Impact of phase errors at the conjugate step on the propagation of intensity and phase shaped laser beams

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ABSTRACT

We investigate the phase conjugating element of a two element Fourier transform beam shaping scheme and the impact this element has on the resulting propagation. We show that there are stricter limitations placed on the system when using such a phase correcting element, and that even at 10× the previous limits one can observe intensity errors due to inaccurate phase conjugation.

Keywords: Beam shaping, phase conjugation, geometric optics, propagation, diffraction.

1. INTRODUCTION

For single–mode laser beams with a Gaussian intensity profile it is possible to transform the beam's intensity into a uniform profile with steep skirts. Examples of such profiles include the well known super–Gaussian and flat–top intensity profiles. In a seminal work in this field, Romero and Dickey[1] used the Fresnel approximation to design a system comprising a diffractive optical element and a Fourier transforming lens to convert the beam from one intensity distribution into another. In general, this problem cannot be solved exactly, so one must be satisfied with an approximate solution. One of the outcomes of this study was the introduction of a dimensionless parameter β, which is a measure of how well the geometrical optics approximation holds when solving such problems:

$$\beta = \frac{2\pi r_0 R_0}{\lambda f}, \tag{1}$$

where r_0 is the incoming beam radius, R_0 is the transformed beam radius, f is the focal length of the lens used and λ is the wavelength in vacuum of the laser beam. When β is large, an analytical solution for the diffractive optic was found to be very accurate. In this approach the phase of the laser beam at the transformed plane was left as a free variable, which has an adverse influence on the propagation of the resulting uniform intensity beam. When this beam is to be delivered to a target at a single plane, this is not an issue. However, when the laser beam is to maintain its shape over an extended distance, as is the case in some applications, then a phase conjugating element is required at the transform plane.

In this study we consider the phase conjugating element in such a laser beam shaping scheme, and consider the properties of this element when the geometric approximation is valid and when it is not. In both cases we also consider the impact such an element has on the resulting propagation of the transformed beam, and suggest a β value for successful transforming of both intensity and phase.

2. FOURIER TRANSFORM BEAM SHAPING SCHEME

In this section we briefly review the key findings of Romero and Dickey[7] for the transformation of a Gaussian beam into a flat–top beam by means of a diffractive optic and lens pair, and then consider the beam behaviour after passing through the second element, the phase conjugating step. Figure 1 illustrates the elements used in this lossless Fourier transform beam shaping scheme: the first phase–only diffractive optical element (A), together with a lens (B) result in an intensity transformed beam at the focal plane of the lens. Since the phase is arbitrary at this plane, a phase conjugating element (C) is used to correct for the phase – i.e., to generate a planar wavefront. The key concept in this transforming scheme is to design the diffractive element (A) so that the Fourier transform of the resulting beam is the desired intensity. Since a

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lens will create such a Fourier transform at its focal plane, one need only place a lens immediately after element A in order to create the desired profile. By changing the focal length of the lens, the resulting laser beam at the Fourier plane may be scaled in size, without any changes in the flat–top profile. Consequently choosing different focal lengths will lead to size tuneability of the final flat–top beam.

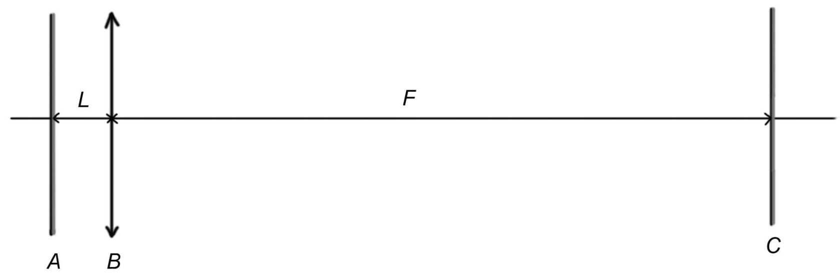


Figure 1: Three element laser beam shaping scheme: A and B (the lens) transform the intensity, while C is used to correct the phase of the transformed beam. The distance from B to C is equal to the focal length of lens B.

The calculations which were done by Romero and Dickey in [7] have some limitations due to the application of the stationary phase approximation. When $\beta > 30$, the actual flat–top beam will be very close to the ideal. In the sections to follow we wish to explore whether this criteria is both necessary and sufficient in order to ensure accurate phase conjugation at the focal plane.

2.1. Intensity transforming elements (A and B)

Following the method of Romero and Dickey[7], we deal with the Fourier transformation or Fraunhofer approximation of the Fresnel diffraction integral and assume that elements A and B are infinitely close together. Consequently the Fresnel integral can be written as a Fourier integral in the form:

$$U_1(\xi) = \frac{\exp(ikf)\exp(ik\xi^2/2f)}{i\lambda f}\int_{-a}^{a} U_0(\eta)\exp(i(\phi_A(\eta) - k\eta\xi/2f))d\eta\,, \tag{2}$$

where $U_0(\eta)$ is the incoming field distribution, $\phi_A(\eta)$ is the desired function of the diffractive element A, $U_1(\xi)$ is the transformed field and $k = 2\pi/\lambda$ is the wave vector. The integration is done over some finite aperture a which in general is written in terms of the starting Gaussian width w. Note that this analysis is for the 1–D case as an example.

We can apply the stationary phase approximation to the integral in (2) because the phase term is large and results in a strongly oscillating function inside the integral. This results in a simple analytical expression for the final field in terms of the desired phase function:

$$U_1(\xi) = \frac{-\sqrt{2\pi i}\exp(ikf)\exp(ik\xi^2/2f)\exp(i\phi_A(\xi))U_0(\xi)}{i\lambda f\sqrt{\phi_A''(\xi)}}\,. \tag{3}$$

From the knowledge that the first derivative of the rapid changing function must be zero, and knowing the starting and desired functions, we can find the phase of the diffractive element A as:

$$\phi_A(\xi) = \varepsilon\frac{\pi}{\lambda f}w\sqrt{\pi}(w\exp(-\xi^2/w^2)/\sqrt{\pi} + \xi erf(\xi/w))\,, \tag{4a}$$

where

$$\varepsilon = \int_{-\infty}^{\infty}\exp(-\xi^2/w^2)d\xi \Bigg/ \int_{-\infty}^{\infty}\exp(-\xi^N/w^N)d\xi\,. \tag{4b}$$

Here we have solved for the transformation of a Gaussian of width w into a super–Gaussian of order N; as $N \to \infty$, so the super–Gaussian function tends towards a perfect flat–top beam.

2.2 Phase conjugating element (C)

Since (3) describes the transformed field, its argument holds the phase information of the field at this plane. Thus the complex conjugate of this function will be the phase function we desire for element C in order to convert the arbitrary phase of the flat–top beam into a planar wavefront. Thus the phase of element C can be written as (where we have ignored constant terms):

$$\phi_C(\xi) = \exp\left\{ -i\frac{k\xi^2}{2f} - i\varepsilon\frac{\pi}{\lambda f} w\sqrt{\pi}\left[\frac{w}{\sqrt{\pi}} \exp\left(-\frac{\xi^2}{w^2} \right) + \xi \operatorname{erf}\left(\frac{\xi}{w} \right) \right] \right\}. \tag{5}$$

Such a transformation will decrease the intensity fluctuations of the resulting propagating field, and we assume for the moment that such a beam propagates with the least disturbances to its intensity. The solution in (5) is only accurate when the geometric approximation holds, i.e., for β large. However, we can solve for the phase of the second element exactly by employing the exact diffraction integral (in the paraxial and Fresnel approximations) and placing in the kernel the phases of elements A and B, as well as the starting field. It was discussed previously that the phase of element A given by (4a) only delivers the correct result when $\beta > 30$. We can now investigate how the actual phase of element C compares to the analytical approximation in (5), and in particular, if the same limitation on β is both necessary and sufficient in the complete transforming problem. The results of this are shown below in Figure 2.

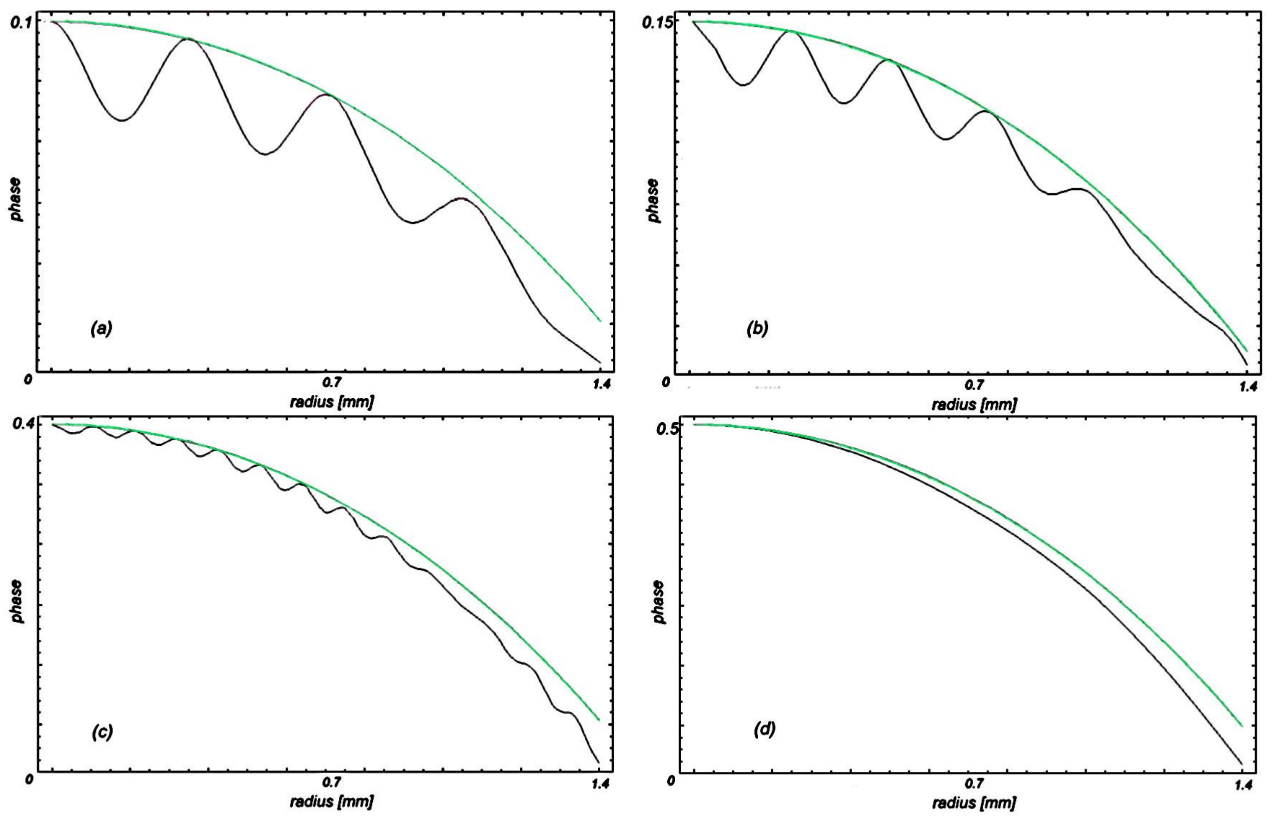


Figure 2: Comparison of solutions for the phase of element C using both the stationary phase approximation (green) and the exact diffraction solution (black). The graphs were calculated using $a = 2w$ for: (a) $\beta = 61$, (b) $\beta = 88$, (c) $\beta = 175$, and (d) $\beta = 300$.

Figures 2 (a)–(d) compare the full diffraction–calculated phase with the phase calculated using the geometric approximation, given by (5). In these graphs the initial aperture radius was held at twice the Gaussian radius, that is, $a = 2w$. At low β this leads to "ringing" on the full diffraction solution for the phase, as the wave nature of light dominates. At large β the geometric limit holds, and the diffraction effects are minimised. However, even at $\beta = 300$ when the aperture effects are negligible, the geometric approximation differs from the full diffraction solution. Qualitative comparison of the phase graphs in Figure 2 suggests that when $\beta > 300$ the differences in phase profiles is minimal. This value of β is approximately 10× times the limit of validity for element A.

For example, consider a helium neon laser with incoming and outgoing beams of diameter 0.5 cm, and a transforming lens of focal length 0.5 m. In this case $\beta = 157$ and, as consequence, we would be outside the limit of validity for the phase conjugating element, even though the geometric approximations would seem to be very valid, since such an incoming beam would have a Rayleigh range in the order of 30 m. From Figure 2 (d) one notes that the difference between the two phase functions appears to increase as the distance from the origin increases. We suggest that this is in keeping with the stationary phase approximation, which employs a Taylor expansion about the origin. As the radius increases, so the error introduced by the approximation also increases.

The balance between aperture and β can also be seen in Figure 3, where a fixed β has been chosen but the integrating aperture for the calculations has been varied. Comparison of Figure 2 and 3 highlights the similarities of the two cases, illustrating the dominance of the wave nature of light at low β, and small apertures.

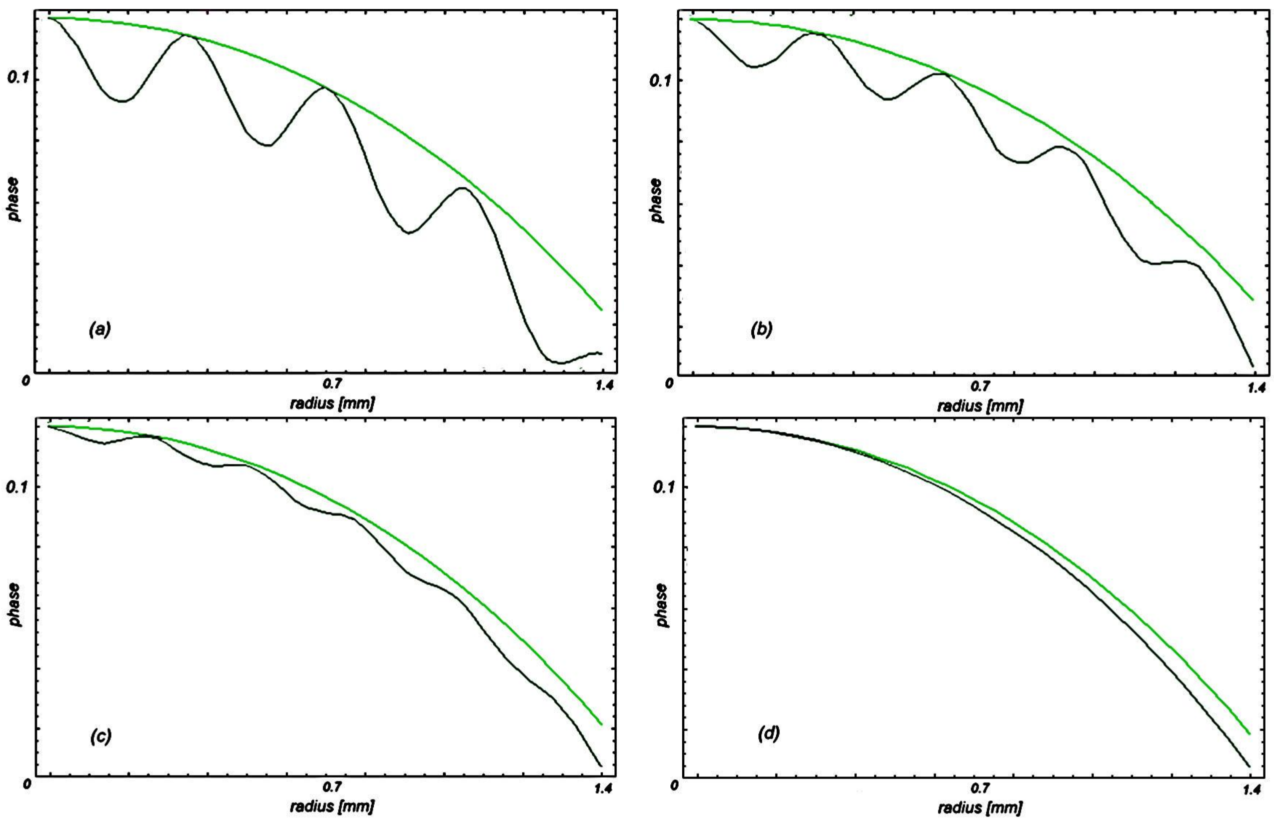
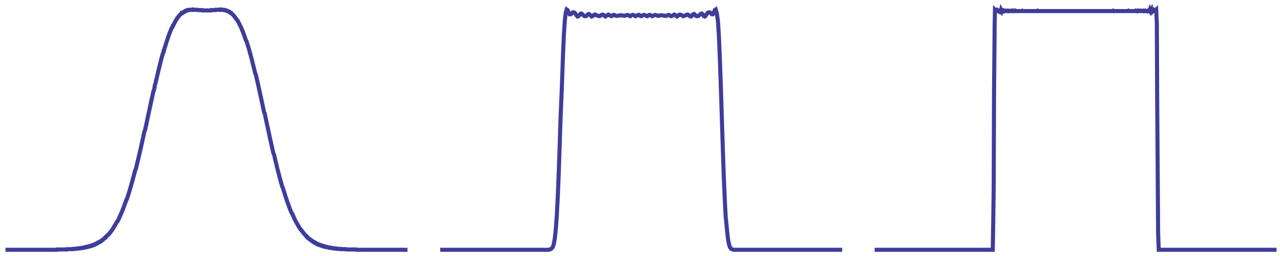


Figure 3: Comparison of solutions for the phase of element C using both the stationary phase approximation (green) and the exact diffraction solution (black). The graphs were calculated using $\beta = 61$ for: (a) $a = 2w$, (b) $a = 2.25w$, (c) $a = 2.75w$, and (d) $a = 4w$.

3. PROPAGATION RESULTS

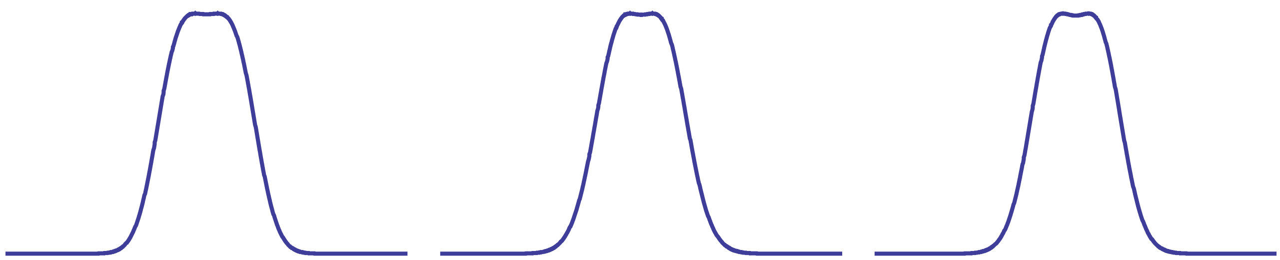
Having noted the differences in the phase of element C discussed above, we now turn our attention to the impact that this element has on the resulting propagation of the transformed field. We consider the propagation under three conditions: (i) no conjugating element present, (ii) a conjugating element based on the result in (5) and (iii) a conjugating element based on the full diffraction calculation. In all cases a suitably large initial aperture was chosen. Results are shown graphically in Figures 4 – 7.



(a) (b) (c)

Figure 4: Intensity profile of the flat–top beam at the focal plane of the lens, with (a) $\beta = 3.76$, (b) $\beta = 37.6$ and (c) $\beta = 376$.

Figures 4 (a)–(c) shows the resulting 3 mm radius flat–top beams for large and small β values. In all calculations β was changed by adjusting the focal length of the lens used on the system, and keeping all the laser beam parameters the same (size and wavelength). As β increases, so the steepness of the flat–top edges increases, and the central region of the beam flattens out. Clearly at $\beta > 30$ the beam is very close to the ideal flat–top.

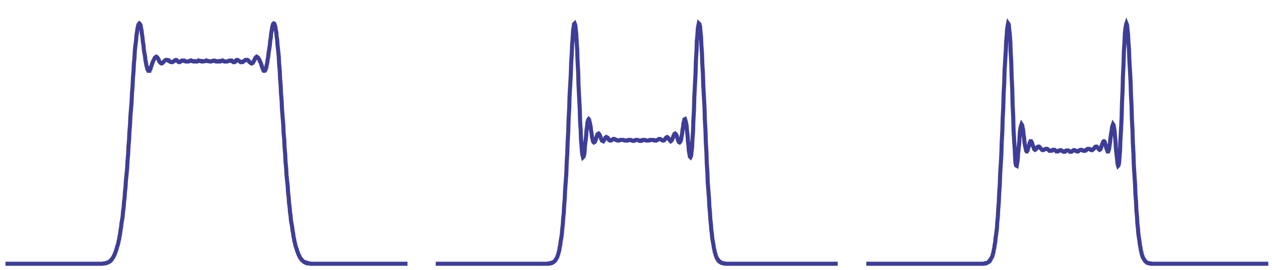


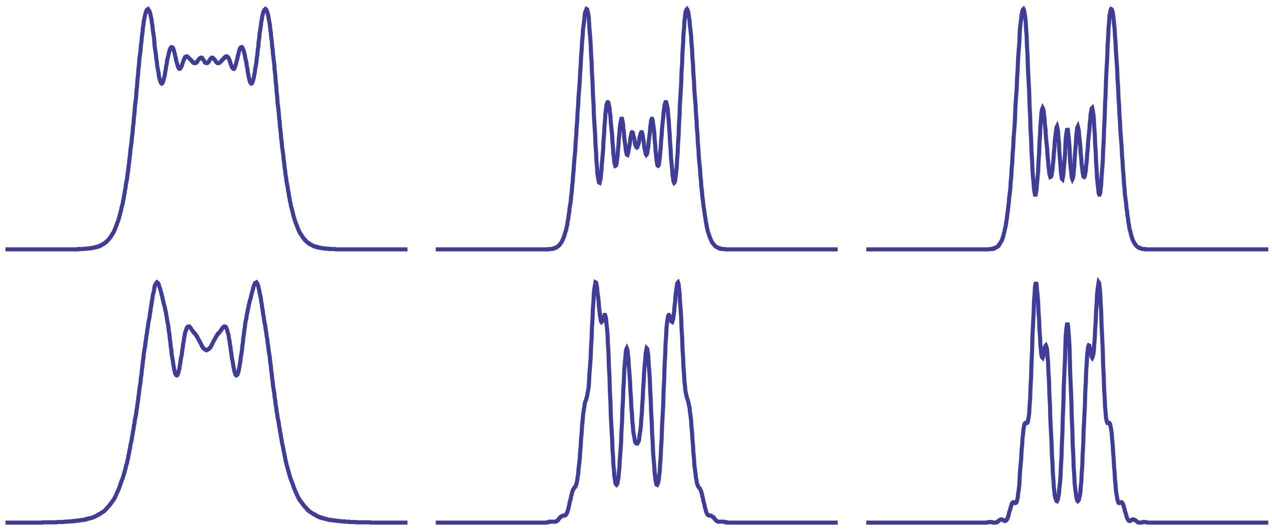
(a) (b) (c)

Figure 5: Intensity profile of the beam in Figure 4 (a) after 0.5 m propagation when: (a) Ideal phase correcting element is present, as calculated using the Fresnel diffraction equation, (b) the phase element calculated using Eq. (5) is present and (c) no second phase element is present. .

Figures 5 (a)–(c) were calculated using $\beta = 3.76$, and shows a 3 mm radius flat–top beam after propagating 0.5 m. In (a) – (c) the beam was, respectively: perfectly phase conjugated; conjugated using an elements calculated by (5) and not conjugated at all. Despite the large differences at low β between the ideal and approximate conjugate phases, there is little impact on the resulting propagation. This is most likely due to the fact that the smoother fat–top has a longer Rayleigh range and propagates farther without shape distortion than beams with higher β. Thus the naturally low divergence of this shape mitigates against the impact of the phase errors.

Figure 6 shows the effect of the phase correcting step for the $\beta = 37.6$ profile. Because $\beta > 30$ the intensity profile at the focal plane of the lens is very close to the ideal (see Figure 4 (b)) flat–top, indicating the validity of the approximations used. However, when one considers the propagation after approximate and exact phase conjugation, the break–down of the validity of such approximations at the phase correcting step is evident. One immediately sees that when the phase element is calculated rigorously using the Fresnel diffraction equation (column (a)), the resulting beam propagates with less intensity changes than in the other cases, and is still approximately flat–top after 0.5 m, with steep edges but large intensity fluctuations in the centre. This field is correctly phase conjugated.



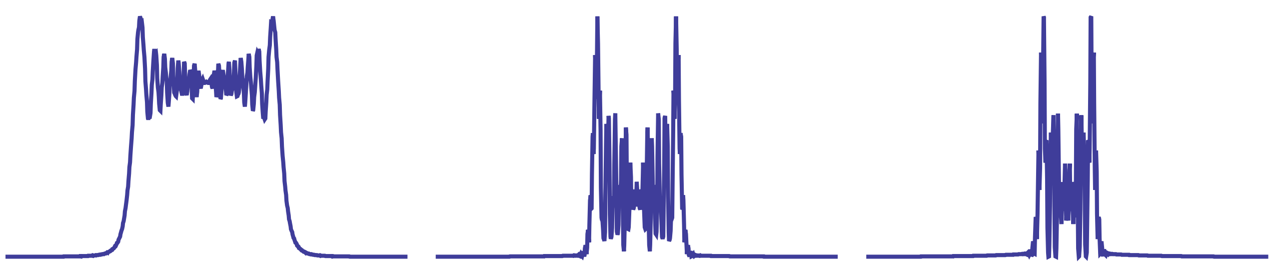


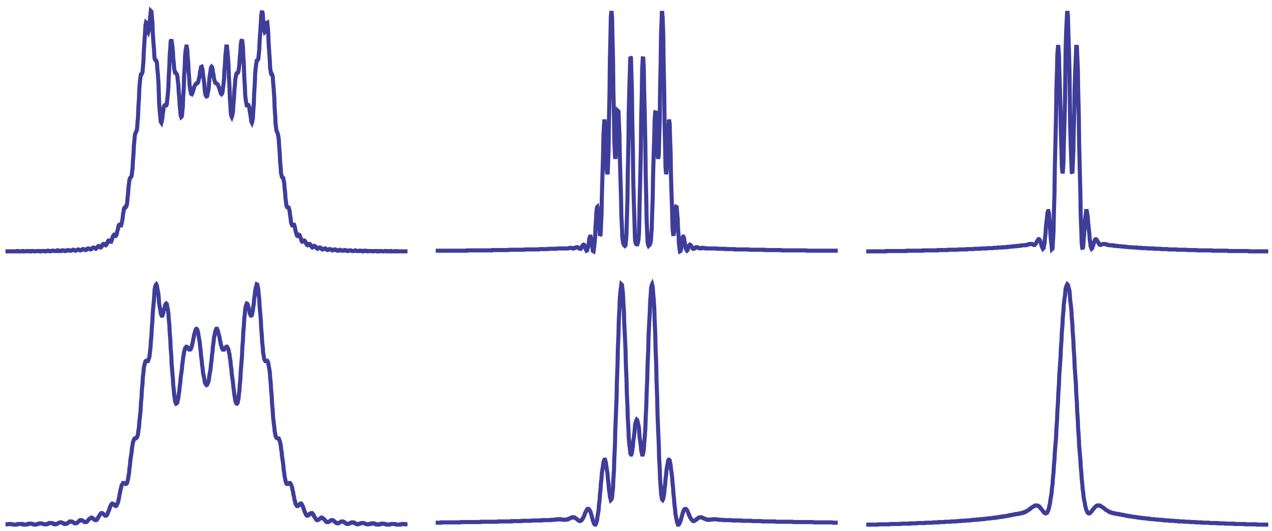
(a) (b) (c)

Figure 6: Intensity profile of the beam in Figure 4 (b) after 0.1 m (first row), 0.25 m (second row) and 0.5 m (third row) propagation when: (a) Ideal phase correcting element is present, as calculated using the Fresnel diffraction equation, (b) the phase element calculated using Eq. (5) is present and (c) no second phase element is present.

However, when the phase element C is calculated using (5), large changes in the intensity pattern are noted for precisely the same input beam. This is shown in column (b), where even after 0.1 m large intensity fluctuations are noted. At 0.5 m the intensity profile is clearly no longer flat–top. Clearly the phase errors due to the approximate conjugate element are large enough to impact on the propagating field, so that although $\beta = 37.6$ is a good enough regime in order to generate a near perfect flat–top at the focal plane, it is not a valid regime for calculating the phase correcting element. Column (c) in Figures 6 shows the propagation of the starting field without any phase correcting step. It is arguable if this situation is any worse than using the inaccurate phase conjugating element, suggesting that there is little benefit to having the phase correcting element in the first place if it is not a perfect conjugating element.

The propagation of the perfect flat–top is shown in Figure 7. Here $\beta = 376$ (see Figure 4 (c)) and the geometric approximations should be exact. However, although the exact and approximate phase profiles are nearly identical close to the origin, they deviate strongly as one increases the radius of the beam (as discussed earlier). Compounding this is the fact that such perfect flat–top beams require high spatial frequencies to describe their intensity profile, and thus are prone to strong divergence and correspondingly rapid intensity changes due to diffraction. Thus the phase conjugating step is critical at such high β values. Comparison of the propagation results with ideal (column (a)) and non–ideal (column (b)) phase correction illustrate this point. When no phase conjugation is used, the intensity starts to approach its far field pattern at 0.5 m (final figure in column (c)).





(a) (b) (c)

Figure 7: Intensity profile of the beam in Figure 4 (c) after 0.1 m (first row), 0.25 m (second row) and 0.5 m (third row) propagation when: (a) Ideal phase correcting element is present, as calculated using the Fresnel diffraction equation, (b) the phase element calculated using Eq. (5) is present and (c) no second phase element is present.

The surprising result is that although the phase profiles of the approximate and ideal elements look very similar at $\beta > 300$, one can still observe differences in the resulting propagation, which we attribute to the increasing phase error with radial distance, and the importance of correcting phase conjugating for such highly divergent fields. Further work is underway to fully understand this and to quantify the impact of the results presented here.

4. CONCLUSION

We have shown that while efficient intensity transformation may be achieved in systems designed for $\beta > 30$, phase transformation with a conjugating element requires $\beta >> 30$. While we have not been able to quantify exactly how large β should be, we have noted through numerical simulation that even at 10× the previous limit one can observe errors due to the phase correcting element. The conjugating step places more restrictions on what may be transformed, but allows one to obtain a flat–top beam with "minimum" divergence, which is particularly useful for experiments where the beam shape should be constant over extended distances.

REFERENCES

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