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Optimal control surface mixing of an unconventional UAV

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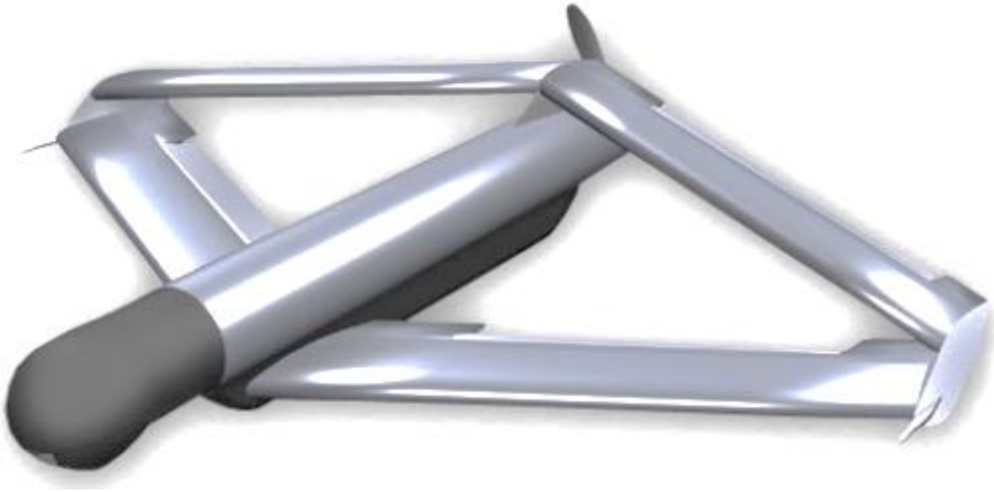
IASSA 2015

Agenda

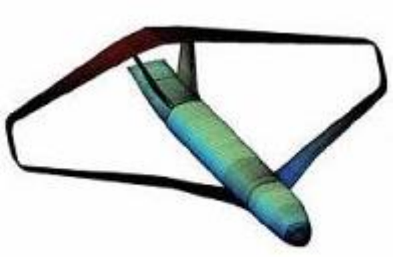
- **Motivation**
- **Approach**
  - Flight Modelling
  - Mixing function
  - Optimisation process
- **Results**
- **Conclusion**

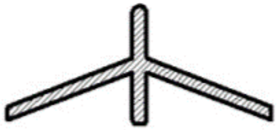


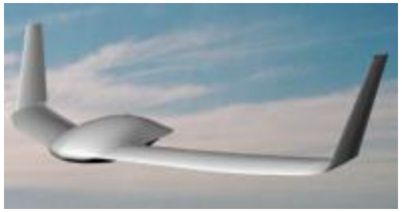
Motivation

Advantages of unconventional aircraft and unconventional control setup:

- Less weight
- Structural strength
- Reduction in wingspan
- Aerodynamic efficiency
- Less induced and parasitic drag

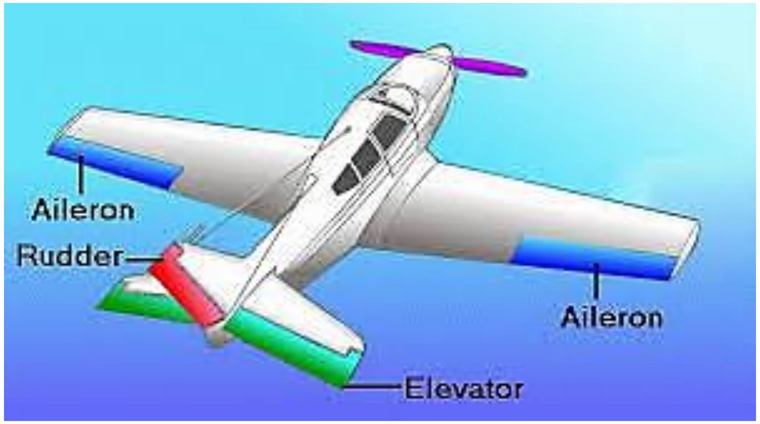






Motivation

Conventional control setup

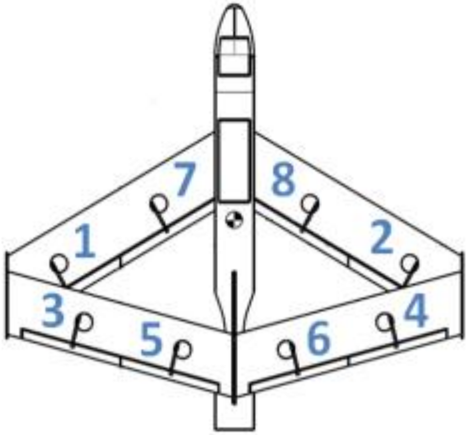


Roll - Aileron

Pitch - Elevator

Yaw - Rudder

Unconventional control setup



Roll
Pitch
Yaw

8 multi-functional control surfaces

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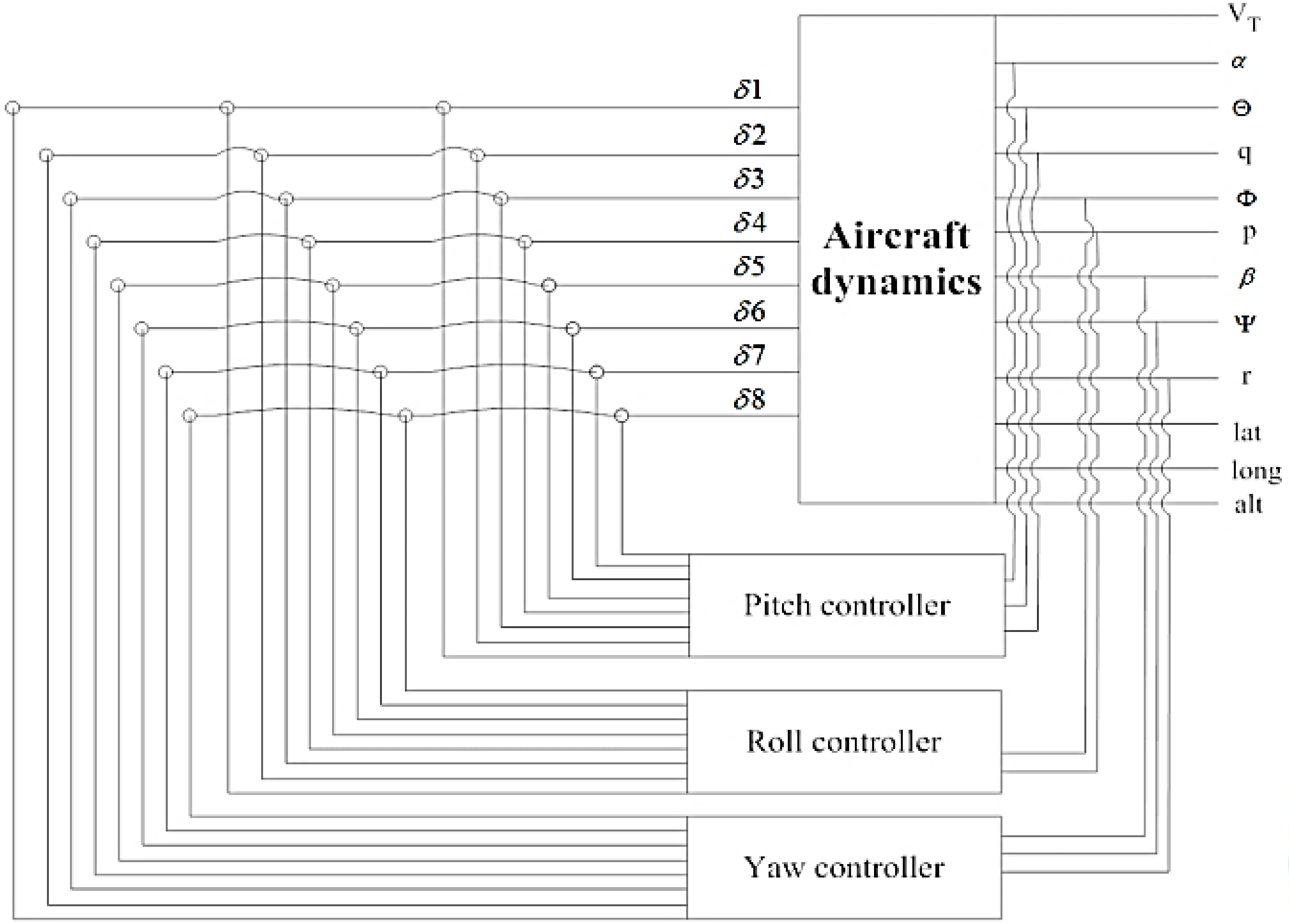
Motivation

Autopilot is responsible for control assignment



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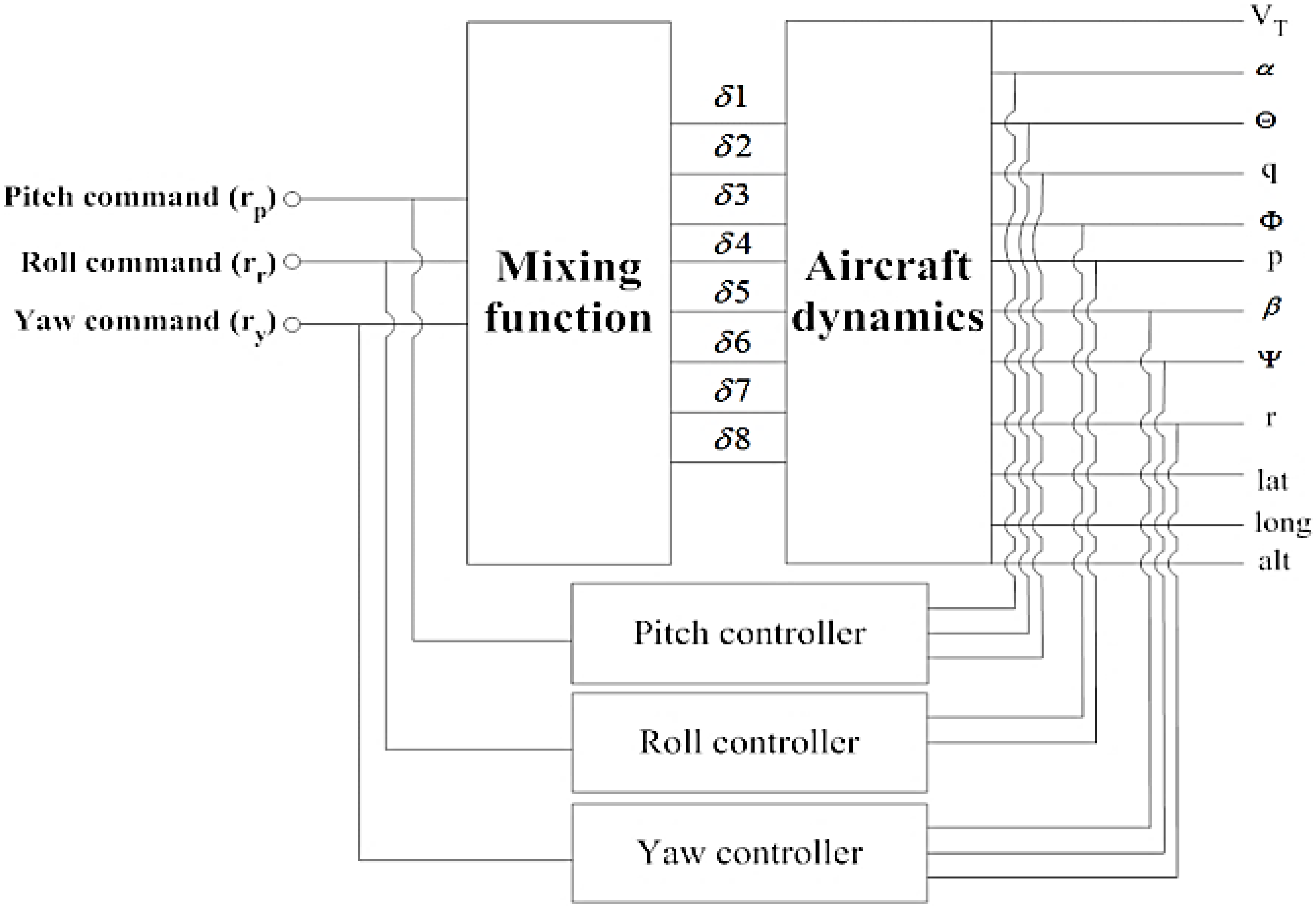
Motivation

Mixing function responsible for control assignment



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Motivation

- Effect conventional **roll, pitch and yaw** control, utilising **8 control surfaces** optimally.
- Considerations:
  - Trim
  - Good response/authority in all three axis
  - Decoupled initial response where possible (e.g. minimise adverse yaw, etc.)
  - Prevent saturation of control surfaces
  - Good flying qualities through entire operational flight envelope

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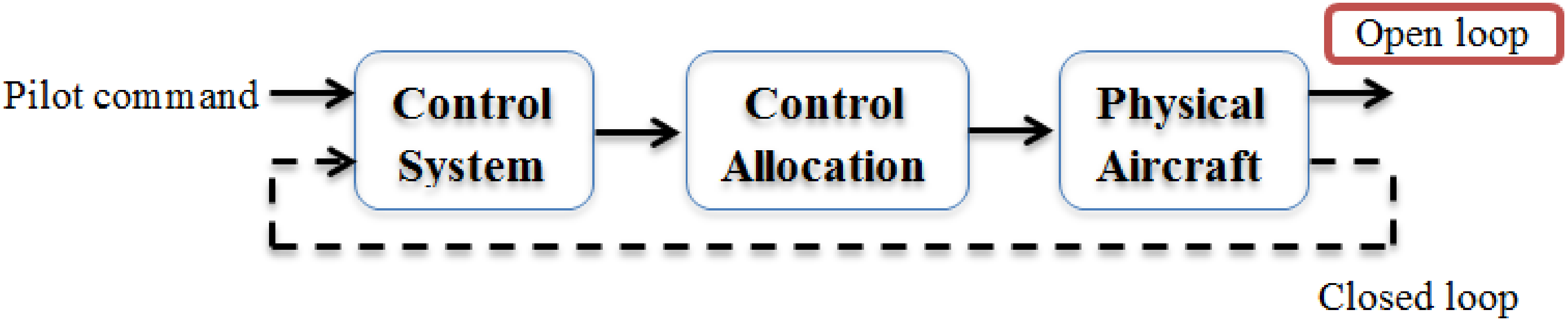
Motivation

- Additional considerations:
  - Open loop control allocation for flight testing and emergency backup



  - Minimal scheduling, and only if required

    (Scheduling as a function or airspeed)

Approach

3 Inputs: Pitch, Roll, Yaw → 8 Control surface deflections

Model the nonlinear, 6 DOF aircraft

↓

Select a suitable mixing function structure

↓

Trim aircraft at a nominal flight condition

↓

Solve for mixing coefficients/parameters

↓

Test if scheduling is required

↓

Perform Robustness study

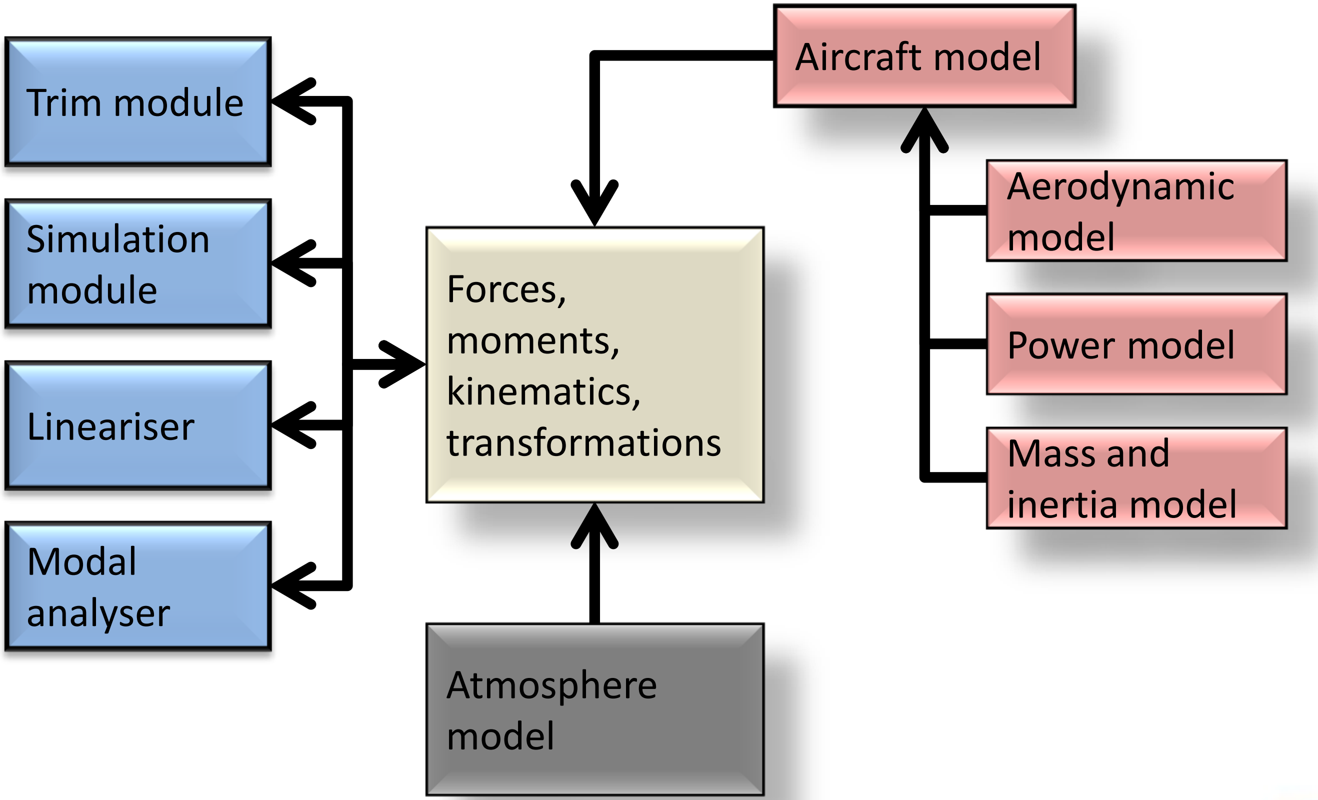
Approach

3 Inputs: Pitch, Roll, Yaw → 8 Control surface deflections

- Model the nonlinear, 6 DOF aircraft
- Select a suitable mixing function structure
- Trim aircraft at a nominal flight condition
- Solve for mixing coefficients/parameters
- Test if scheduling is required
- Perform Robustness study

Flight Dynamics Modelling

- Custom 6-DOF simulation



Trim module

Simulation module

Lineariser

Modal analyser

Forces, moments, kinematics, transformations

Atmosphere model

Aircraft model

Aerodynamic model

Power model

Mass and inertia model

Flight Dynamics Modelling

- Main features:
  - Aerodynamics
    - Static coefficients from wind tunnel data (MDOE)
    - Fully nonlinear, includes coupling and induced effects
    - Dynamic derivatives from vortex lattice and empirical methods
  - Propulsion
    - Custom electric motor
    - Propeller model from measured data
    - Model includes gyroscopic and torque effects

Approach

3 Inputs: Pitch, Roll, Yaw → 8 Control surface deflections

- Model the nonlinear, 6 DOF aircraft
- Select a suitable mixing function structure
- Trim aircraft at a nominal flight condition
- Solve for mixing coefficients/parameters
- Test if scheduling is required
- Perform Robustness study

Mixing function

- Select second order function:

$$\{\delta\} = [A]\begin{Bmatrix} {r_p}^2 \\ {r_r}^2 \\ {r_y}^2 \end{Bmatrix} + [B]\begin{Bmatrix} r_p \\ r_r \\ r_y \end{Bmatrix} + \{Trim\}$$

- Actual control surface deflection in [degrees]:

$$\delta = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7, \delta_8\}^T$$

- Commands:

$$r_p = pitch\ command\ \ -1 \rightarrow 1\ (down \dots up)$$

$$r_r = roll\ command\ \ -1 \rightarrow 1\ (left \dots right)$$

$$r_y = yaw\ command\ \ -1 \rightarrow 1\ (nose\ left \dots nose\ right)$$

- Trim deflections in [degrees]:

$$Trim = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\}^T$$

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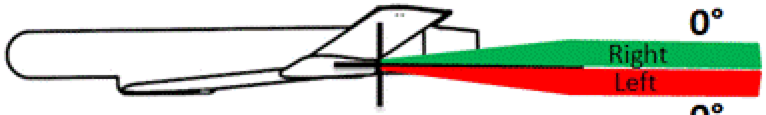
Mixing function

- Characteristics:
    - Constant trim bias vector: can be solved independently from control allocation problem
    - Linear and quadratic terms allow for differential control (e.g. more up on left than down on right and vice-versa) – helps eliminate adverse yaw, etc.



Solution strategy

- Design problem:

Phase 1

  - Solve trim bias vector at nominal flight condition

Phase 2

  - Determine [A] and [B] coefficient matrices while satisfying original control and handling qualities requirements

Phase 1

$$\{\delta\} = [A]\begin{Bmatrix} {r_p}^2 \\ {r_r}^2 \\ {r_y}^2 \end{Bmatrix} + [B]\begin{Bmatrix} r_p \\ r_r \\ r_y \end{Bmatrix} + \{Trim\}$$

Phase 2

Approach

3 Inputs: Pitch, Roll, Yaw → 8 Control surface deflections

Model the nonlinear, 6 DOF aircraft

↓

Select a suitable mixing function structure

↓

Trim aircraft at a nominal flight condition

↓

Solve for mixing coefficients/parameters

↓

Test if scheduling is required

↓

Perform Robustness study

Optimisation phase 1: Trim

- Objective function: Minimise individual control deflections

$$\boldsymbol{f} = \sum_{i=1}^{8} [\boldsymbol{\delta_i}^2]$$

- Utilise equality constraints to enforce trim conditions:

$$\begin{aligned} h_1 &= \dot{p} = 0 \\ h_2 &= \dot{q} = 0 \\ h_3 &= \dot{r} = 0 \\ h_4 &= \dot{\alpha} = 0 \\ h_5 &= \dot{\beta} = 0 \\ h_6 &= \dot{V}_t = 0 \end{aligned}$$

- Can be implemented using any suitable optimiser (e.g. Sequential Quadratic Programming)

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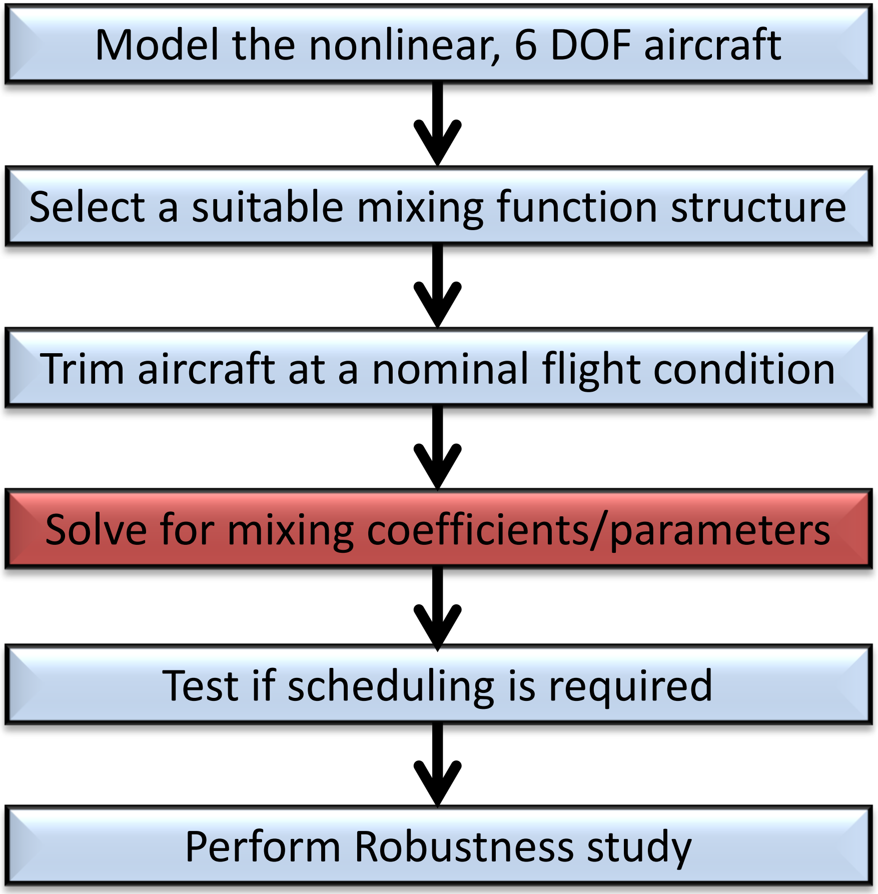
Approach

3 Inputs: Pitch, Roll, Yaw → 8 Control surface deflections



Model the nonlinear, 6 DOF aircraft

Select a suitable mixing function structure

Trim aircraft at a nominal flight condition

Solve for mixing coefficients/parameters

Test if scheduling is required

Perform Robustness study

Optimisation Phase 2 : Control allocation

- **Objective function:** Maximise three rotational responses to individual pitch, roll and yaw inputs
- **Equality constraints:** Minimise coupling between pitch, roll and yaw responses
- **Inequality constraints:** Prevent control surface saturation for all likely combined inputs (e.g. combined roll and pitch inputs)

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Optimisation Phase 2 : Control allocation

- Objective function – maximise:
  - Roll acceleration to a maximum roll input
  - Steady-state sideslip achieved for a maximum yaw input (more consistent results as compared to maximising yaw acceleration)
  - Pitch acceleration to a pitch input

$* \, r_p = pitch\ command$

$* \, r_r = roll\ command$

$* \, r_y = yaw\ command$

$$\mathbf{f} = -\left(\mathbf{w}_1 \dot{p}_{r_p=0;r_r=1;r_y=0} + \mathbf{w}_2 \beta_{r_p=0;r_r=0;r_y=1} + \mathbf{w}_3 \dot{q}_{r_p=1;r_r=0;r_y=0} - \mathbf{w}_4 \dot{q}_{r_p=-1;r_r=0;r_y=0}\right)$$

Optimisation Phase 2 : Control allocation

- Equality constraints - decouple initial response to individual control inputs:

Full roll command ($\boldsymbol{r_r = 1}$):

$$\boldsymbol{h_1 = \dot{q}_{0_{r_p=0;r_r=1;r_y=0}}}$$

$$\boldsymbol{h_2 = \beta_{0_{r_p=0;r_r=1;r_y=0}}}$$

Full yaw command ($\boldsymbol{r_y = 1}$):

$$\boldsymbol{h_3 = \dot{q}_{0_{r_p=0;r_r=0;r_y=1}}}$$

$$\boldsymbol{h_4 = \dot{p}_{0_{r_p=0;r_r=0;r_y=1}}}$$

Optimisation Phase 2 : Control allocation

- Inequality constraints – prevent control saturation for all realistic combined inputs
- Investigated all possible combined inputs, and identified combined inputs applicable to typical UAV flight
- Prevent unnecessary over-constraining:
  - Only applied constraint functions to realistic input combinations

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Optimisation Phase 2 : Control allocation

Possible control input combinations relevant to UAV flight

Pitch	Roll	Yaw	Reqd?	Comment
-1	0	-1	×	Not a realistic input
-1	0	0	✓	Full down elevator
-1	0	1	×	Not a realistic input
-1	1	-1	×	Not a realistic input
-1	1	0	×	Not a realistic input
-1	1	1	×	Not a realistic input
-1	-1	-1	×	Not a realistic input
-1	-1	0	×	Not a realistic input
-1	-1	1	×	Not a realistic input
0	0	-1	✓	Left yaw command
0	0	0	✓	Neutral control
0	0	1	✓	Right yaw command
0	1	-1	✓	Roll + yaw

Pitch	Roll	Yaw	Reqd?	Comment
0	1	0	✓	Right roll command
0	1	1	✓	Roll + yaw
0	-1	-1	✓	Roll + yaw
0	-1	0	✓	Left roll command
0	-1	1	✓	Steady-heading sideslip
1	0	-1	✓	Pos. pitch + yaw
1	0	0	✓	Full positive pitch
1	0	1	✓	Pos. pitch + yaw
1	1	-1	×	Not a realistic input
1	1	0	✓	Pitch + roll
1	1	1	×	Not a realistic input
1	-1	-1	×	Not a realistic input
1	-1	0	✓	Pitch + roll
1	-1	1	×	Not a realistic input

Optimisation Phase 2 : Control allocation

- Total of 14 realistic command combinations
- Inequality constraints can be expressed in terms of:
  - – Coefficient matrix entries (design variables)

$$\{\boldsymbol{\delta}\} = [\boldsymbol{A}] \begin{Bmatrix} \boldsymbol{r_p}^2 \\ \boldsymbol{r_r}^2 \\ \boldsymbol{r_y}^2 \end{Bmatrix} + [\boldsymbol{B}] \begin{Bmatrix} \boldsymbol{r_p} \\ \boldsymbol{r_r} \\ \boldsymbol{r_y} \end{Bmatrix} + \{\boldsymbol{Trim}\}$$

  - – Trim vector entries (from phase 1)

$$\{\boldsymbol{\delta}\} = [\boldsymbol{A}] \begin{Bmatrix} \boldsymbol{r_p}^2 \\ \boldsymbol{r_r}^2 \\ \boldsymbol{r_y}^2 \end{Bmatrix} + [\boldsymbol{B}] \begin{Bmatrix} \boldsymbol{r_p} \\ \boldsymbol{r_r} \\ \boldsymbol{r_y} \end{Bmatrix} + \{\boldsymbol{Trim}\}$$

- Select maximum allowable control surface deflection in degrees (*k*)

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Optimisation Phase 2 : Control allocation

- Complete set of inequality constraints:

$$
\begin{aligned}
g(1) &= (x(3) - x(15) + T_{12})^2 - k^2 \\
g(2) &= (x(3) + x(15) + T_{12})^2 - k^2 \\
g(3) &= (x(2) + x(3) + x(14) - x(15) + T_{12})^2 - k^2 \\
g(4) &= (x(2) + x(3) + x(14) + x(15) + T_{12})^2 - k^2 \\
g(5) &= (x(2) + x(14) + T_{12})^2 - k^2 \\
g(6) &= (x(2) - x(14) + T_{12})^2 - k^2 \\
g(7) &= (x(2) + x(3) - x(14) - x(15) + T_{12})^2 - k^2 \\
g(8) &= (x(2) + x(3) - x(14) + x(15) + T_{12})^2 - k^2 \\
g(9) &= (x(1) + x(3) + x(13) - x(15) + T_{12})^2 - k^2 \\
g(10) &= (x(1) + x(3) + x(13) + x(15) + T_{12})^2 - k^2 \\
g(11) &= (x(1) + x(13) + T_{12})^2 - k^2 \\
g(12) &= (x(1) - x(13) + T_{12})^2 - k^2 \\
g(13) &= (x(1) + x(2) + x(13) + x(14) + T_{12})^2 - k^2 \\
g(14) &= (x(1) + x(2) + x(13) - x(14) + T_{12})^2 - k^2
\end{aligned}
$$

Results

Mixing function results:

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \end{Bmatrix} = \begin{bmatrix} -13.0566 & 2.3778 & -3.2467 \\ -13.0566 & 2.3778 & -3.2467 \\ 9.8355 & 0.4442 & 2.6698 \\ 9.8355 & 0.4442 & 2.6698 \\ 6.7903 & -5.0488 & -0.0268 \\ 6.7903 & -5.0488 & -0.0268 \\ -10.2273 & -6.4153 & 2.7788 \\ -10.2273 & -6.4153 & 2.7788 \end{bmatrix} \begin{Bmatrix} {r_p}^2 \\ {r_r}^2 \\ {r_y}^2 \end{Bmatrix} + \begin{bmatrix} 17.8122 & 2.3778 & -27.6222 \\ 17.8122 & -2.3778 & 27.6222 \\ -23.2785 & 13.8872 & 16.1128 \\ -23.2785 & -13.8872 & -16.1128 \\ -27.4756 & 0.0000 & -5.0220 \\ -27.4756 & 0.0000 & 5.0220 \\ 23.4091 & 19.5971 & -10.4029 \\ 23.4091 & 19.5971 & 10.4029 \end{bmatrix} \begin{Bmatrix} r_p \\ r_r \\ r_y \end{Bmatrix} + \begin{Bmatrix} 0.7408 \\ 0.9968 \\ -3.4925 \\ -2.7357 \\ -4.4211 \\ -4.1108 \\ 3.5589 \\ 3.7138 \end{Bmatrix}$$

Objective function weights:

w_1	w_2	w_3	w_4
1.0	1.0	1.1	0.9

Approach

3 Inputs: Pitch, Roll, Yaw → 8 Control surface deflections

Model the nonlinear, 6 DOF aircraft

↓

Select a suitable mixing function structure

↓

Trim aircraft at a nominal flight condition

↓

Solve for mixing coefficients/parameters

↓

Test if scheduling is required

↓

Perform Robustness study

Scheduling

Mixing function was designed at three different airspeeds:

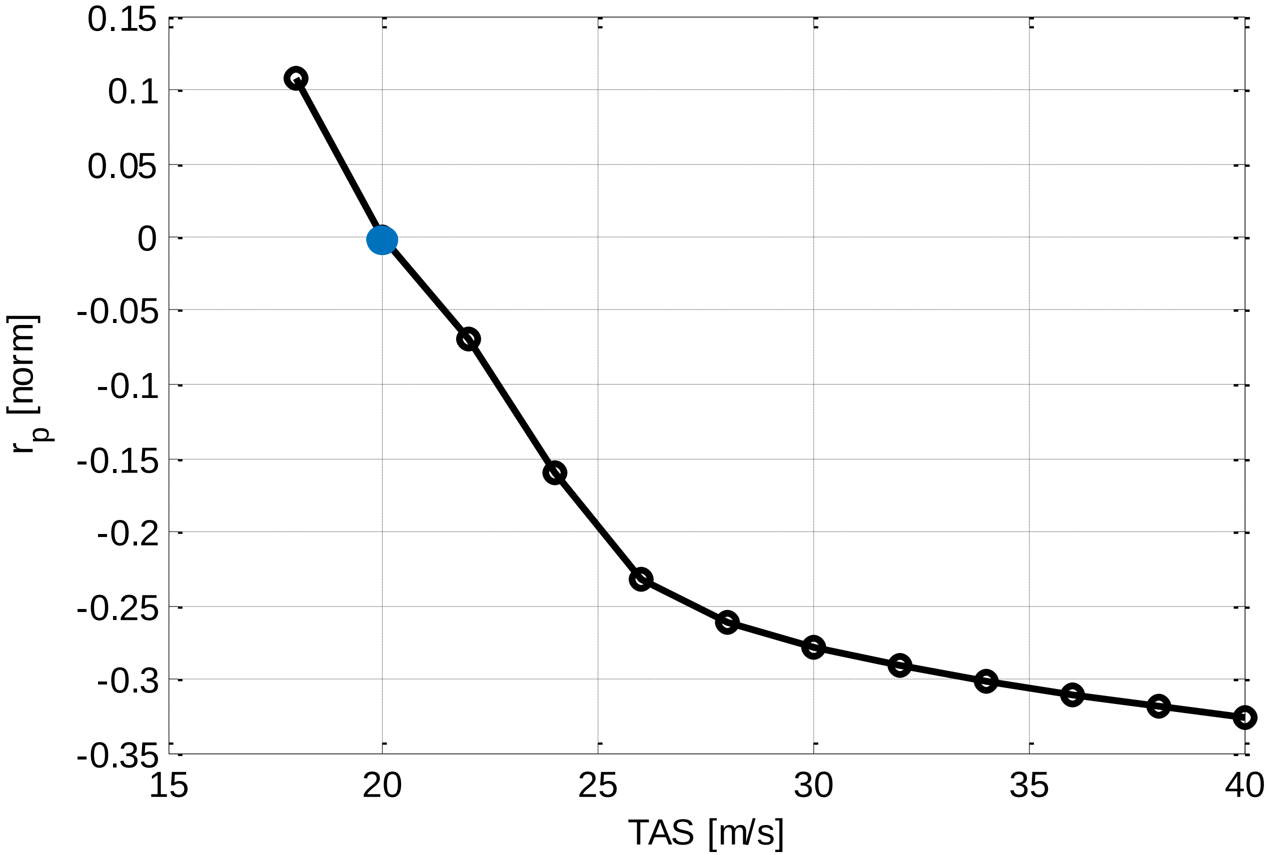
- Airspeed of 20 m/s
- Airspeed of 30 m/s
- Airspeed of 40 m/s

The scheduling was tested through:

- Evaluating the amount of control authority required to trim the aircraft at off-design conditions
- Evaluating the dynamic response of the aircraft at off-design conditions

Mixing function designed at 20 m/s

Maximum of 33% pitch command required to trim the aircraft.



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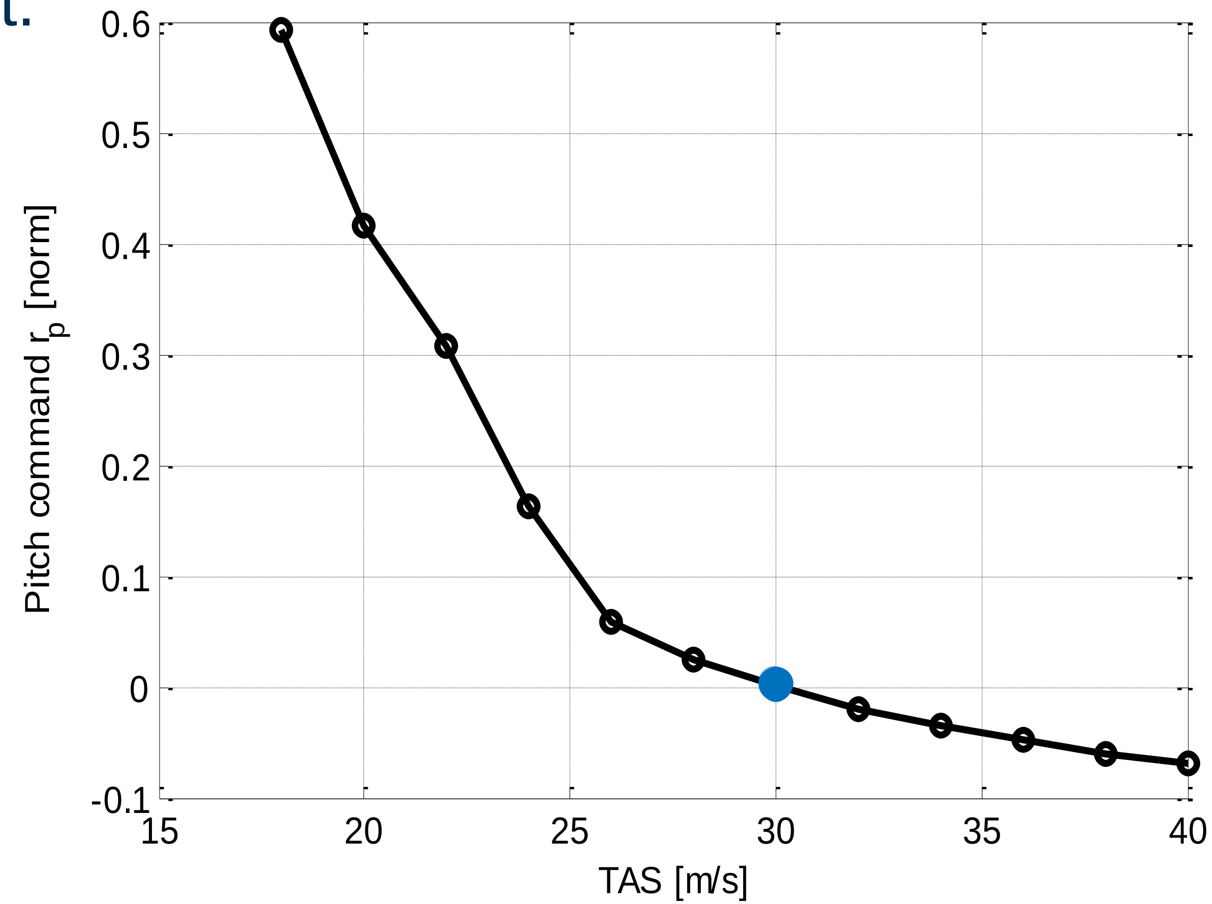
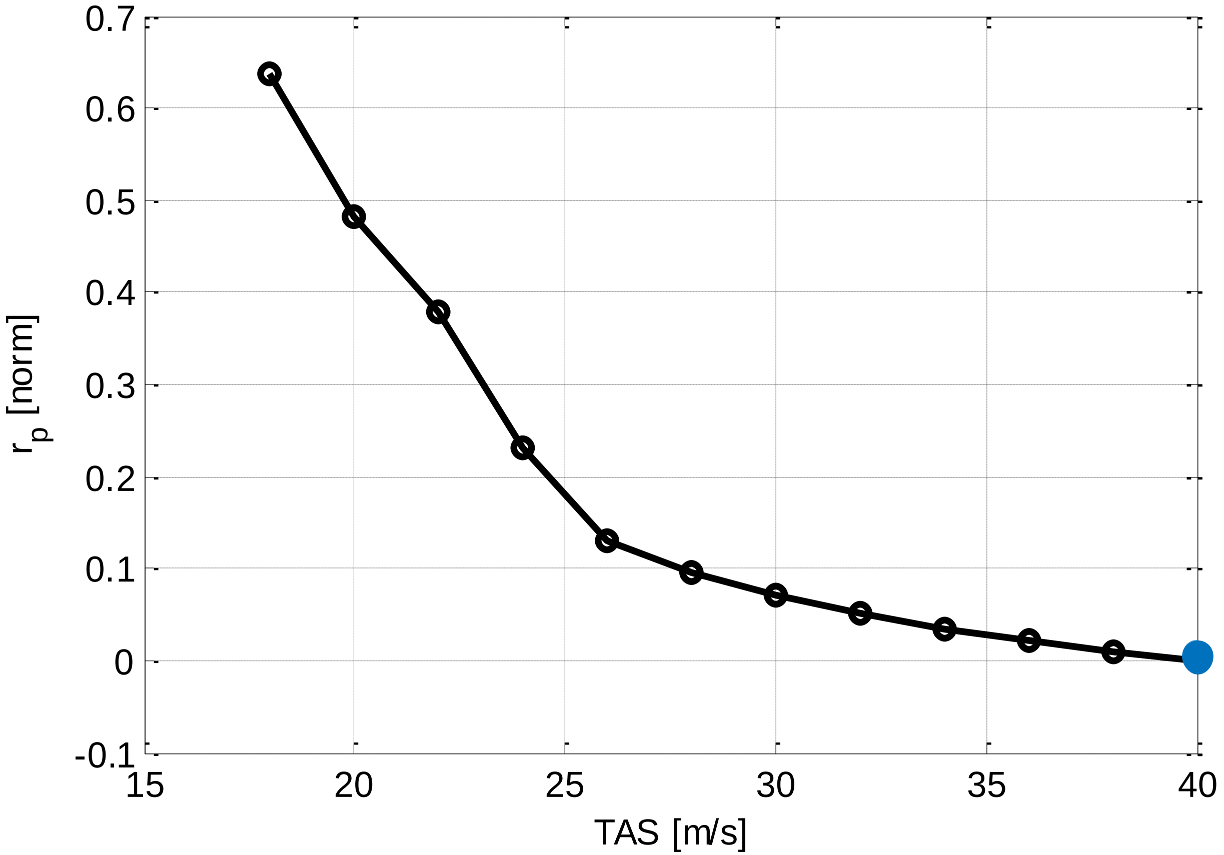
Mixing function designed at 30 m/s

Maximum of 60% pitch command required to trim the aircraft.



Mixing function designed at 40 m/s

Maximum of 63% pitch command required to trim the aircraft.



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Dynamic response of the aircraft

Mixing function designed at 20 m/s

TAS [m/s]	q [°/s]	r_p [norm]	$\dot{n}z$ [g]
18	4.4	0.503	-1.1171
20	11.66	0.6305	-1.3854
25	27.4	0.5860	-2.1731
30	41.3	0.6101	-3.1386
35	54.08	0.6223	-4.2779
40	66.13	0.6121	-5.5893
45	77.8	0.6407	-7.0803

- Maximum obtainable pitch rate
- Load factor on the aircraft

Dynamic response of the aircraft

Mixing function designed at 30 m/s

TAS [m/s]	q [°/s]	r_p [norm]	$\dot{n}z$ [g]
18	-	-	-
20	1.671	0.8566	-1.0491
25	13.74	0.8541	-1.5966
30	24.11	0.8685	-2.2673
35	33.48	0.8624	-3.0591
40	42.24	0.8640	-3.9733
45	50.56	0.8511	-5.0071

- Maximum obtainable pitch rate
- Load factor on the aircraft

Dynamic response of the aircraft

Mixing function designed at 40 m/s

TAS [m/s]	q [°/s]	r_p [norm]	$\dot{n}z$ [g]
18	5.6	1	-1.1555
20	12.5	1	-1.4145
25	28.33	1	-2.2110
30	42.25	1	-3.1834
35	55.1	1	-4.3327
40	67.29	1	-5.6595
45	79	1	-7.1608

- Maximum obtainable pitch rate
- Load factor on the aircraft

Approach

3 Inputs: Pitch, Roll, Yaw → 8 Control surface deflections

Model the nonlinear, 6 DOF aircraft

↓

Select a suitable mixing function structure

↓

Trim aircraft at a nominal flight condition

↓

Solve for mixing coefficients/parameters

↓

Test if scheduling is required

↓

Perform Robustness study

Robustness study

The following assumptions were made regarding the actuator failures:

- Single actuator failure at a time
- Actuator fail at zero degree deflection (δ = 0°)
- Results for mixing function designed at 30 m/s

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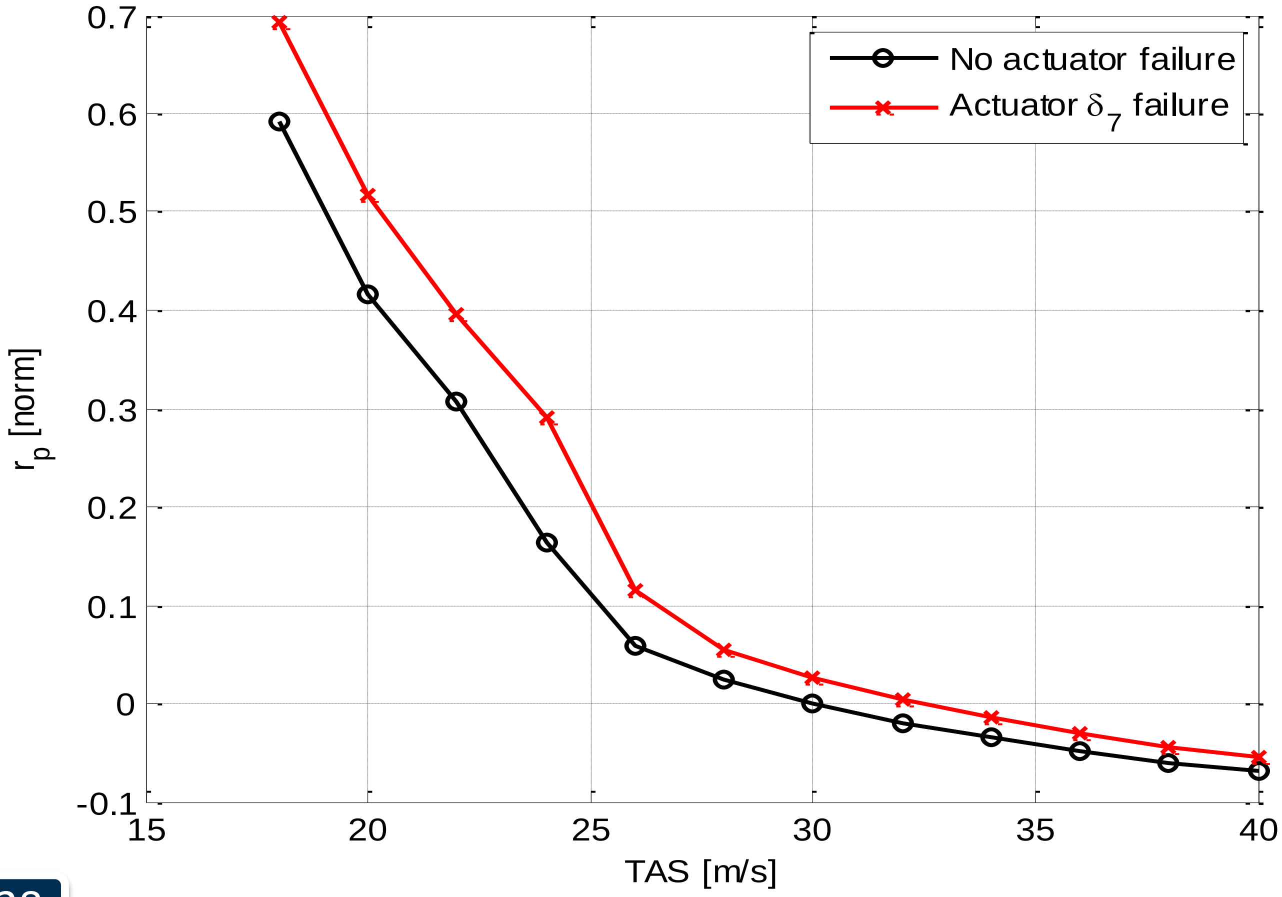
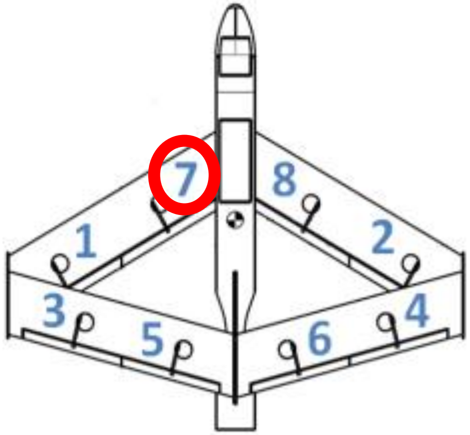
Robustness study

Simulated failure of inner actuator, front wing





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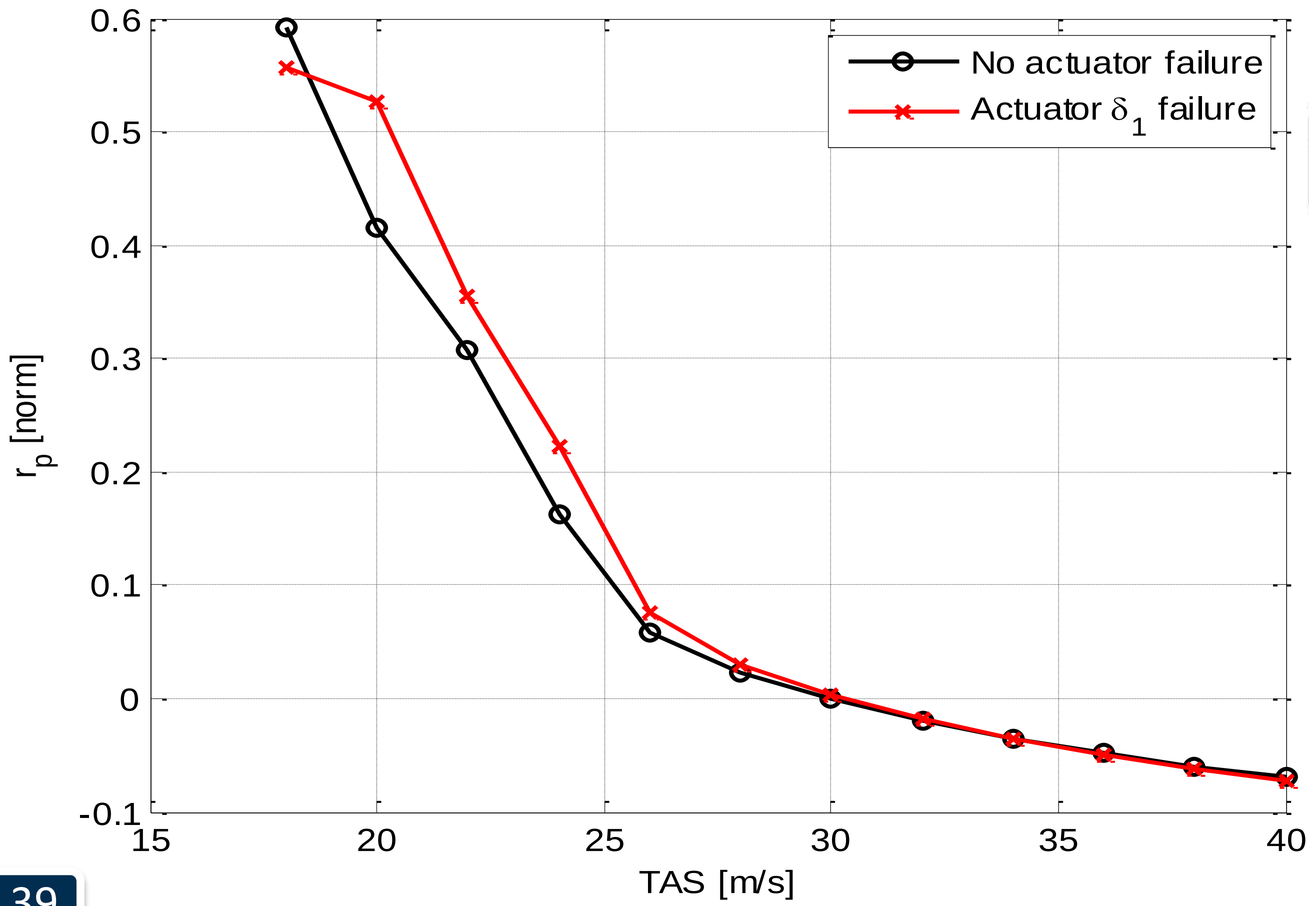
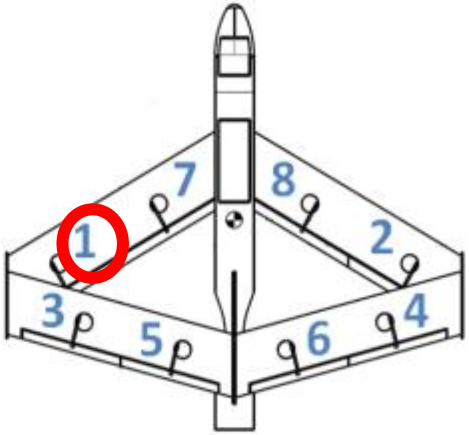
Robustness study

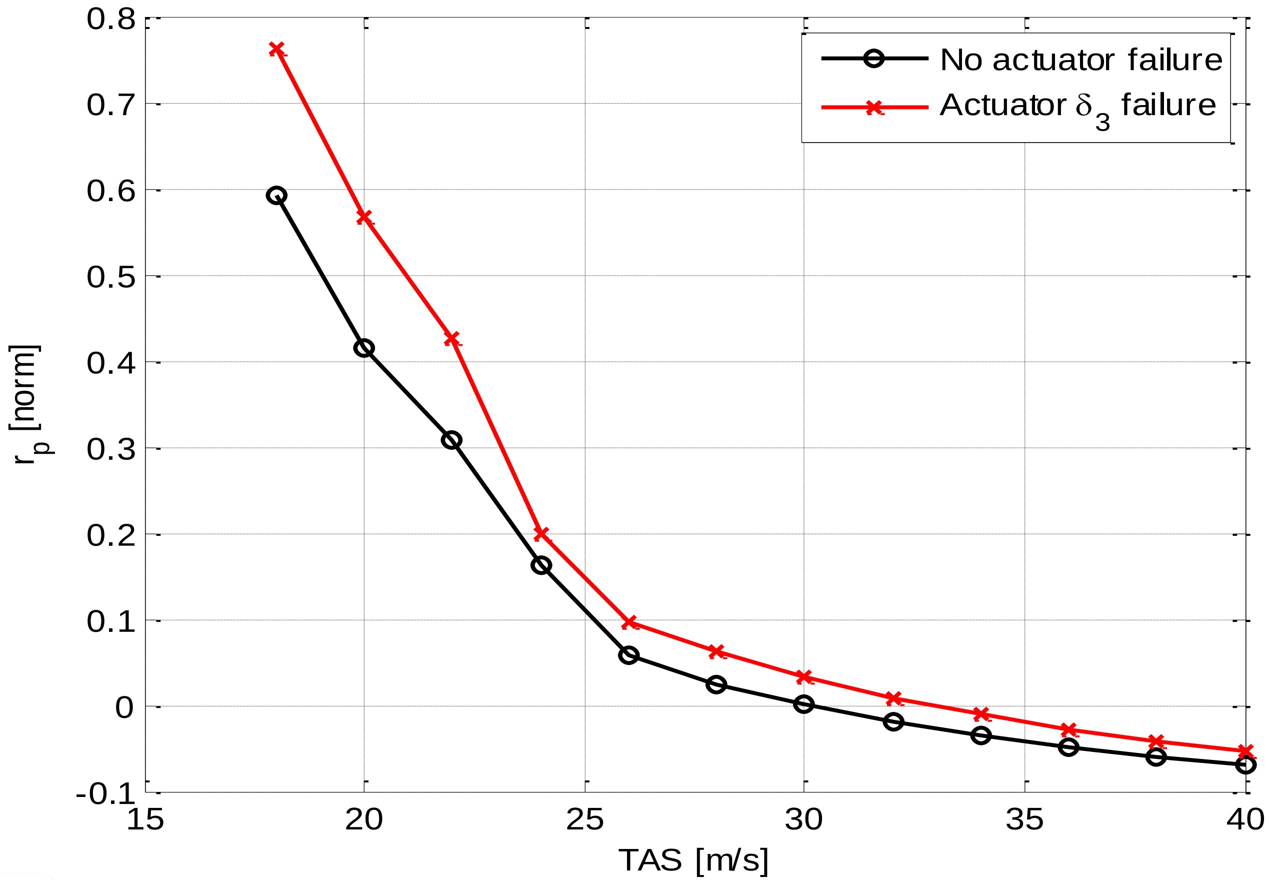
Simulated failure of outer actuator, front wing

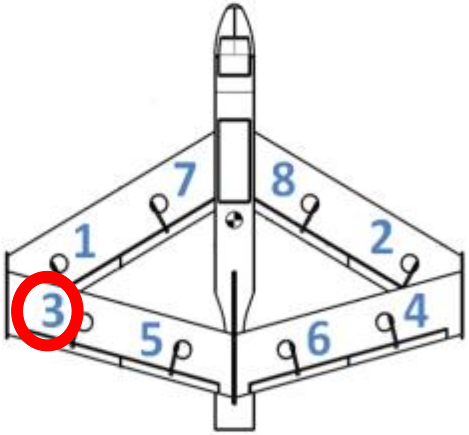




Robustness study

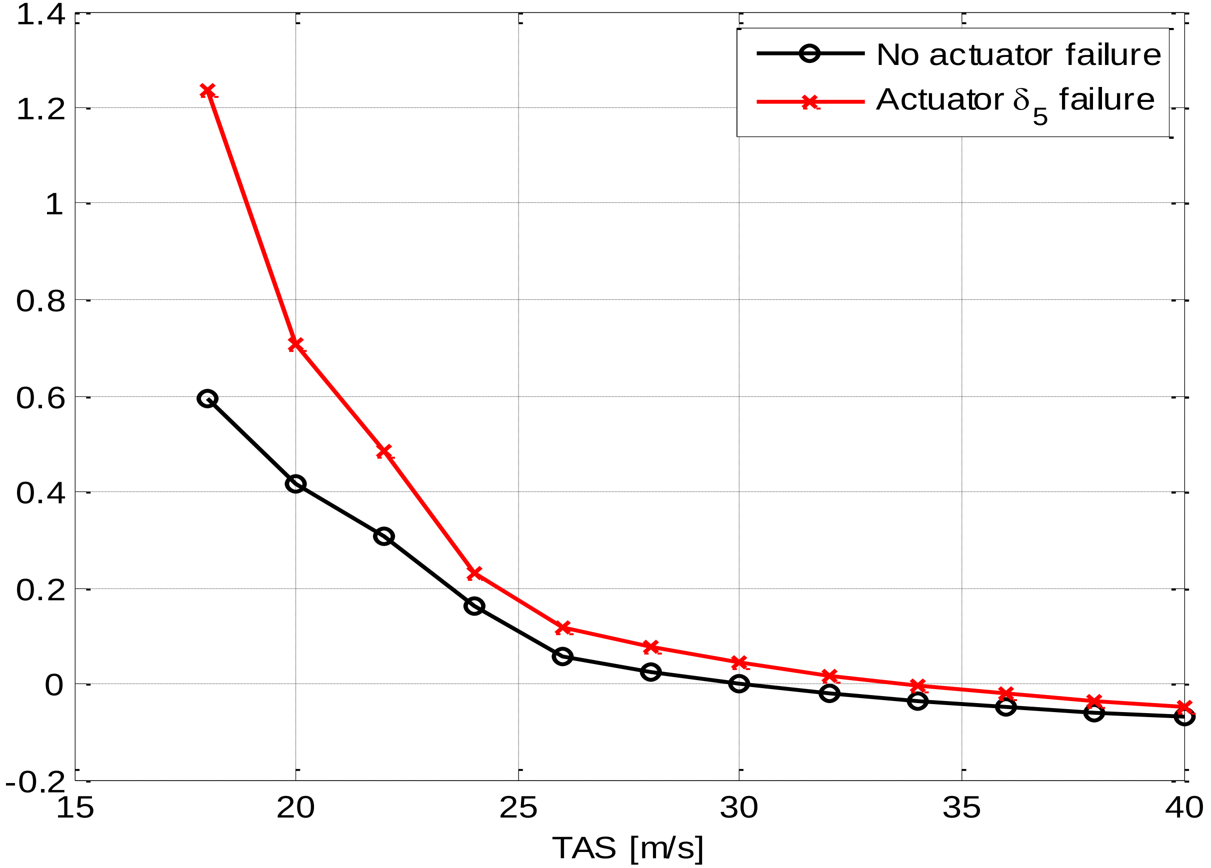
Simulated failure of outer actuator, rear wing

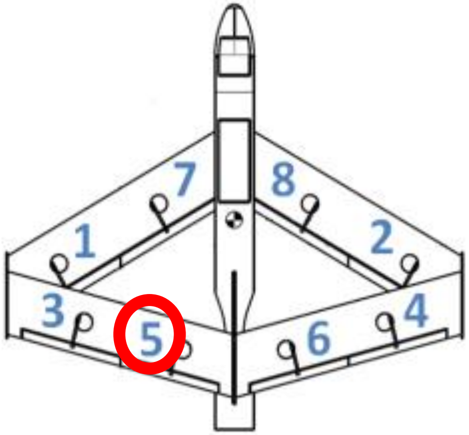




Robustness study

Simulated failure of inner actuator, rear wing





Conclusion

- A methodology to efficiently allocate controls was developed and demonstrated.
- The resulting aircraft response was demonstrated to be satisfactory, all design requirements were met.
- Pitch control authority through the entire flight envelope was found to be sufficient.
- Scheduling as a function of airspeed was investigated, use of single mixing function is satisfactory.
- The aircraft could still be trimmed in all cases except when actuator failure occur on the inner control surfaces on the rear wing.

Questions

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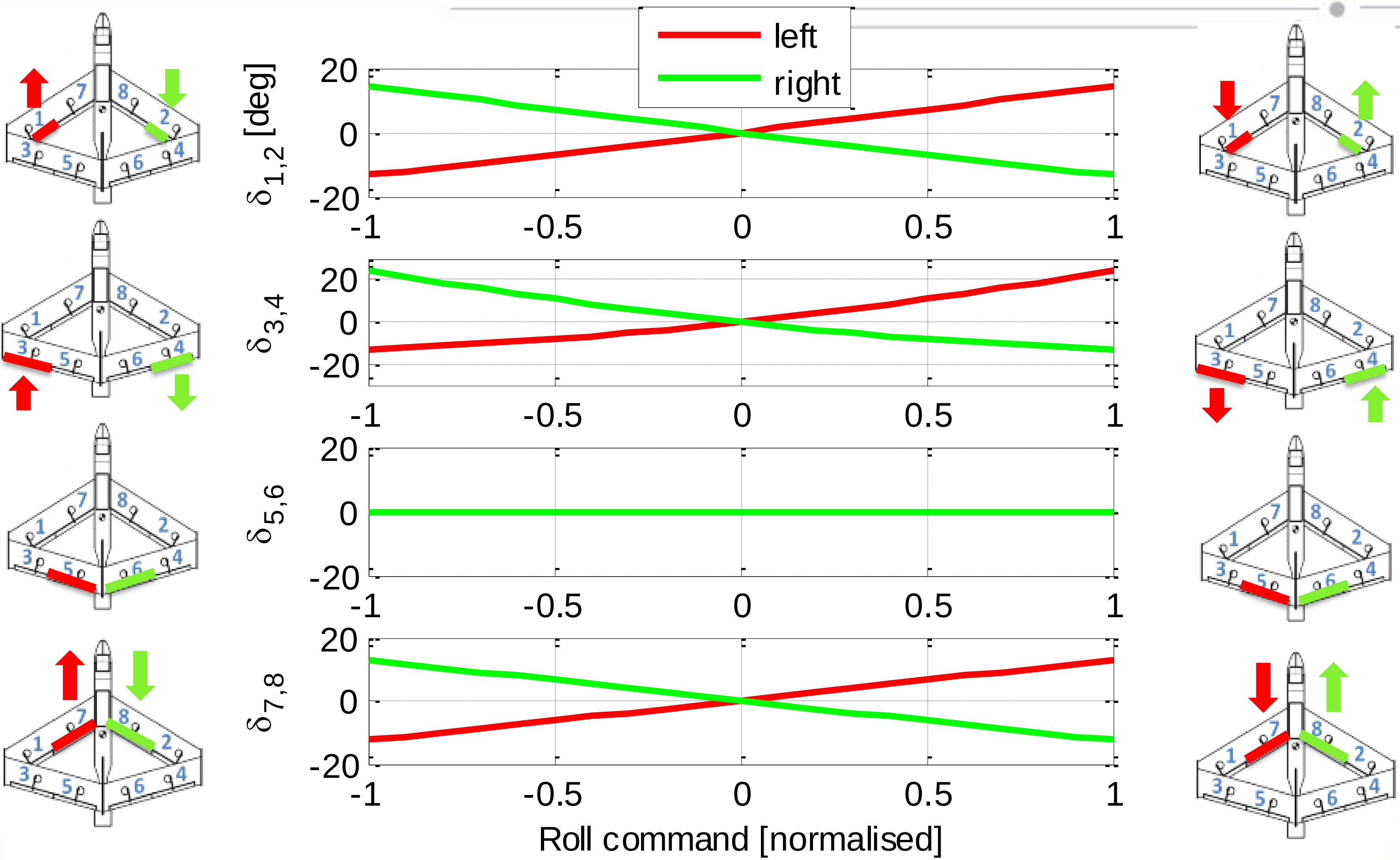
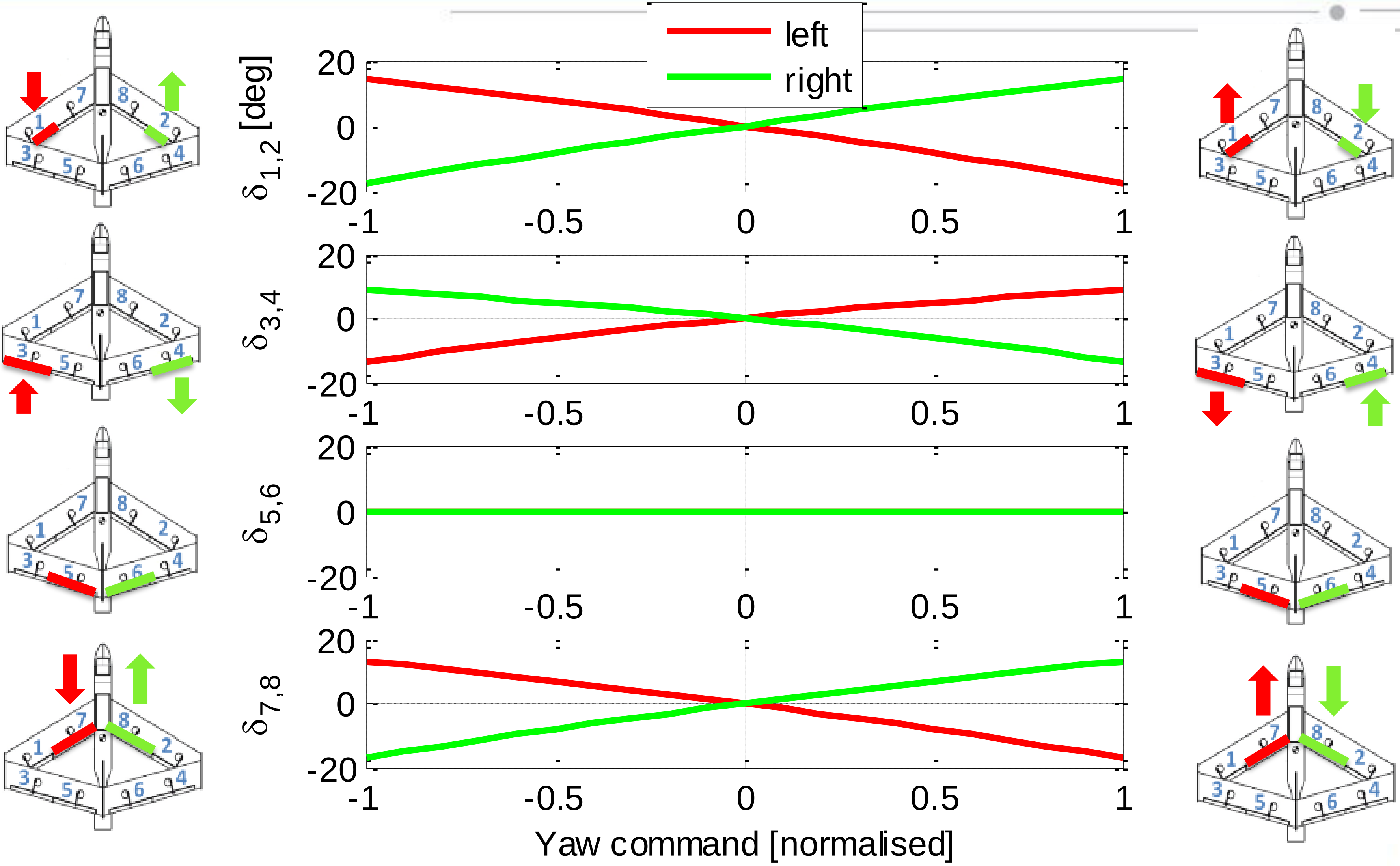
Elizna Miles (emiles@csir.co.za)

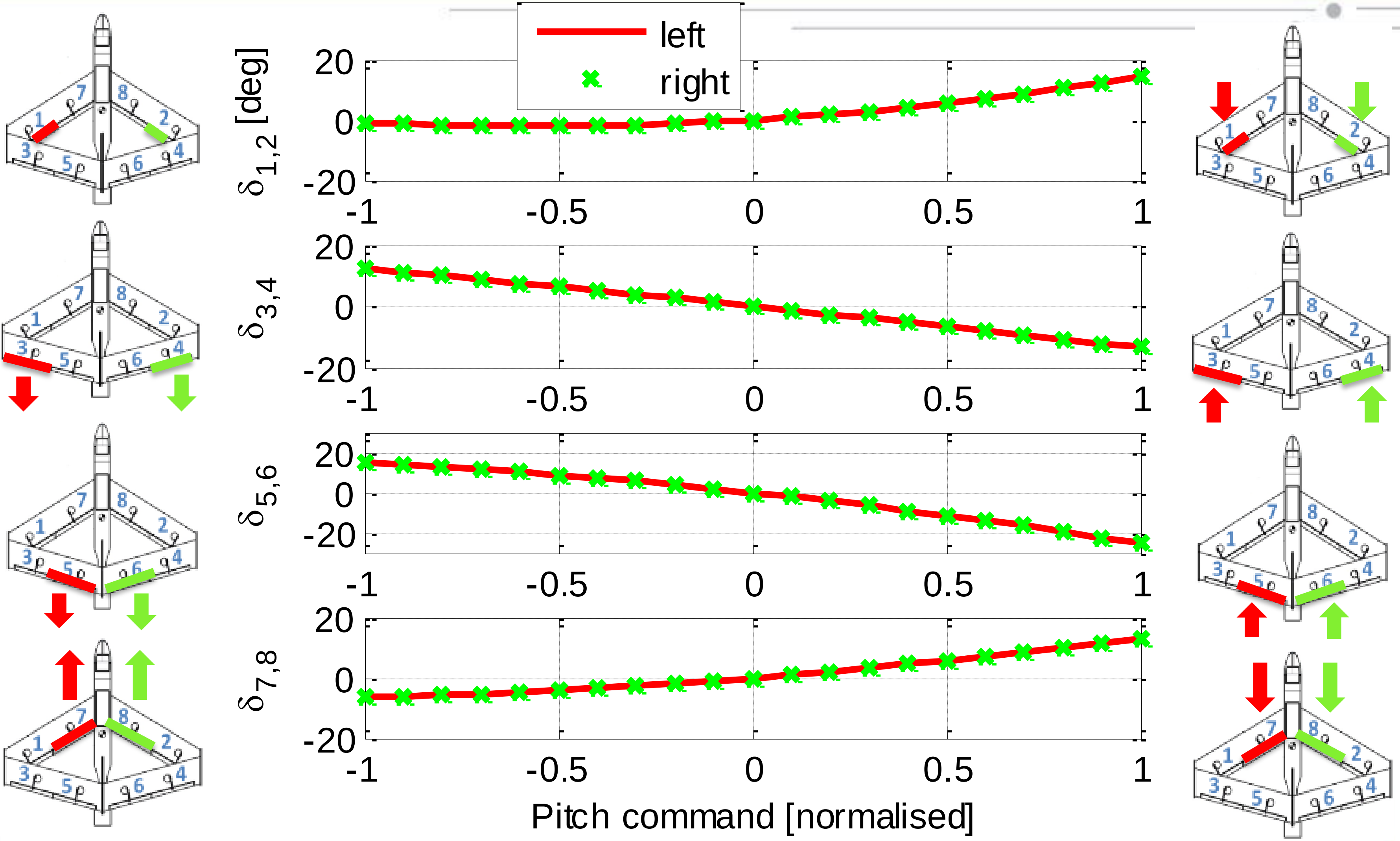
Roll control allocation



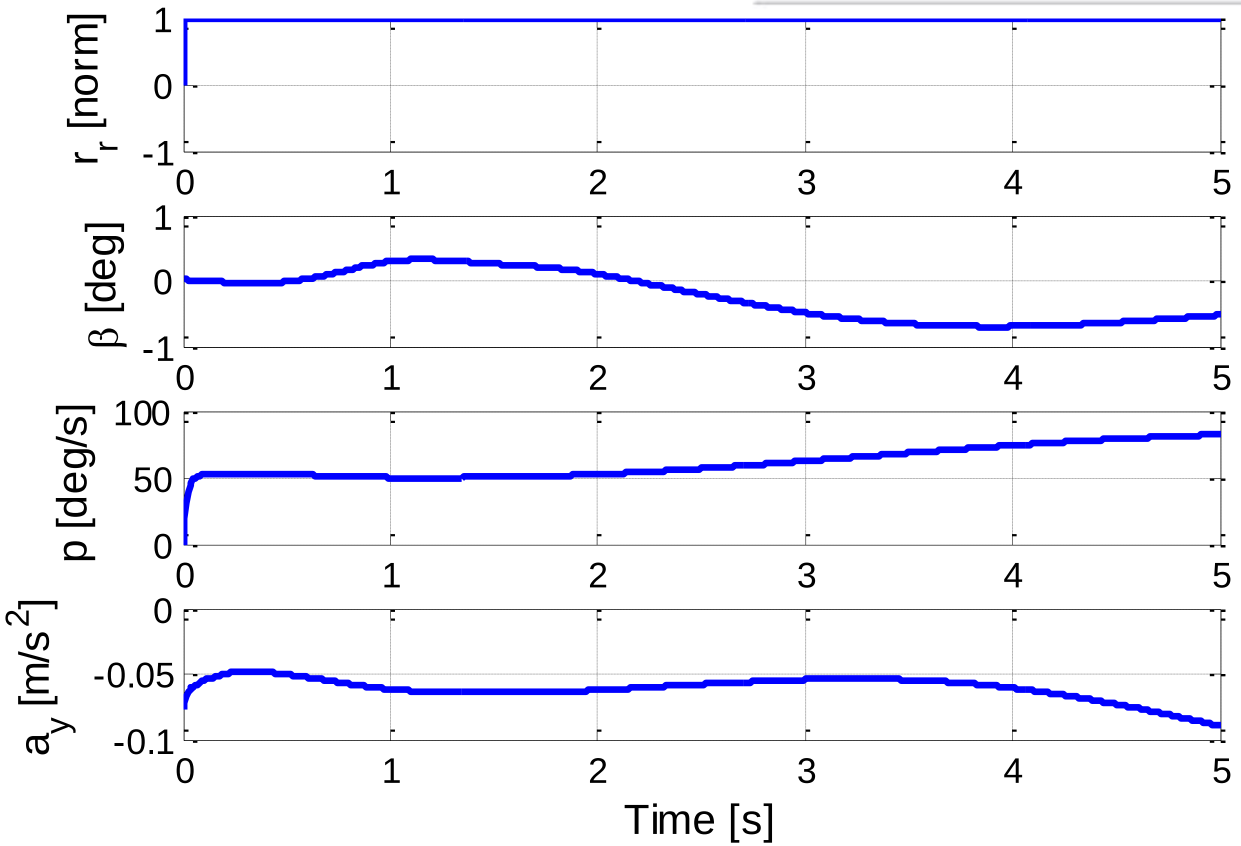
Yaw control allocation



Pitch control allocation

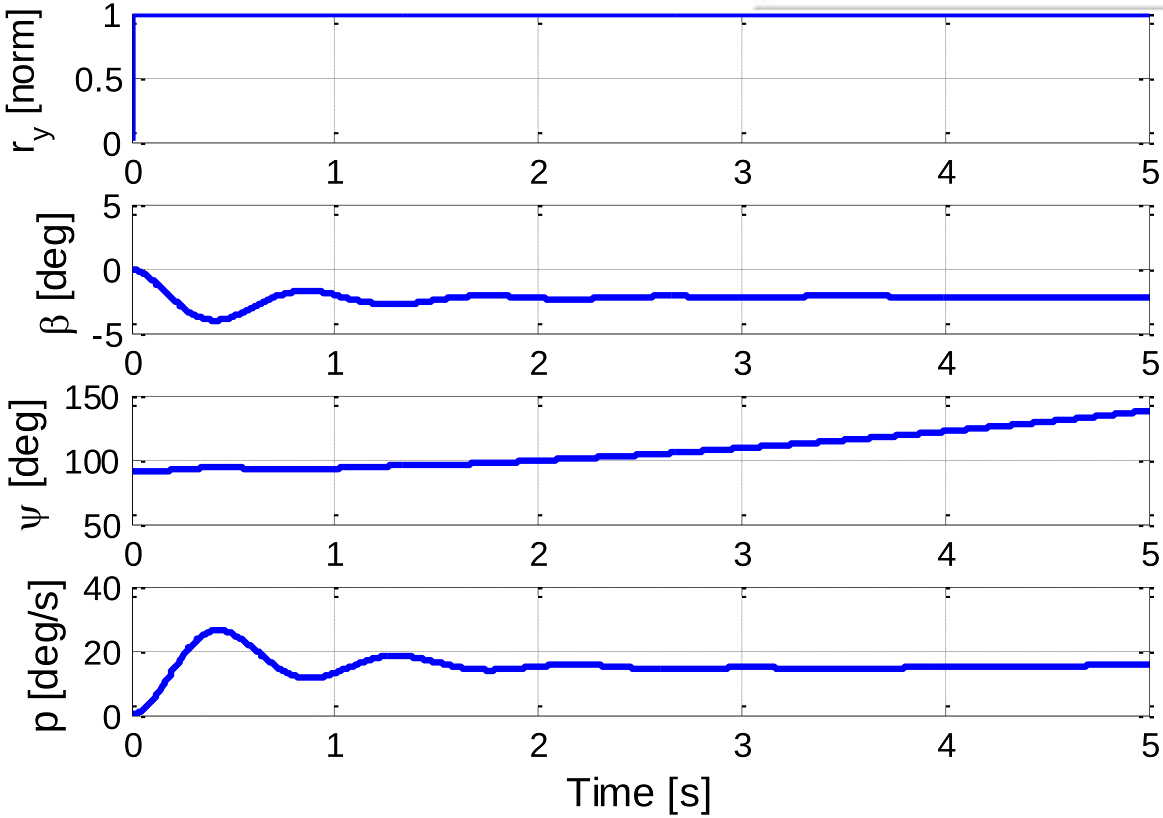


Response to step roll input



- Demonstrating a good roll response of 50 deg/s

Response to step yaw input



- Demonstrating sufficient yaw authority

Mixing function

$$\{\delta\} = [A]\begin{Bmatrix} {r_p}^2 \\ {r_r}^2 \\ {r_y}^2 \end{Bmatrix} + [B]\begin{Bmatrix} r_p \\ r_r \\ r_y \end{Bmatrix} + \{Trim\}$$

- Coefficient matrices:
  - Repeat some entries with appropriate signs to enforce symmetry (reduce number of unknown variables)

$$[A] = \begin{bmatrix} x(1) & x(2) & x(3) \\ x(1) & x(2) & x(3) \\ x(4) & x(5) & x(6) \\ x(4) & x(5) & x(6) \\ x(7) & x(8) & x(9) \\ x(7) & x(8) & x(9) \\ x(10) & x(11) & x(12) \\ x(10) & x(11) & x(12) \end{bmatrix} \qquad [B] = \begin{bmatrix} x(13) & x(14) & x(15) \\ x(13) & -x(14) & -x(15) \\ x(16) & x(17) & x(18) \\ x(16) & -x(17) & -x(18) \\ x(19) & x(20) & x(21) \\ x(19) & -x(20) & -x(21) \\ x(22) & x(23) & x(24) \\ x(22) & -x(23) & -x(24) \end{bmatrix}$$

Optimisation Phase 2 : Control allocation

- Normalised objective function

$$f(x) = -\left(\begin{array}{c} 2w_1 \dfrac{\dot{p}}{abs(\dot{p}^{max})} - 2w_2 \dfrac{\beta}{abs(\beta^{max})} + \\ w_3 \dfrac{\dot{q}_{pos}}{abs\left((\dot{q}_{pos})^{max}\right)} - w_4 \dfrac{\dot{q}_{neg}}{abs\left((\dot{q}_{neg})^{max}\right)} \end{array} \right)$$

- Advantages of normalisation
  - Avoid numerical instability
  - Objectives are of the same order magnitude
  - Weight selection is more intuitive