The Precision and Accuracy of Mechanistic-Empirical Pavement Design

Theyse H. L.
CSIR Built Environment, Pretoria, Gauteng, South Africa

ABSTRACT: The availability of mechanistic-empirical pavement design methods is increasing internationally. Although mechanistic-empirical design does offer some insight into pavement behaviour and performance, at least more so than empirical design methods, they remain mere models of the real-life pavement and are not perfect by far. Recently these design methods have evolved to include two aspects, the basic engineering knowledge and models incorporated in the method and the computational simulation techniques that are often used to introduce spatial and time variability in the design process. It is the opinion of the author that the introduction of the computational simulation techniques has shifted the focus of researchers and the developers of mechanistic-empirical design methods from the core engineering models that determine the accuracy of these methods to the simulation techniques that are merely computer coding exercises but do not improve the accuracy of the design methods. The paper provides an overview of concepts such as variability, precision, accuracy, bias or error and design risk in the context of pavement design. A simple classification of mechanistic-empirical design methods is also provided and the main components of these design methods are discussed in general. The effects of variability and error on the design accuracy and design risk are lastly illustrated at the hand of a simple mechanistic-empirical design problem, showing that the engineering models alone determine the accuracy of these design methods.

KEY WORDS: Mechanistic-empirical design, Computational simulation, Variability, Accuracy, Design risk.

1. INTRODUCION

It is the duty of engineers to design facilities that will perform a certain function at a required service level for a given period of time subject to the specific demands placed on the facility. In general, engineering design is therefore an attempt to balance the “supply” provided by the facility with the “demand” placed on the facility. The definition and quantification of the supply and demand will vary depending on the specific field of engineering and the design problem. In the case of pavement engineering there is a complex interaction between the demand and supply that needs to be considered during the pavement design process. Figure 1 attempts to illustrate this complex interaction in a simplistic manner.

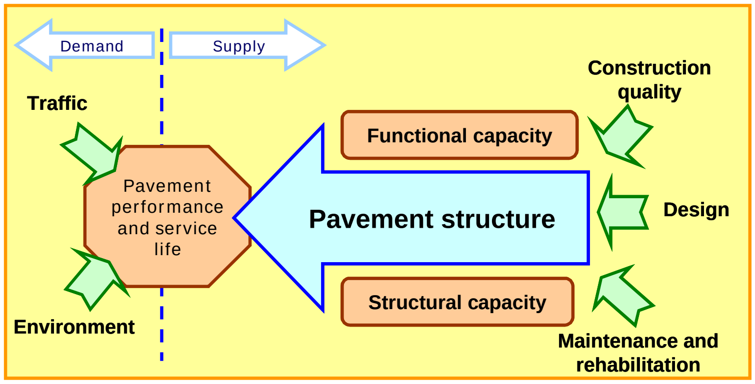


Figure 1. Simplistic representation of demand and supply in the pavement design context

The extent to which the demand and supply sides are balanced during the design process determines the performance and ultimately the service life of the pavement. Classical mechanistic-empirical design methods focussed very much on balancing the design traffic, expressed in terms of "equivalent standard axles" with the structural capacity of the pavement expressed in terms of "standard axles" with sometimes very little regard for the other demand and supply factors affecting the pavement performance and service life. Modern mechanistic-empirical design methods (NCHRP, 2004) have a holistic approach by attempting to simulate the variability of all the factors and the interaction between these factors to estimate the service life of the pavement.

2. BACKGROUND INFORMATION

2.1. Statistical concepts of variability, accuracy and risk applied to pavement design

The behaviour and performance of pavements are variable by nature because of variations in the demand placed on the pavement in terms of traffic and environmental loading as well variation in the characteristics of the pavement such as layer thickness and material quality variation. The concepts of variability, precision, accuracy, error or bias and design risk therefore have to be accommodated in the pavement design process. These concepts are explained at the hand of a classical design approach separating the design traffic, measured in terms of "equivalent axle loads" from the structural capacity, measured in terms of "standard axle loads".

Suppose a section of pavement is observed under controlled traffic loading of a single load magnitude, from the time of construction until a predefined terminal structural condition is reached. If the experimental section is sufficiently long to be subdivided and the number of load repetitions to reach the terminal condition (structural capacity) is recorded on each subdivision, a range of structural capacity observations will be generated that may be represented in histogram format or a probability density function (pdf) may be fitted to the sample of observations. The variability of the actual pavement performance will determine how "wide" the distribution of observations is. The more precise the process is, the less the variation will be and the narrower the spread of the observations. If an attempt is made to model the structural capacity of the observed pavement section using any design method that allows for input and response variability, not necessarily a mechanistic-empirical design method, a second distribution of modelled structural capacity values is generated. The precision of the performance modelling process may not necessarily be the same as that of the actual pavement performance process. The deviation of the central tendency of the modelled structural capacity sample from the central tendency of the observed structural capacity sample is a measure of the accuracy or bias of the performance model. If the observed and modelled distributions are approximately normally distributed the mean of the distributions could be used as a measure of central tendency but if one or both of the distributions are skew, the median (50^{th} percentile) of the distributions provides a better indication of the central tendency. These concepts are summarised in Figure 2.

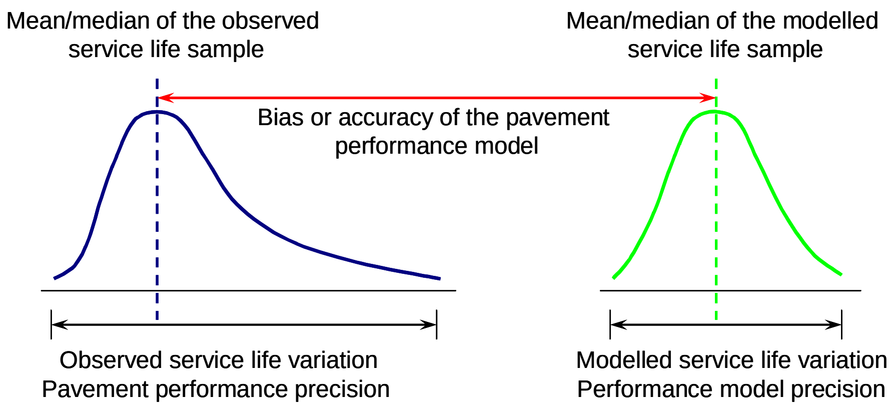
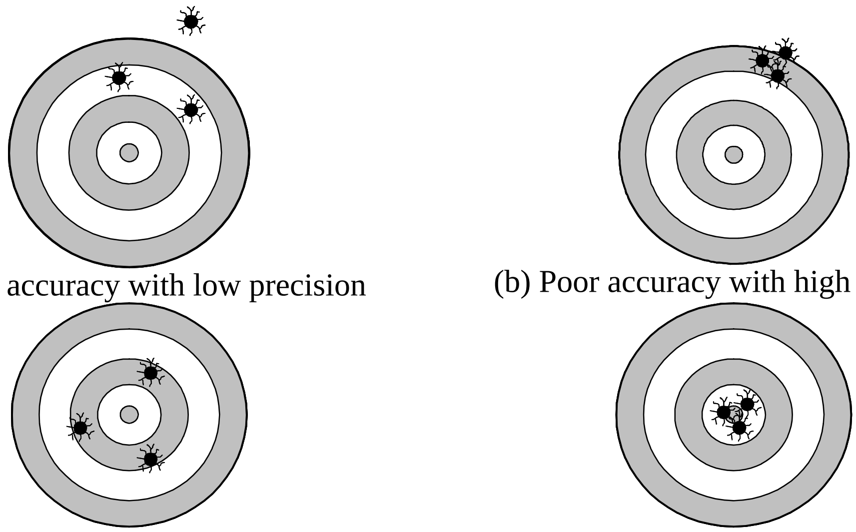


Figure 2. Summary of the concepts of variability, precision and accuracy applied to the pavement design process

The accuracy of a pavement performance (or design) model can therefore only be assessed if a sample of actual observed structural capacities is available. The precision of the performance model may, however, be assessed in absence of an observed structural capacity distribution by calculating statistical parameters such as confidence or prediction limits from the modelled distribution. The ultimate pavement design method or performance model should have no bias and the same precision as that of the actual pavement performance process. Figure 3 illustrates the possible combinations of accuracy and precision. Given the variation in the observed performance of in-service pavements, the real-life pavement performance process is not expected to have a high precision and a performance model or design method with a high accuracy and low precision such as the combination illustrated in Figure 3(c) will probably be the desired option.

Unfortunately, sufficiently large samples of pavement service life or structural capacity observations for a particular pavement under a fixed set of conditions rarely exist in pavement engineering. It is therefore not possible to quantify the bias or accuracy of design methods on a regular basis but reality checks may be done on a limited scale. If the aspects of the design method that dominates the accuracy of the method can be identified, researchers and developers could focus on the refinement of these aspects thereby improving the accuracy of the design method.

Using the classical approach of separating the design traffic from the structural capacity estimation a design traffic distribution may also be generated. The design risk may then be quantified by sampling the structural capacity and design traffic distributions and subtracting the design traffic from the structural capacity. If the structural capacity estimate exceeds the design traffic estimate, the result of the calculation will exceed zero, the supply exceeds the demand and the design is successful. On the other hand, if the result of the calculation is negative, the demand exceeds the supply and failure occurs. By repeating this process a sufficient number of times a survival histogram is created. The area below the negative tail of this histogram represents the probability of failure defines the design risk. This process is illustrated graphically in Figure 4. (AASHTO, 1993)



(a) Poor accuracy with low precision
(b) Poor accuracy with high precision
(c) Good accuracy with low precision
(d) Good accuracy with high precision

Figure 3. Possible combinations of accuracy and precision

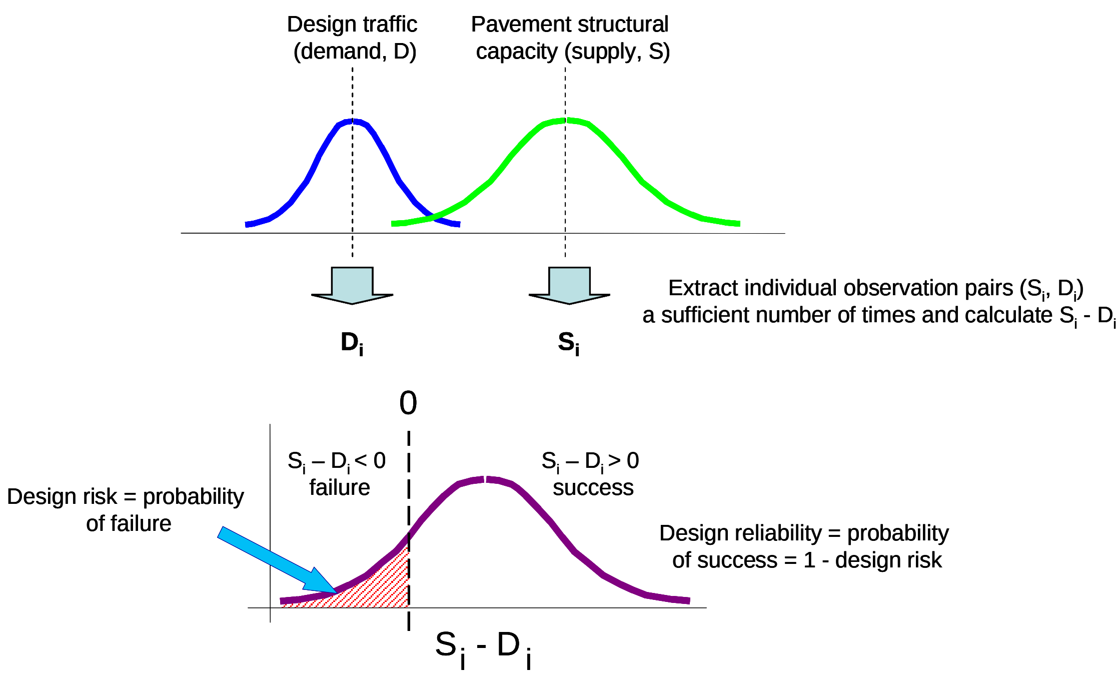


Figure 4. Design risk calculation for a classical design approach

It is clear from this formulation of the design risk that the design traffic and structural capacity estimates plays an equal role in determining the design risk. The same effort therefore needs to be applied to the design traffic and structural capacity. In modern mechanistic-empirical design methods, the traffic input and pavement performance model are combined in a single simulation process and a service life distribution is generated. In this case the design risk is assessed by comparing the service life distribution (supply) with the desired service life (demand).

2.2. Classification of Mechanistic-Empirical Design Methods

A distinction is made between the following types of mechanistic-empirical design methods:

- Classical mechanistic-empirical design methods;
  - Design methods that make single point estimates of the pavement's structural capacity and the cumulative design traffic;
- Probabilistic mechanistic-empirical design methods;
  - Design methods that provide for time-independent variation of input parameters such as layer thickness, material properties, traffic load, contact stress and traffic wander as well as variation in the transfer functions or damage models to generate distributions of the pavement's structural capacity and the cumulative design traffic. These methods allow for the calculation of the design risk according to the method shown in Figure 4;
- Cumulative damage mechanistic-empirical design methods that incorporate time dependent variation in traffic, daily temperature variation and seasonal environmental variation in addition to the time independent variation accommodated probabilistic design methods. Further distinction is made between two types of cumulative damage methods:
  - Linear recursive methods utilising Miner's Law to calculate the damage for each analysis increment;
  - Non-linear or incremental recursive methods for which the damage models or transfer functions are calibrated to allow for the non-linear accumulation of damage.

All the methods listed above use the same formulation of the mechanistic-empirical models as the classical design method except the non-linear recursive method. Fairly complex modelling is therefore possible using the basic method but such modelling is only worthwhile if the method yields realistic/accurate results. The damage modelling concepts involved in each of these methods are briefly explained.

2.2.1 Classical mechanistic-empirical design methods

The damage models or transfer functions of classical mechanistic-empirical design methods are formulated only in terms of the terminal condition for each of the distress mechanism allowed by the method. It is therefore assumed that the pavement deteriorates from a condition of no distress at the onset of loading to a condition of terminal distress when the structural capacity of the pavement is reached. No information is contained in the damage model or transfer function in terms of how the damage accumulates during loading and a linear accumulation can be assumed at best.

Classical mechanistic-empirical design methods mostly treat the individual distress mechanisms for different pavement layers as being independent. The distress mechanism that reaches a terminal condition first therefore determines the structural capacity of the pavement. The classical mechanistic-empirical design methods therefore have a critical layer approach with the most critical layer determining the structural capacity of the pavement as a whole.

2.2.2 Probabilistic mechanistic-empirical design methods

Probabilistic mechanistic-empirical design methods may use the exact same pavement response and damage model formulations as classical mechanistic-empirical design methods. The only difference being that the variability associated with the pavement performance problem is introduced in terms of the input variables of the method and the variability in the set of data form which the damage model is calibrated. Instead of providing a single point estimate of the structural capacity of the pavement, a structural capacity distribution is obtained.

2.2.3 Cumulative damage, linear recursive mechanistic-empirical design methods

Linear recursive mechanistic-empirical design methods use the same damage model formulations as classical mechanistic-empirical design methods. However, these methods allows for the introduction of time-dependent traffic variation and environmental variation on a daily and seasonal basis by using Miner's Law for accumulating the damage. The method therefore relies on the assumption that the accumulation of damage for a single combination of load and environmental conditions is linear.

If the number of load repetitions applied at a specific combination of environmental conditions and load level is less than the structural capacity for that combination, the damage contribution is calculated from Miner's Law as the ratio of the number of load repetitions at a specific combination of environmental conditions and load level to the structural capacity for that combination. The total damage is calculated by adding the incremental damage from each analysis increment. The terminal condition is reached when the total damage reaches a value of one. In this case, because of the inclusion of time-dependent variation, the structural capacity of the pavement cannot be expressed in terms of standard axles and is mostly expressed in time units. However, because of the assumed linearity in the model the load sequence or load history does not have an effect on the level of damage at the end of the application of the individual load cases.

2.2.4 Cumulative damage, non-linear recursive mechanistic-empirical design methods

Non-linear recursive mechanistic-empirical design methods do not assume linear accumulation of damage and do not use the same damage model formulations as classical mechanistic-empirical design methods. These methods require that the formulation of the damage models not only include the terminal condition but also the full non-linear progression from no distress to terminal distress. If two load histories are applied to a pavement, each with the same number of load repetitions per load level but the sequence of loading is changed, the calculated total damage for the two load history paths are not equal if a non-linear recursive method is used.

2.3. Components of Mechanistic-Empirical Design

Mechanistic-empirical design methods consist of several processing components with a stress/strain analysis engine consisting of either a continuum mechanics model (solved by integral transformation or finite element techniques) or a particulate media model at their core. A number of engineering models are layered over the stress-strain analysis engine including input models such as resilient modulus models and damage models on the output side. In modern design methods the engineering models may again be encapsulated by simulation models introducing spatial variability and time-dependency.

Figure 5 shows the integration of the stress/strain analysis engine and engineering models. If this core is collapsed into a single component such as illustrated in Figure 6, the simulation models form pre- and post-processing elements preparing the input data according to the variability and time-dependency of the input data and presenting the results in a meaningful and statistically appropriate manner.

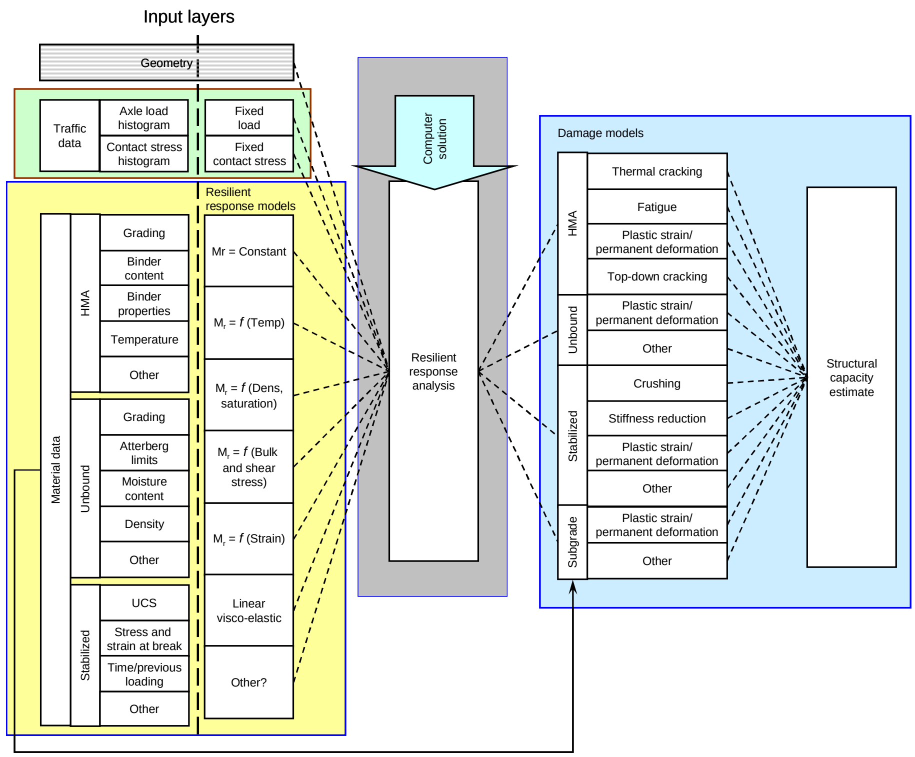


Figure 5. Example of the engineering components of a mechanistic-empirical design method

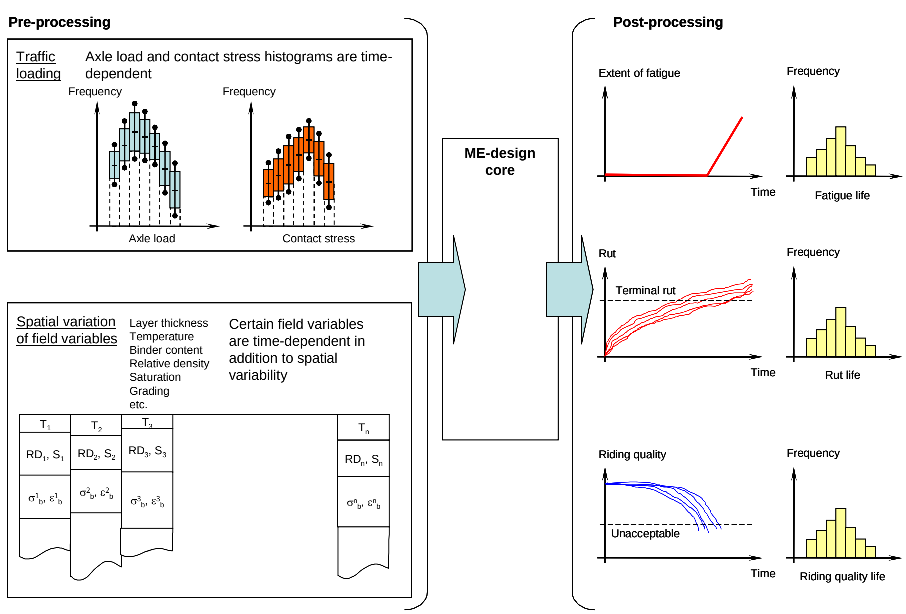


Figure 6. An example of simulation modelling applied to mechanistic-empirical design

The simulation models which are increasingly found in modern mechanistic-empirical design methods have received much attention recently but the author strongly believes that the accuracy of mechanistic-empirical design methods is determined in full by the stress/strain analysis engine and engineering models. The analysis engine and engineering models are mathematical expressions of the engineering knowledge regarding the immediate response and long-term distress of the pavement when subjected to loading. If these models are not accurate the design method is unlikely to be accurate. The simulation models are, however, mere computational exercises requiring programming skills to introduce spatial variability and time-dependency in the design method and do not contribute to the accuracy of the method.

3. THE EFFECTS OF VARIABILITY AND ERROR ON ACCURACY AND RISK

The effects of variability and error on the calculation of design accuracy and risk are illustrated with an example using a probabilistic mechanistic-empirical design method. As mentioned earlier, accuracy can only be assessed if the estimated structural capacity is measured against an observed benchmark. Unfortunately, sufficiently large actual observations are rarely available. A modelled benchmark was therefore created using the South African mechanistic-empirical design method (Theyse et al, 1996) to illustrate the concepts involved. Error was consequently introduced into the design method and the mechanistic-empirical analysis was repeated using different settings for the variability parameters. The effects of variability and error on the calculation of design accuracy and risk were evaluated at the hand of the results from this process.

3.1. The benchmark

The basic pavement structure used in the analysis is shown in Figure 7. Although typical of many pavements in South Africa it merely serves as a modelling example in this context.

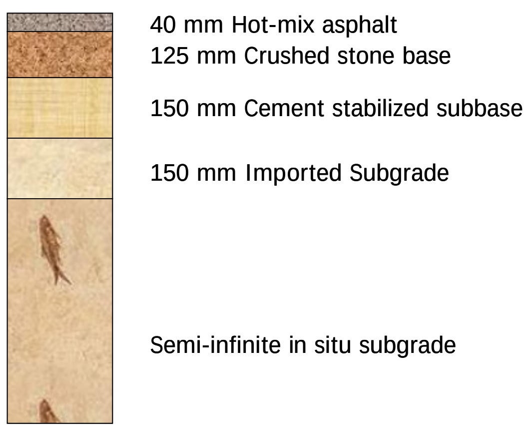


Figure 7. The pavement structure used for modelling purposes

The pavement was modelled in two phases with the cement stabilized subbase having a resilient modulus of 2000 MPa in phase 1 and 300 MPa in phase 2. The resilient modulus of the hot-mix asphalt was set at 2500 MPa, the imported subgrade at 120 MPa and the in situ subgrade at 70 MPa with the Poisson's Ratios 0.45 for the asphalt layer and 0.35 for all the other layers.

Layer thickness tolerances of ±10 and 25 mm were applied to the wearing course and pavement layers. The design load was set at a dual wheel-load of 20 kN per wheel and 520 kPa contact stress.

A total of 1000 simulations were run using a Monte Carlo process with normal distributions (coefficient of variability of 20 %) applied to the resilient modulus, Poisson's Ratio, contact stress and wheel-load variables. Triangular distributions were used for the layer thickness values with the minimum and maximum layer thicknesses determined by the layer thickness tolerances. Only the fatigue life of the asphalt wearing course according to Equation 1 (Theyse et al, 1996) was used as a measure of the structural capacity of the pavement for the purpose of the paper.

$$\log N_f = 17.10 - 4.454 \log \varepsilon_t \qquad [1]$$

Where N_f = fatigue life (number of repetitions)

ε_t = tensile strain at the bottom of the layer ($\mu\varepsilon$)

3.2. Modelling cases

Three additional cases were modelled to investigate the effects of variability and error. In the first case, the coefficient of variability of the benchmark pavement was reduced to 10 % and the layer tolerances were reduced to ±5 and 15 mm for the wearing course and pavement layers respectively. All other parameters were kept the same as for the benchmark.

An incorrect estimation of the resilient modulus of the asphalt wearing course and a slight modification of the fatigue damage model was introduced as "error" in the two subsequent modelling cases. The resilient modulus of the wearing course was overestimated at 3000 MPa while the damage model was changed according to Equation 2.

$$\log N_f = 16.99 - 4.236 \log \varepsilon_t \qquad [2]$$

With the variables as defined in Equation 1.

Given these two "errors" in the engineering models, the modelling process was repeated with the same coefficients of variance and layer tolerances as used for the benchmark case.

3.3. Modelling results

The most convenient way of presenting the results from the individual modelling cases is by frequency and cumulative distribution histograms of the structural capacity as shown in Figure 8 for the benchmark case. This structural capacity distribution represents the "true" structural capacity for the benchmark pavement. Similar structural capacity distributions were generated for the other modeling cases. Table 1 provides a summary of the median of each of the structural capacity distributions and the bias associated with each case.

Table 1. Median and bias of the structural capacity distributions

		Error		
		No	Yes	Bias or inaccuracy (%)
Precision of input variables (Coefficient of variance, CoV)	20 %	4,5 million	13 million	189
	10 %	4,5 million	12 million	
Bias or inaccuracy (%)		0,0		167

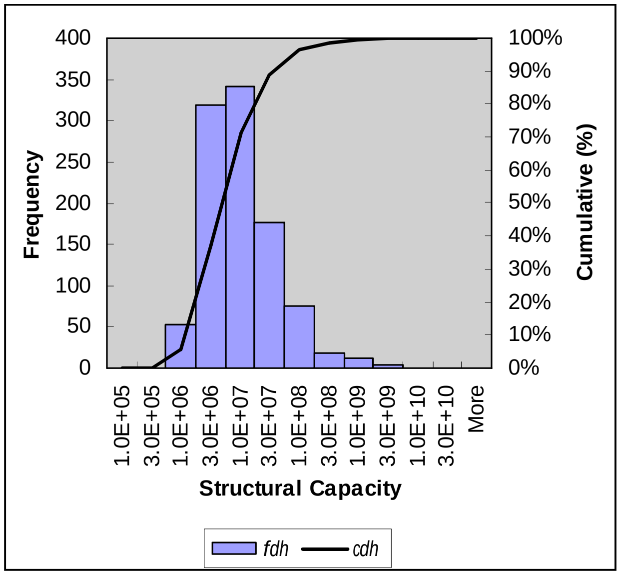


Figure 8. True structural capacity distribution for the benchmark pavement

The highlighted cell in Table 1 represents the benchmark case with a median (50^{th} percentile) structural capacity of 4,5 million axle loads. The accuracy or bias of the other modelling cases is measured against this reference. If the pavement performance process is assumed to be more precise than what it actually is but no error is included in the engineering models (no error, 10 % CoV), the bias or inaccuracy of the design process remains 0 %. If the precision of the pavement performance process is modelled correctly but the engineering models of the design method contains the error described previously, the median structural capacity is estimated to be 13 million, a 189 % over-estimation compared to the true structural capacity. If both the precision and engineering models are incorrect, the bias is practically the same as for the case with the error in the engineering models only. These results show that the accuracy of the design method is extremely sensitive to errors in the engineering models of the mechanistic-empirical design method while the changes in the precision of the computational simulation have little effect on the accuracy of the design method.

In order to asses the estimation of the design risk given the above modelling cases, a design traffic distribution of 1000 estimates was generated given variations in the input data to the design traffic calculation process. Figure 9 shows the distribution histogram for the design traffic. The process illustrated in Figure 4 was followed to determine the survival histogram for each modelling case of which an example is shown in Figure 10.

The survival histogram in Figure 10 consists of a negative tail where the individual design traffic estimate exceeds the structural capacity estimate (failure occurs) and a positive tail where the structural capacity estimate exceeds the design traffic estimate and the design is successful. The point of interest is the cumulative percentage of the negative tail of the distribution representing the number of cases for which failure will occur (the design risk). The design risk for all the modelling cases is summarized in Table 2.

Given the design traffic distribution and the true structural capacity distribution of the benchmark pavement, 60 % of the cases failed. If the precision of the design process is over-estimated by setting a small coefficient of variation for the input variables but no error is included in the engineering models, the risk of failure is slightly under-estimated at 54 %. The risk of failure is, however, under-estimated by far in the cases where error is included in the engineering models regardless of the level of precision that is applied to the input variables. In a real design situation the design engineer will not be aware of the true design risk and will base his assessment of the risk on the modelled design risk which was shown to be substantially under-estimated for relatively minor errors introduced in the engineering model of this specific analysis case.

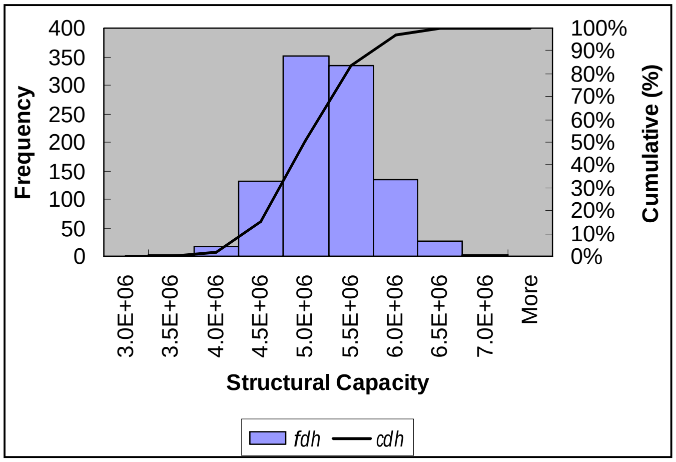


Figure 9. Design traffic distribution histogram

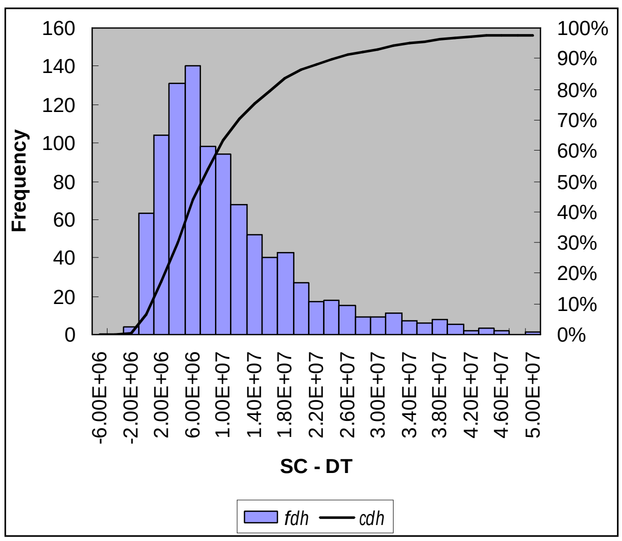


Figure 10. Survival histogram for the case with error included in the design method and with a precision of 10 % CoV applied to the input variables

Table 2. Design risk estimates

		Error in design method	
		No	Yes
Precision of input variables	20 %	60 %	7 %
(Coefficient of variance, %)	10 %	54 %	19 %

4. CONCLUSIONS AND RECOMMENDATIONS

The availability of mechanistic-empirical pavement design methods is increasing rapidly. These methods have also evolved from the fairly basic classical methods that focused largely on obtaining a single structural capacity estimate to the modern methods that attempt to simulate the complete supply and demand process in the pavement design context. Computational simulation techniques have been introduced in these mechanistic-empirical methods to introduce spatial variability and time-dependency. These simulation techniques lend reality to the design process by resembling the characteristics of the real-life pavement performance process and are therefore crucial to be included in the design process.

While these simulation techniques model the precision of the pavement performance process, the accuracy of the design process and the calculation of the design risk are not improved by the mere introduction of the simulation routines. The accuracy of the structural capacity estimation and design risk calculation are determined by the validity and accuracy of the engineering models used in the design method as was illustrated using an example. Any error in the engineering models is reflected out of proportion in the structural capacity estimate because of the mostly logarithmic formulation of the engineering models. Researchers and developers should therefore not neglect to ensure that the stress/strain analysis and damage models included in the design method are realistic and accurate. Utmost care, effort and critical investigation should therefore be applied during the development of these engineering models.

5. REFERENCES

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