

The Suction Pressure, Yield Strength and Effective Stress of Partially Saturated Unbound Granular Pavement Layers

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ABSTRACT: The predominant mode of distress for unbound pavement layers is the permanent deformation of the layer either through gradual deformation or rapid shear failure of the layer. Several researchers have related both these forms of permanent deformation to the ratio of the imposed stress over the shear strength of the material. Conventionally, the shear strength of the material was characterised with the Mohr-Coulomb shear strength parameters. This paper further develops a recent yield strength model that includes an approximation of the suction pressure in the formulation of the model. The suction pressure approximation is extended to include the effect of density on the suction pressure. The calibration of the suction pressure approximation and yield strength model using conventional tri-axial test results are presented for a sand and calcrete mixture. The soil-water characteristic curve derived from the suction pressure approximations is validated using actual matric suction measurements. The effects of suction pressure and effective stress on pavement design calculations are also illustrated.

KEY WORDS: Unbound material, Partially saturated, Yield strength, Matric suction, Suction pressure.

1. INTRODUCCION

Partially saturated unbound granular layers are present in all pavements. In certain applications these layers are found fairly deep down in the pavement structure where the imposed stresses have dissipated to such an extent that they do not pose any design problems. In other applications such as low-volume roads and even high-volume roads in certain countries, these layers are often found in the base layer of the pavement and are subject to high stress conditions. In these cases it is necessary to model the strength, resilient and permanent deformation response of the material accurately.

The refinement and calibration of a recent yield strength model for partially saturated unbound granular material that includes an approximation of the suction pressure is done using conventional tri-axial test results for a windblown sand and calcrete mixture. The suction pressure is shown to have a significant magnitude, equal to and exceeding that of the external confinement pressure usually applied during tri-axial testing. The suction pressure therefore has

a significant effect on the modelling of pavements, especially on modelling the resilient response of unbound granular material and cannot be neglected during the pavement design process.

2. BACKGROUND INFORMATION

2.1. The Permanent Deformation of Unbound, Granular Material

The predominant distress mechanism of unbound pavement layers is the permanent deformation of the layer under repeated loading. Several researchers have studied the permanent deformation of unbound material and pavement layers. In terms of describing the permanent deformation behaviour of unbound material, the shakedown theory is currently gaining popularity (Wellner, et al., 2002, Werkmeister, et al., 2004) although the concepts involved in the theory have been described as early as the late 1970s (Maree, 1978).

The shakedown theory is, however, merely a description of the observed permanent deformation behaviour of unbound material and is not specific in terms of the critical parameter that controls the permanent deformation of the material. In this regard, a number of studies have related the permanent deformation of unbound material to the imposed stress expressed as a ratio of the ultimate strength of the material (Maree, 1978; Huurman, 1997, van Niekerk et al., 1998 and Theyse, 2000). Equation 1 provides the formulation of the Stress Ratio (SR) in the format used by Theyse (2000) which is equivalent to the inverse of the Factor of Safety used by Maree (1978). Equation 2 provides the formulation of the Stress Ratio in the format used by Huurman (1997) and van Niekerk et al. (1998). Both equations are given in terms of the yield stress (σ_1^y) and then in terms of the Mohr-Coulomb shear strength parameters, Cohesion (C) and Internal Angle of Friction (ϕ). In principal, both formulations therefore fundamentally rely on the yield strength (σ_1^y) to be known.

$$SR = \frac{\sigma_1^a - \sigma_3}{\sigma_1^y - \sigma_3} \quad [1]$$

$$= \frac{\sigma_1^a - \sigma_3}{\sigma_3 \left(\tan^2 \left(45^\circ + \frac{\phi}{2} \right) - 1 \right) + 2C \tan \left(45^\circ + \frac{\phi}{2} \right)}$$

$$SR = \frac{\sigma_1^a}{\sigma_1^y} \quad [2]$$

$$= \frac{\sigma_1^a}{\sigma_3 \tan^2 \left(45^\circ + \frac{\phi}{2} \right) + 2C \tan \left(45^\circ + \frac{\phi}{2} \right)}$$

Where σ = principal stress (kPa)

τ = shear stress (kPa)

ϕ = angle of internal friction ($^\circ$)

C = cohesion (kPa)

σ_1^y = yield stress (kPa)

σ_1^a = applied major principal stress (kPa)

σ_3 = minor principal stress or confining pressure for the tri-axial test (kPa)

In the above work, the calculation of the stress ratio is based only on the externally imposed minor and major principal stress ignoring any possible residual compaction stress and suction pressure in partially saturated granular material. It will be shown that the magnitude of the suction pressure in a partially saturated material may equal and exceed the minor principal stress caused by an external load and the suction pressure cannot be ignored.

2.2. Effective Stress and Suction Pressure in Partially Saturated Material

Bishop (Bishop, 1959) formulated the concept of effective stress for partially saturated material. Assuming that the pore water and air pressure do not affect the shear stress, the effective normal stress may be written in scalar form given by Equation 3 (Heath, 2002).

$$\begin{aligned}\sigma'_n &= \sigma_n - [u_a - \chi(u_a - u_w)] \\ &= (\sigma_n - u_a) + \chi(u_a - u_w)\end{aligned}\quad [3]$$

Where σ'_n = effective normal stress (kPa)

σ_n = total normal stress (kPa)

u_a = air pressure (kPa)

u_w = water pressure (kPa)

χ = Bishop parameter related to the degree of saturation

The second term in Equation 3 represents the suction pressure, p_{suc} (Equation 4) and consists of two components, the Bishop parameter and the difference between the pore air and water pressure called the matric suction. The matric suction is a function of the degree of saturation of the material and the relationship between the matric suction and degree of saturation over the possible range of saturation values from 0 to 1 is called the soil-water characteristic curve.

$$p_{suc} = \chi(u_a - u_w) \quad [4]$$

The suction pressure is an equal all-round pressure because the pore air and water pressures are the same in all directions and the normal stress imposed by the external load is superimposed on the suction pressure already present because of the partially saturated condition of the material. Figure 1: shows a graphical interpretation of the effect of the suction pressure on the effective stress in a partially saturated material.

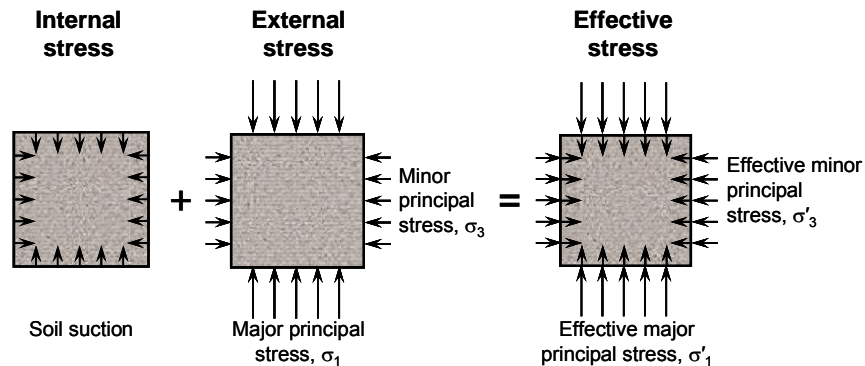


Figure 1: Effective stress in a partially saturated granular material

The Bishop parameter has the same limits as the degree of saturation ($0 \leq \chi \leq 1$) and according to Heath (2002), the Bishop parameter may be reasonably approximated by the degree of saturation. The soil-water characteristic curve may be determined using porous pressure plates,

vacuum desiccators or filter paper methods. However, as mentioned by Heath (2002), the suction pressure which is the product of these two parameters is of main importance especially from a soil mechanics point of view. Heath (2002) therefore approximated the suction pressure curve with the formula in Equation 5 and calibrated the suction pressure curve from tri-axial test data using a back-calculation or reversed calibration process.

$$\frac{p_{suc}}{p_{atm}} = \frac{\theta}{v_r} n_1 \left(\frac{1}{\theta^{n_2}} - \frac{1}{v_r^{n_3}} \right)^{n_3/n_2} \quad [5]$$

Where p_{suc} = suction pressure (kPa)
 p_{atm} = atmospheric pressure (101,3 kPa)
 θ = volumetric water content (volume of water to total volume)
 v_r = void ratio (volume of voids to volume of solids)
 n_1 to n_3 = regression parameters

Considering the behaviour of non-cohesive sand, the suction pressure has to be zero when the saturation is zero and approaching unity with a peak at some intermediate saturation level. Theyse et al (2006) therefore suggested the simplified approximation of the suction pressure curve given by Equation 6 which has these characteristics.

$$p_{suc} = \frac{\rho S}{e^{\omega S}} \quad [6]$$

Where S = degree of saturation (ratio of voids filled with water to total voids)
 e = base of the natural logarithm
 ρ, ω = model coefficients

The function in Equation 6 has the following desirable characteristics that may be derived mathematically:

- The optimum saturation level (S_{opt}) at which the suction pressure reaches a maximum is given by the inverse of the model parameter ω ;
- The maximum suction pressure (p_{suc}^{max}) achieved at the optimum saturation level (S_{opt}) is given by $\rho/\omega e$.

The suction pressure formulation given in Equation 6 is further refined in this paper and included in the formulation of a yield strength model for partially saturated material.

2.3. The Need for a Yield Strength Model

The accurate calculation of the Stress Ratio from either the formulation in Equation 1 or 2 requires that both the imposed stress and yield strength be known precisely. In pavement engineering, the externally imposed stress may be calculated using an integral transformation solution of a multi-layer, linear elastic system or a finite element solution of more complex systems. Although there are many problems associated with the calculation of the externally imposed stress in a pavement system, this aspect is not the focus of the paper and it is assumed that the stress in a pavement caused by an external load is known accurately. The remaining component that needs to be estimated accurately is the yield strength of the material.

In general, the yield strength of a unbound granular material is determined by the material quality (grading, activity of the fines, crushing strength, etc.), the density and degree of saturation of the material and the level of confinement of the material.

The current approach of estimating the yield strength, at least for pavement engineering, is to determine the Mohr-Coulomb shear strength parameters at predetermined density and saturation levels. The yield strength is then calculated from these Mohr-Coulomb parameters and the imposed minor principal stress. The problems associated with this approach such are:

- The Mohr-Coulomb shear strength parameters are only valid for specific density and saturation levels while the density and saturation of unbound layers vary in real-life pavements;
- It is difficult to prepare 3 or 4 tri-axial specimens at almost the same density and saturation levels resulting in variation of the tri-axial test results;
- The yield stress is not linear in terms of the level of confinement as implied in the linear Mohr-Coulomb failure envelope.

It is for these reasons that Theyse et al (2006) formulated a yield strength model that is valid for a range of density and saturation levels, is non-linear in terms of the density, saturation and confinement pressure and incorporates the effect of the suction pressure on the yield strength. This model is listed in Equation 7 and was calibrated very accurately from tri-axial data for a number of road-building materials.

$$\sigma_1^y = \frac{e^{aRD}}{e^{bS}} \left(\sigma_3 + \frac{\rho S}{e^{\omega S}} \right)^c - \frac{\rho S}{e^{\omega S}} \quad \text{if } \sigma_3 > -\frac{\rho S}{e^{\omega S}}$$

$$\sigma_1^y = -\frac{\rho S}{e^{\omega S}} \quad \text{if } \sigma_3 = -\frac{\rho S}{e^{\omega S}} \quad [7]$$

$$\sigma_1^y = \text{Undefined} \quad \text{if } \sigma_3 < -\frac{\rho S}{e^{\omega S}}$$

Where σ_1^y = yield stress or yield strength, compression is positive (kPa)
S = degree of saturation (ratio of voids filled with water to total voids)
RD = relative density (ratio of volume filled with solids to total volume)
 σ_3 = minor principal stress, compression is positive (kPa)
e = base of the natural logarithm
a,b,c = model coefficients
 ρ, ω = suction pressure model coefficients

3. MODIFICATION, CALIBRATION AND APPLICATION OF THE MODEL

3.1. Modification of the Suction Pressure Model

The suction pressure model suggested by Theyse et al (2006) did not include the effect of density in the formulation of the suction pressure as the model used by Heath (2002) does. A minor modification was therefore made to the suction pressure model in Equation 6 by introducing density in the model (Equation 8). The revised suction pressure model was introduced in the yield strength model from Equation 7 and the yield strength model (Equation 9) was recalibrated

for a number of road-building materials using the same procedure as previously used by Theyse et al (2006).

$$p_{suc} = \frac{\rho S}{e^{\omega S/RD}} \quad [8]$$

$$\begin{aligned} \sigma_1^y &= \frac{e^{aRD}}{e^{bS}} \left(\sigma_3 + \frac{\rho S}{e^{\omega S/RD}} \right)^c - \frac{\rho S}{e^{\omega S/RD}} \quad \text{if } \sigma_3 > -\frac{\rho S}{e^{\omega S/RD}} \\ \sigma_1^y &= -\frac{\rho S}{e^{\omega S/RD}} \quad \text{if } \sigma_3 = -\frac{\rho S}{e^{\omega S/RD}} \\ \sigma_1^y &= \text{Undefined} \quad \text{if } \sigma_3 < -\frac{\rho S}{e^{\omega S/RD}} \end{aligned} \quad [9]$$

With the variables as defined previously

Mathematically, the suction pressure formulation in Equation 8 has the following characteristics:

- The optimum saturation level (S_{opt}) at which the suction pressure reaches a maximum is given by RD/ω ;
- The maximum suction pressure (p_{suc}^{max}) achieved at the optimum saturation level (S_{opt}) is given by $\rho RD/\omega e$.

This results in an increase in both the optimum saturation level and maximum suction pressure with an increase in relative density.

3.2. Calibration and Characteristics of the Yield Strength Model

The calibration and characteristics of the yield strength model from Equation 9 is illustrated for a windblown sand and calcrete mixture from the east coast of southern Africa, which was also used by Theyse et al (2006) to calibrate the model given by Equation 7. Table 1 provides a summary of the material properties of the sand and calcrete mixture.

The calibration data for the yield strength models from Equations 7 and 9 are given in Table 2. The model given by Equation 9 provides a slightly better fit to the data than the model in Equation 7. The main differences between the models are:

- The model in equation 9 provides for multiple suction pressure curves depending on the density of the material as illustrated in Figure 2;;
- Sections through the yield strength model at a constant saturation level originated at a single point for the original model in Equation 7, illustrated in Figure 3:(a) as opposed to the multiple origins of the revised model in Equation 9, illustrated in Figure 3:(b).

Table 1. Material properties of the sand and calcrete mixture

Grading	Grading modulus		1.1
	Maximum particle size (mm)		13.2
	Percentage passing 0.075 mm		9.6
Atterberg limits	Liquid Limit		NP
	Plastic Limit		-
	Plasticity Index		NP
	Bar Linear Shrinkage		0
CBR	Swell (%)		-
	Relative density	100.0	66
		95.0	41
		90.0	13
Density characteristics	mod. AASHTO	Maximum dry density (kg/m^3)	1832
		Optimum moisture content (%)	11.5
	Volumetric	Apparent relative density	2.623
		Bulk relative density	-
		Water absorption	-

Table 2. Yield strength model calibration results

Model	Model parameters					Model fit			Suction characteristics		
	ω	ρ	a	b	c	R^2	SEE*	RMSE**	RD	S_{opt}	p_{suc}^{max} (kPa)
Eq. 7	17.3	18949	3.6	0.087	0.831	0.992	42	8	-	5.8 %	404
Eq. 9	12.1	18697	3.4	0.058	0.857	0.994	37	7	68 %	5.6 %	386
									70 %	5.8 %	399
									72 %	6.0 %	410

Notes: * Standard Error of Estimate

** Root Mean Square Error

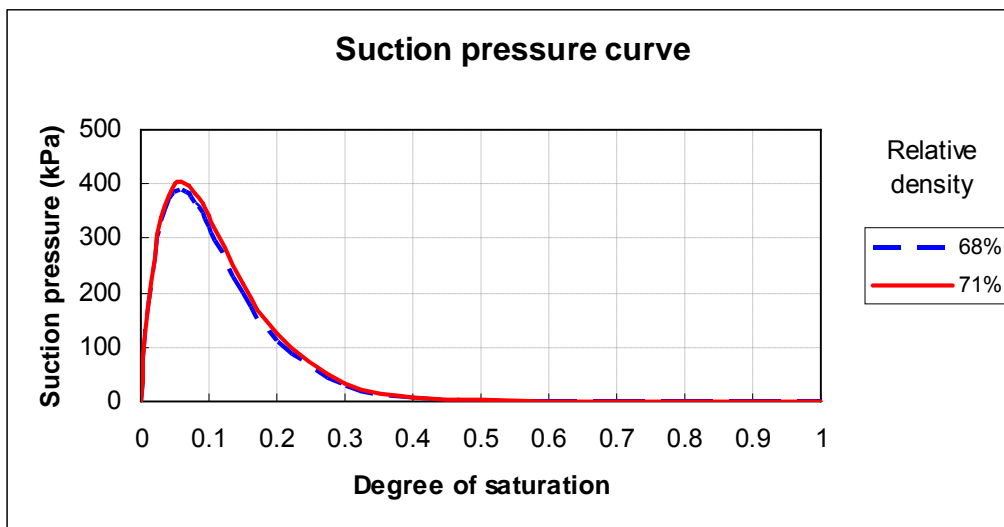


Figure 2: Suction pressure curves at different densities

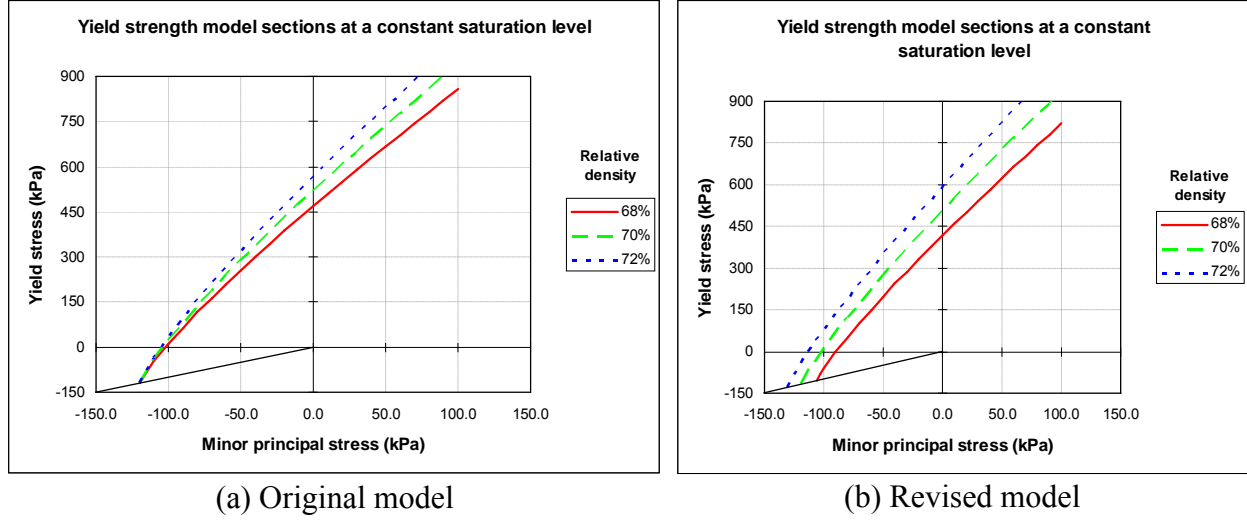


Figure 3: Sections through the original and revised yield strength models at 20 % saturation

Using Equations 5 and 8 it is possible to obtain approximations of the soil-water characteristic curves by approximating the Bishop parameter with the degree of saturation. The formulations for the soil-water characteristic curves are then provided by Equations 10 and 11 respectively.

$$\frac{\text{Matric suction}}{p_{atm}} = n_1 \left(\frac{1}{\theta^{n_2}} - \frac{1}{v_r^{n_3}} \right)^{n_3/n_2} \quad [10]$$

$$\text{Matric suction} = \frac{\rho}{e^{\omega S/RD}} \quad [11]$$

With the variables as defined previously

The matric suction of the sand and calcrete mixture was also measured using the filter paper method and three filter paper calibrations by Leong et al (2002), Van Genuchten (1980) and Fredlund and Xing (1994). The measured matric suction values were compared to the soil suction curves calibrated from the tri-axial data using the Heath (2002) approximation (Equation 10) and the approximation presented in this paper (Equation 11). The comparison is shown in **Error! Reference source not found.**

Both approximations fit the data well with the Heath approximation fitting the data well at very low saturation levels but not that well at intermediate saturation levels. Road materials are likely to operate at intermediate levels of saturation where Equation 11 provides a better approximation of the matric suction for the particular data set.

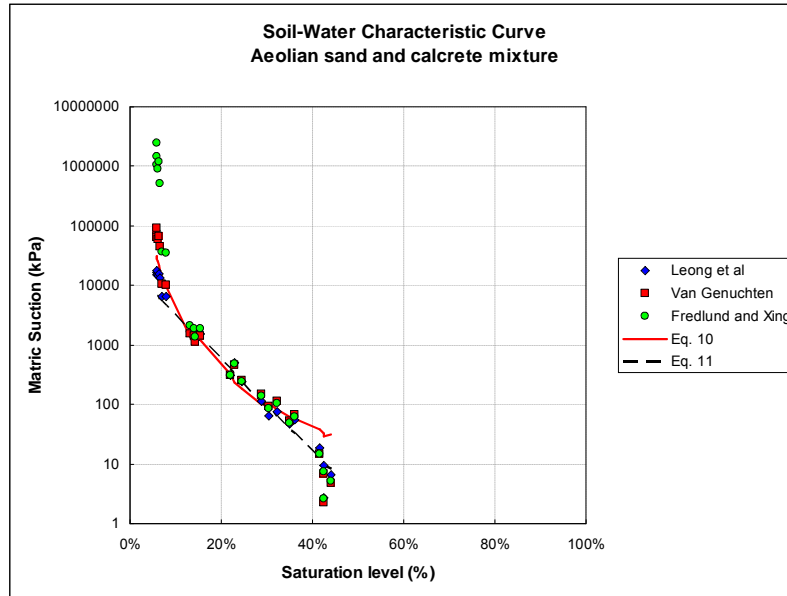


Figure 4: Comparison of the soil-water characteristic curves and filter paper matric suction results

3.3. Application of the Yield Strength Model and Suction Pressure in Pavement Design

The work presented in this paper impacts on mechanistic-empirical design of unbound pavement layers in terms of the permanent deformation behaviour and the resilient response of the material when subjected to external loading. As mentioned earlier, many researchers relate the permanent deformation of unbound granular material under repeated loading to the Stress Ratio imposed on the material which is the ratio of the applied stress to the yield strength of the material. Once the yield strength model is calibrated for a particular material it allows for estimating the yield strength of this material at any combination of relative density, saturation and minor principal stress and hence enables the calculation of the Stress Ratio in the unbound layer when the pavement is subjected to external loading. In terms of the resilient response of unbound material, the suction pressure affects the effective stress which in turn impacts on the stress dependent resilient modulus of the material.

3.3.1 Yield strength and stress ratio calculations

The most efficient application of the yield strength model, calibrated for a particular material, is to calculate the yield strength directly from the calibrated model for a given set of density, saturation and minor principal stress values using Equation 9. In this case, the calculated yield strength represents the total stress and not the effective yield strength. Using a shear stress formulation of the Stress Ratio such as the formulation in Equation 1, which is independent of effective stress as the shear stress is not affected by the suction pressure, the Stress Ratio may be calculated directly from the modelled yield strength and major principal stress imposed by the external load.

After being presented with the model, many experienced pavement design engineers commented that they would not be satisfied with such a once-off calculation that cannot be related to the usual Mohr-Coulomb shear strength parameters that they are familiar with.

However, once calibrated for a particular material the model allows for generating a set of yield strength values over a range of minor principal stress values for specific levels of density and saturation. The set of minor and maximum allowable major principal stress values generated in this way may be processed in the conventional manner to obtain the Mohr-Coulomb shear strength parameters for the material at the specific density and saturation values relevant to the design. The Stress Ratio is then calculated from the Mohr-Coulomb parameters using the second part of either Equation 1 or 2. The advantage of this approach is that it is consistent with the conventional approach of calculating the Stress Ratio and yields Mohr-Coulomb shear strength parameters that are applicable to the density and moisture conditions relevant to the design and are well understood and easily interpreted by pavement engineers.

This process was applied to the calibrated yield strength model for the sand and calcrete mixture to generate the Mohr stress circles and linear Mohr-Coulomb failure envelope shown in Figure 5: at a relative density of 70 % and a degree of saturation of 20 %. The yield strength and subsequently the Stress Ratio, may therefore be calculated from Equation 12 for this particular combination of density and saturation for any given minor principal stress.

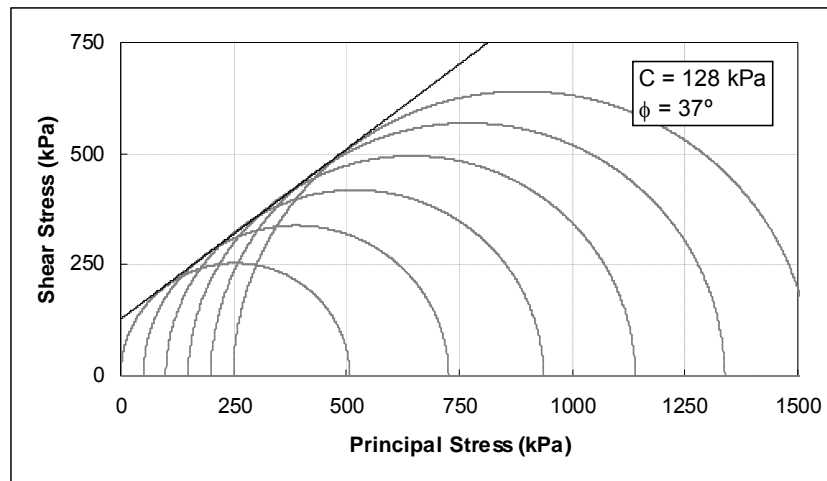


Figure 5: Mohr stress circles and Mohr-Coulomb failure line generated from the yield strength model

$$\sigma_1^y = \sigma_3 \tan^2\left(45^\circ + \frac{\phi}{2}\right) + 2C \tan\left(45^\circ + \frac{\phi}{2}\right) \quad [12]$$

3.3.2 Resilient response modelling

The stress dependency of the resilient response of unbound granular material is well accepted. Using a stress dependent resilient modulus model such as the one in Equation 13, modified form Uzan (1985), the suction pressure has the effect of increasing the bulk stress and hence the resilient modulus of the material.

$$\frac{M_r}{p_{atm}} = K_0 \left(\frac{p}{p_{atm}} \right)^{K_1} \left(\frac{q}{p_{atm}} \right)^{K_2} \quad [13]$$

Where M_r = Resilient modulus (MPa)
 p_{atm} = atmospheric pressure (101,3 kPa)
 p = Mean bulk stress, $p = (\sigma_1 + \sigma_2 + \sigma_3)/3$ (kPa)
 q = Deviator stress $q = (\sigma_1 - \sigma_3)$ (kPa)
 K_0 to K_2 = model parameters

Using a relative density of 70 % and degree of saturation of 20 % for the sand calcrete mixture as an example, the suction pressure calculated from Equation 8 equals 119 kPa. Assuming a major principal stress of 300 kPa and a tensile minor principal stress of -20 kPa (tensile stresses are often predicted in unbound granular layers when a multi-layer, linear elastic solution is used) and using the calibrated yield strength model and a resilient modulus model of the type shown in Equation 13 which was also calibrated for this material, the impact of the effective stress on the resilient modulus and Stress Ratio calculation is shown in Table 3.

Table 3. Resilient modulus and Stress Ratio calculation

Stress regime	Principal stress (kPa)		Resilient modulus calculation			Stress Ratio calculation		
			Bulk stress (kPa)	Deviator stress (kPa)	Resilient modulus (MPa)	Yield stress (kPa)	Stress Ratio	
							Eq. 1	Eq.2
Total stress	Minor	-20	260	320	90	416	73 %	72 %
	Major	300						
Effective stress	Minor	99	618	320	281	535	73 %	78 %
	Major	419						

The resilient modulus calculated from the stress imposed by the external load alone (the total stress as is routinely used in pavement engineering) is only 32 % of the resilient modulus based on the effective stress including the suction pressure. (In this example both cases exclude the over-burden stress as it is assumed equal for the two cases.) The yield strength based on total stress is only 78 % of the yield strength based on effective stress. The Stress Ratio calculated from Equation 1 is, however, independent of whether the total or effective stress is used because the deviator stress (shear stress) formulation used in Equation 1 is not affected by the suction pressure. If the Stress Ratio formulation from Equation 2 is used, there is a slight difference between the Stress Ratios based on the total and effective stress because this formulation is based on the normal stress which is affected by suction pressure according to Equation 3.

4. CONCLUSIONS AND RECOMMENDATIONS

Although difficult to comprehend and measure directly, suction pressure exists in partially saturated unbound granular material and has a significant effect on the response of the material, especially the resilient response and yield strength of the material. Neglecting the effect of suction pressure could lead to significant modelling inaccuracy in pavement design.

Introducing a suction pressure approximation in a yield strength model for partially saturated unbound granular material allows the yield strength of the material to be modelled very accurately. The suction pressure approximations used by Heath (2002) and the one presented in this paper both seem to allow for the accurate modelling of the yield strength of the material.

The yield strength model presented is applicable to materials with no true cohesion. The yield strength model may, however, be readily extended to materials with true cohesion such as cement, foamed and emulsified bitumen treated materials by adding a constant term to the suction pressure curve. It is recommended that the calibration of such a yield strength model should be investigated for such stabilized materials.

The calibration and characteristics of the proposed approximations of the soil-water characteristic curve, the suction pressure curve and the yield strength model presented in this paper were shown for a windblown sand and calcrete mixture. The calibration of the model has been extended to include other road-building materials and it is suggested that the model parameters should be related to the basic material properties of these of materials to allow for the general use of the model in mechanistic-empirical pavement design.

The validation of the suction pressure model using actual matric suction measurements should also be extended beyond the specific case presented in this paper.

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