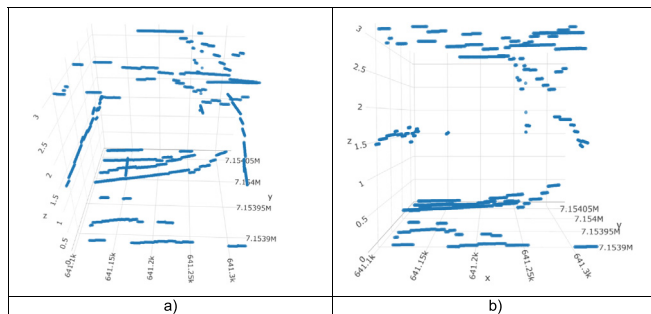


Fig. 4. 3D point pattern representation of the line pattern in Fig. 4. The images in a) and b) show two different angles of the 3D representation.

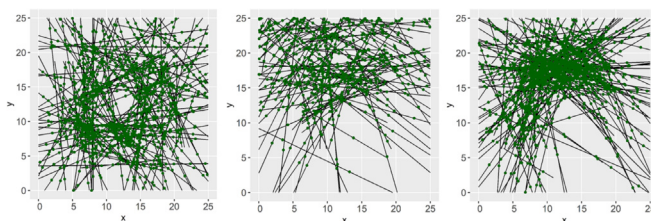


Fig. 5. Examples of second-order non-stationary line patterns.

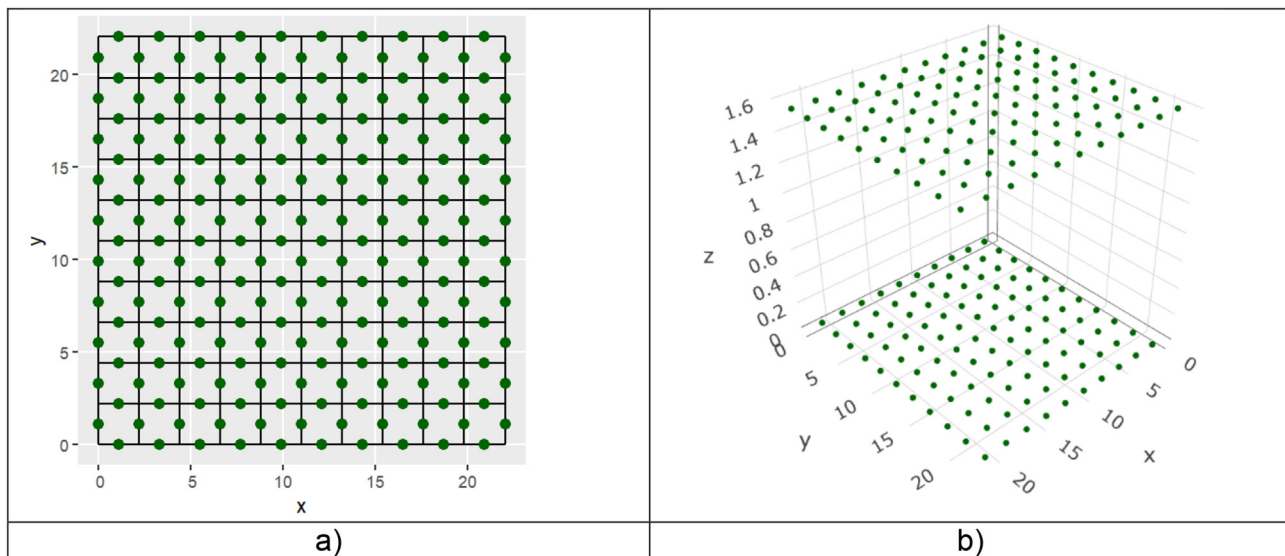


Fig. 6. An example of a first-order stationary, second-order non-stationary line pattern. The line pattern with its 2D point representation is shown in a). b) The 3D point representation.

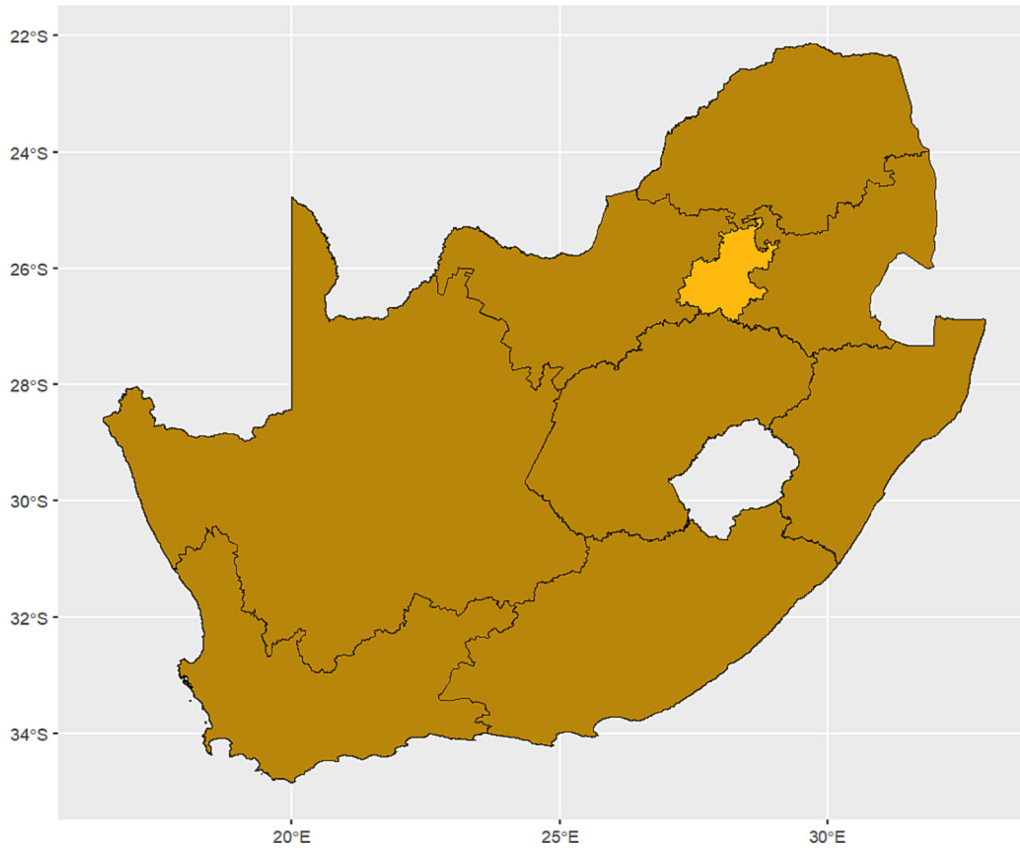


Fig. 7. A map of South Africa with the Gauteng province highlighted. The study area for the application is contained within Gauteng province, the country's central economic region, with a growing immigrant and local population.

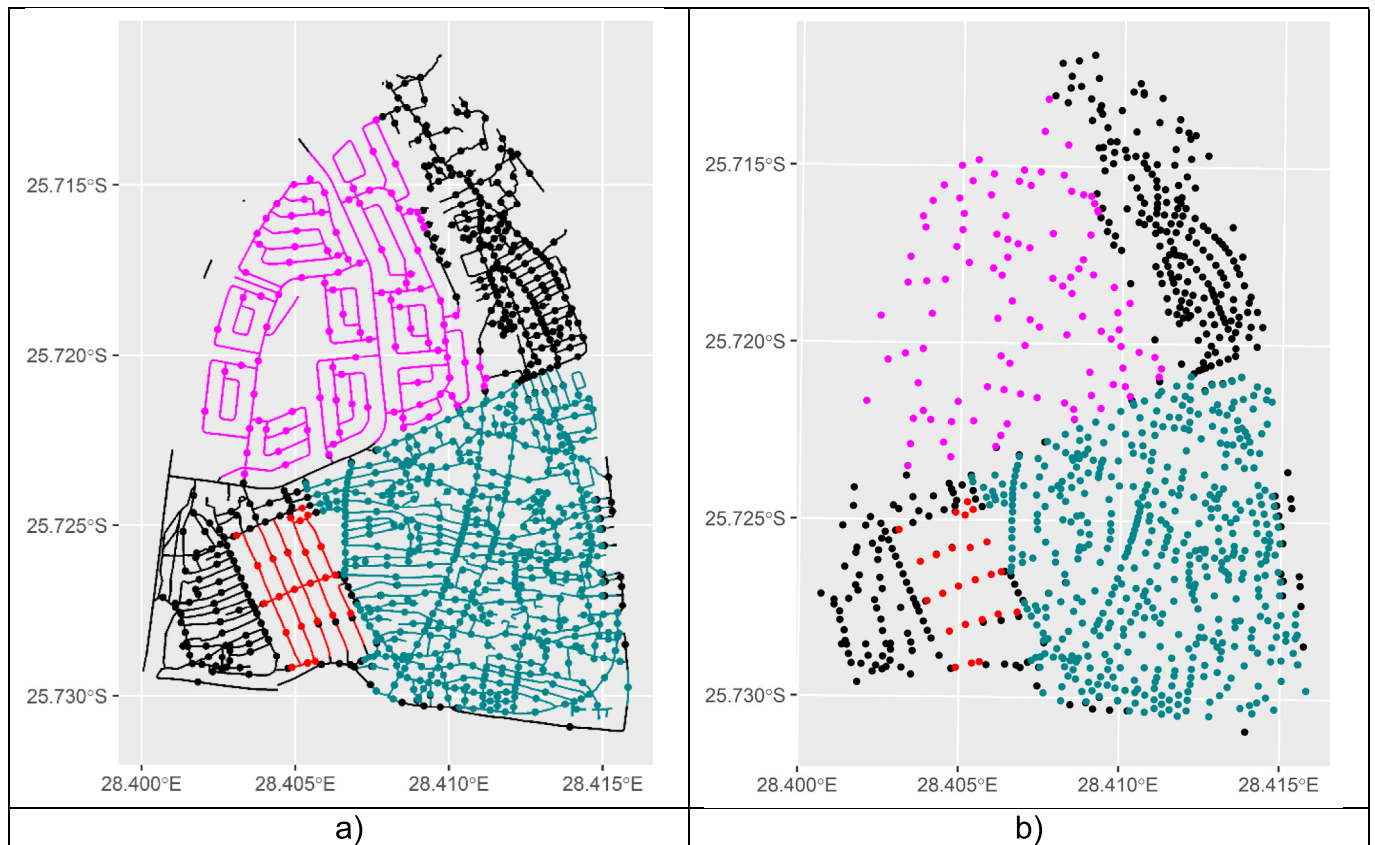


Fig. 8. The study area within Mamelodi, South Africa. Different colours denote different sub-networks on which the homogeneity hypothesis will be tested. a) Roads in the study area. The pink roads represent formal roads with an organic structure. The red roads are formal roads with a grid-like structure. The green roads are informal roads, and are generally shorter and closer together than formal roads. b) The 2D point representation of the roads.

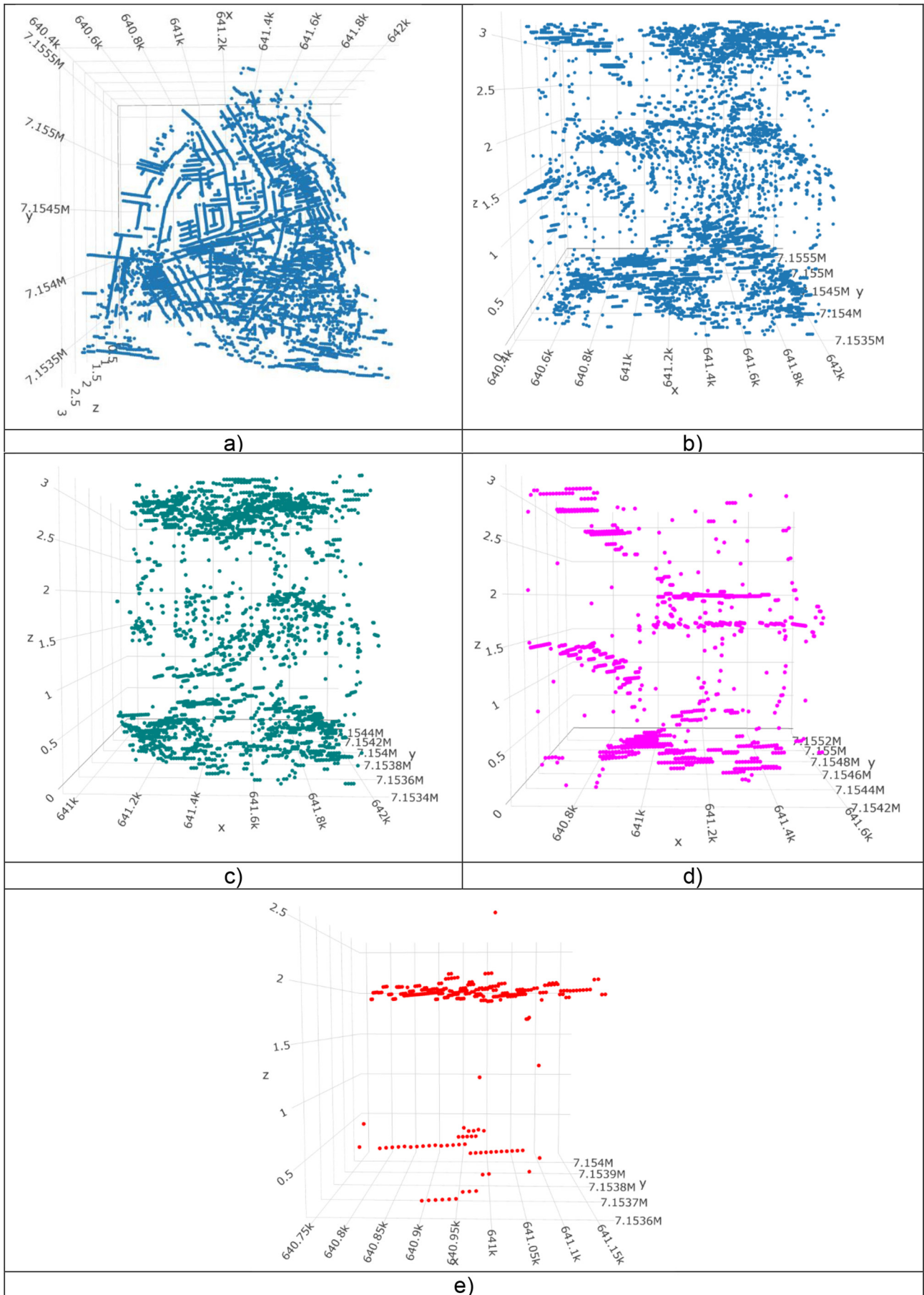


Fig. 9. 3D point representations of the road networks in Fig. 8. a)–b) Two angles of the 3D point representation of the overall road network. c)–e) 3D point representations of the informal roads, organic formal roads, and grid-like formal roads respectively.

Table 1
p-values of the homogeneity tests conducted on the roads within the Mamelodi study area.

Area	# quadrats	χ^2 test p-value (2D)
Overall area	13 × 13	0.001
Formal, organic roads	4 × 4	0.203
Informal roads	9 × 9	0.588
Formal, grid-like roads	2 × 2	0.574

CRedit authorship contribution statement

Renate Thiede: Conceptualization, Methodology, Software, Writing – original draft. **Inger Fabris-Rotelli:** Conceptualization, Methodology, Writing – review & editing, Supervision. **Pravesh Debba:** Supervision, Writing – review & editing. **Christopher W. Cleghorn:** Supervision, Writing – review & editing.

Data availability

The data is publicly available from OpenStreetMap and StatsSA.

Acknowledgments

Funding

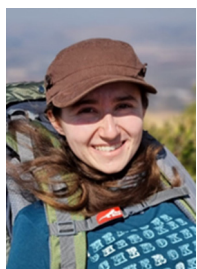
This work is based on research supported by the National Research Foundation of South Africa under Grant 137785 and CoE-MaSS ref. #2022–018-MAC-Road; and an NRF-SASA Academic Statistics Bursary. Opinions expressed and conclusions arrived at are those of the author and are not necessarily to be attributed to the NRF.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Further reading

- [1] C. Kraamwinkel, I. Fabris-Rotelli, A. Stein, Bootstrap testing for first-order stationarity on irregular windows in spatial point patterns, *Spatial Statistics* 28 (2018) 194–215.
- [2] Kunene, Spatial Dependency between a Linear Network and a Point Pattern, Masters Dissertation University Of Pretoria, 2020.
- [3] R.A.A. Nobrega, C.G. O'Hara, J.A. Quintanilha, Detecting roads in informal settlements surrounding Sao Paulo City by using object-based classification, *Proceedings Of The 1st International Conference On Object-based Image Analysis (OBIA 2006)*, Salzburg, Austria 2006, July, pp. 4–5.
- [4] R.F. Potthoff, M. Whittinghill, Testing for homogeneity: II. The Poisson distribution, *Biometrika* 53 (1/2) (1966) 183–190.
- [5] R.G. Ramos, B.F. Silva, K.C. Clarke, M. Prates, Too fine to be good? Issues of granularity, uniformity and error in spatial crime analysis, *J. Quant. Criminol.* 37 (2) (2021) 419–443.
- [6] L. Van Der Meer, L. Abad, A. Gilardi, R. Lovelace, Sfnetworks: Tidy Geospatial Networks, URL: <https://CRAN.R-project.org/package=sfnetworks> 2021 R Package Version 0.5.2.
- [7] A.S. Wigley, N. Tejedor-Garavito, V. Alegana, A. Carioli, C.W. Ruktanonchai, C. Pezzulo, Z. Matthews, A.J. Tatem, K. Nilsen, Measuring the availability and geographical accessibility of maternal health services across sub-Saharan Africa, *BMC Med.* 18 (1) (2020) 1–10.



Renate Thiede is a PhD candidate and lecturer in the Department of Statistics at the University of Pretoria, South Africa. She obtained her MSc in Mathematical Statistics from the University of Pretoria in 2019. Her research interests include spatial statistics, road network modelling and geoinformatics. She seeks to build a better future by applying rigorous statistical research to address real-world societal and environmental problems.



Prof Inger Fabris-Rotelli is currently an associate professor in the Department of Statistics at the University of Pretoria. She holds a PhD Mathematical Sciences (2013). Her research interests are in spatial statistics and GIS, as well as remote sensing and general image processing. She served on the executive of the South African Statistical Association (SASA) from 2012 until 2018, and as a director on the ICCSSA (Institute of Certificated and Chartered Statisticians in South Africa) board from 2019. She is the current president of SASA and the CEO of ICCSSA. Among others she is also a member of ISI, ISPRS and IMS internationally, and the Golden Key Society, SASA, S2A3, SAMS, ICCSSA (registered as a Chartered Statistician from 2019 - 2023). She is a SACNASP council member for the period 2021 to 2025, is a Registered Professional Natural Scientist with SACNASP, and has a National Research Foundation Y2 rating in recognition of her research. She is an Abe Bailey Fellow (2007 tour award), and a 2018 fellow of the TUKS Young Researcher Leadership Program (TYRLP). She was selected as a BRICS Young Scientist 2020 (in Artificial Intelligence).



Professor Pravesh Debba completed an MSc degree in Biostatistics at Hasselt University in Belgium in 1998. He completed his PhD in Spatial Statistics in 2006 from the International Institute for Geo-Information Science and Earth Observation ITC and Wageningen University in the Netherlands. Professor Debba is currently the manager for Inclusive Smart Settlements and Regions and Acting Centre Manager for the Energy Centre within Smart Places at the Council for Scientific and Industrial Research (CSIR). Professor Debba has been the president of the South African Statistical Association (SASA) from 2010–2011 and Chairperson of the Institute for Certificated and Chartered Statisticians of South Africa (ICSSA) from 2014–2018. He was appointed as the chair of the Body of Trustees of ICCSSA from 2018–2022. His field of applied research is in the areas of Remote Sensing, GIS, Spatial Statistics, Space-Time Models, Design of Optimal Sampling Schemes. His most recent awards include the 2020 SASA-SAS Thought Leadership Award, 2019 ICCSSA Charter Award, 2018 CSIR CEO Award, 2016 CSIR Management Excellence Award, 2016 and 2014 CSIR Collaboration Award.



Christopher W. Cleghorn received his Masters and PhD degrees in Computer Science from the University of Pretoria, South Africa, in 2013 and 2017 respectively. He is an Applied Science manager at Amazon Web Services and an Associate Professor in the School of Computer Science and Applied Mathematics, at the University of the Witwatersrand. His research focuses are within both optimization and machine learning. Prof Cleghorn annually serves as a reviewer for numerous international journals and conferences in domains ranging from swarm intelligence and neural networks to mathematical optimization.