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Comparison between SAFE forced response and Abaqus/Explicit solutions of a rail excited at a cut-off frequency

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Abstract

In this paper, we present a comparison between two different methods for modelling the response of an elastic waveguide to a point force excitation. Both frequency ranges which contain, as well as those that do not contain, mode cut-offs are considered. Firstly, an Abaqus/Explicit model is presented. In order to simulate an infinitely long waveguide, reflections from free ends are eliminated using absorbing boundary conditions. The Abaqus/Explicit time domain results are compared to time domain results computed using a semi-analytical finite element (SAFE) method forced response solution. The SAFE method is formulated and solved in the frequency domain, and encounters numerical difficulties when transforming results to the time domain if cut-off frequencies are excited. These difficulties are due to the fact that the steady state solution at the cut-off frequency is unbounded. Selected methods to alleviate these numerical difficulties are presented and compared. A frequency domain comparison between the two models is also performed. The SAFE method computes the amplitude of each mode, as a function of frequency and therefore modal contributions are naturally separated. In order to perform the comparison, a procedure to extract modal amplitudes from the time domain Abaqus/Explicit results is required. A method previously used to extract modal amplitudes from experimental results is used for this purpose. Good agreement between the SAFE and Abaqus/Explicit results is demonstrated in both the time and the frequency domains.

Keywords: Guided waves; SAFE forced response; Abaqus/Explicit; cut-off frequency; mode extraction

1. Introduction

Guided wave ultrasound (GWU) is well suited for inspection and monitoring applications of elongated structures such as plates, rods, pipes and rails [1]. Guided waves can propagate long distance, especially when compared to conventional ultrasonic inspection (up to kilometers in some cases [2]). In order to design a GWU-based non-destructive evaluation (NDE) system, it is necessary to understand how guided waves are excited and how they propagate. A conventional time-domain finite element analysis can be carried out for this purpose. However, time-domain simulations are generally very numerically expensive (if possible at all) especially at high frequencies and over significant distance, due to the fine spatial and temporal discretization required. Furthermore, since the analysis is carried out in the time domain, modal information is not obtained directly and has to be extracted in some way. Due to these drawbacks, the semi-analytical finite element (SAFE) method [3, 4, 5] has become a popular analysis and design tool in the GWU community.

SAFE forced response analyses are generally carried out in the frequency domain, with the steady state response being computed at specific frequencies. The SAFE method naturally computes results based on their modal contributions. Long range responses can efficiently be estimated since the propagation direction is treated analytically. For transient simulations, a Fourier transform is used to convert the forcing function from the time to the frequency domain. An inverse Fourier transform can then be used to convert the response from the frequency domain back to the time domain. One complication with this type of analysis occurs when modes of propagation are excited at or close to their cut-off frequency. This situation produces an unrealistically large response if no damping is present since the steady state response is computed at each frequency step. This behavior of propagating waves excited close to their cut-off frequency is analogous to driving a vibrating structure at its natural frequency.

The focus of this paper is on the forced response of guided waves when modes are excited close to cut-off frequency. The resonance-like behaviour encountered when exciting a mode of propagation close to its cut-off frequency is studied and addressed. Several methods to deal with the unrealistically large time domain response are compared. The response in the frequency domain is also considered. The SAFE forced response is compared with results from a time domain solution computed using the commercial finite element package Abaqus/Explicit. This comparison represents a verification that the SAFE method accurately solves the idealised forced response problem. It is assumed that Abaqus/Explicit model produces the correct time-domain solution, and the verification of the SAFE results is performed relative to the Abaqus/Explicit results.

2. Problem formulation

The problem being considered is depicted in Figure 1(a), and involves determining the response to a harmonic point force applied to the web of a UIC60 rail. In figures 1(b-d) the dispersion curves for the UIC60 rail in the frequency range of interest are also depicted. These dispersion curves, computed using the SAFE method, have been separated in symmetric and anti-symmetric modes for convenience in the development which follows. Also depicted in the figures is the limit on phase velocity which is used to filter modes of propagation close to cut-off, as detailed in Section 3.

In this section, the SAFE forced response formulation is presented. This formulation is based on the SAFE formulation proposed by Gavric [3] and uses a subtle variation of the transformation proposed by Damljanovic et al. [5, 6]. Since these formulations are relatively well know, only the salient features of our formulation will be presented here. More detail regarding the conventional SAFE formulation can be found in, for example, [4, 7].

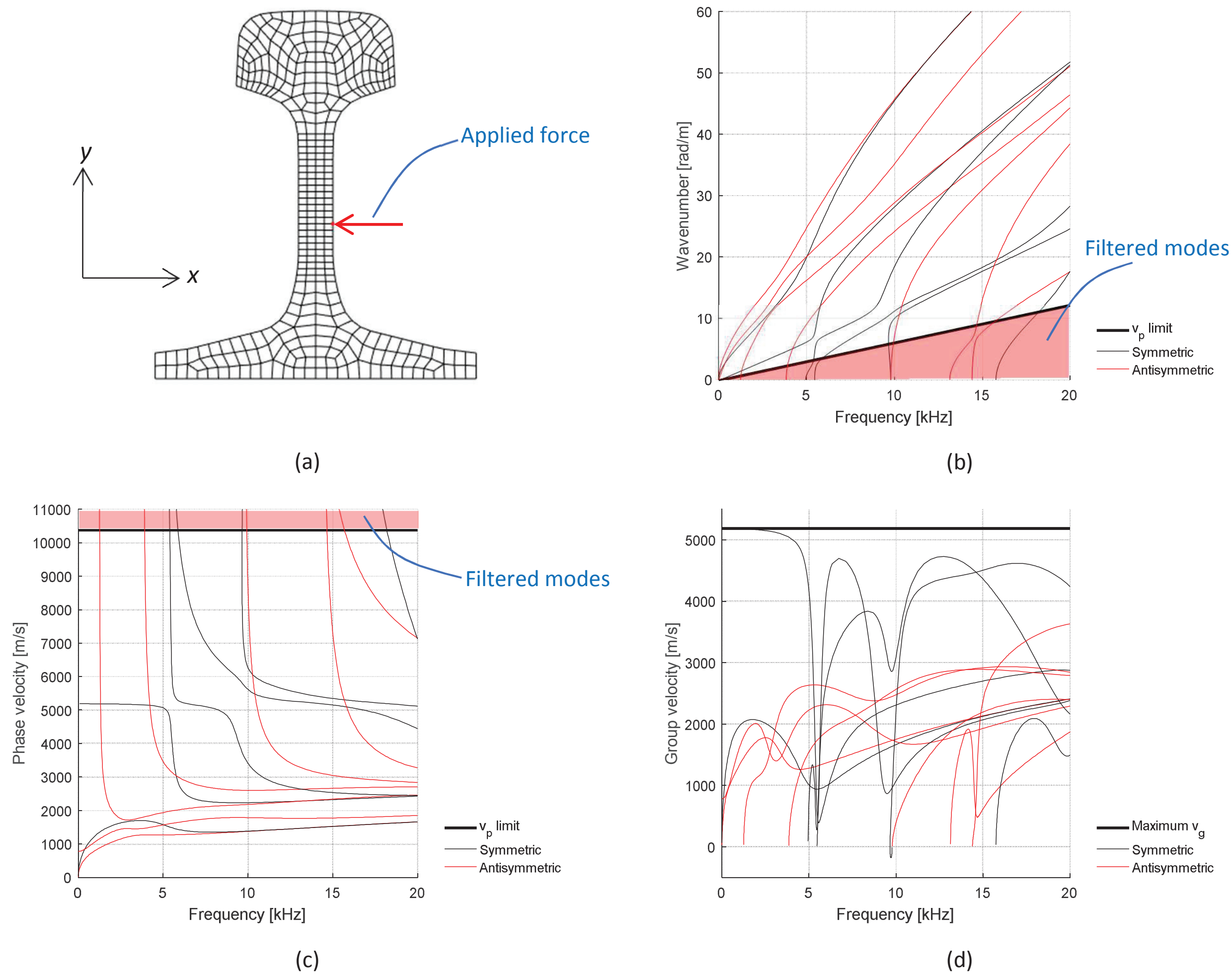


Fig. 1. (a) UIC60 rail SAFE mesh with applied force and associated (b) wavenumber-frequency, (c) phase velocity-frequency, (d) group velocity-frequency, dispersion curves

2.1. SAFE formulation

As stated, the semi-analytical finite element (SAFE) formulation employed in this paper is based on that proposed by Gavric [3]. This formulation is convenient since it results in symmetric stiffness matrices. However, since only free vibrations were considered in [3], some detail needs to be added for the forced response problem. A one-dimensional waveguide with arbitrary cross-section in the $x-y$ plane and with wave propagation in the $z-$direction is considered, see Figure 1(a). The displacements are assumed to take the form:

$$
\begin{aligned}
u_x(x,y,z,t) &= u_x(x,y)e^{-j(\kappa z-\omega t)} = \hat{u}_x(x,y)e^{-j(\kappa z-\omega t)} \\
u_y(x,y,z,t) &= u_y(x,y)e^{-j(\kappa z-\omega t)} = \hat{u}_y(x,y)e^{-j(\kappa z-\omega t)} \\
u_z(x,y,z,t) &= u_z(x,y)e^{-j(\kappa z-\omega t)} = j\,\hat{u}_z(x,y)e^{-j(\kappa z-\omega t)},
\end{aligned}
\tag{1}
$$

where $je^{-j(\kappa z-\omega t)} = e^{-j(\kappa z-\omega t-\pi/2)}$. These displacement equations can be written in vector form as:

$$
\boldsymbol{u}(x,y)e^{-j(\kappa z-\omega t)} = \boldsymbol{T}\,\hat{\boldsymbol{u}}(x,y)e^{-j(\kappa z-\omega t)} \tag{2}
$$

where displacements in the physical coordinate system (i.e. the coordinate system of the conventional 3D finite elements) are denoted $\boldsymbol{u}$. The transformed (SAFE) displacements, which presuppose a 90° phase shift between in-plane and out of plane displacements are denoted $\hat{\boldsymbol{u}}$. A transformation matrix $\boldsymbol{T}$ which converts the SAFE displacements to physical displacements has been introduced, similar to that introduced by Damljanovic et al. [5], and is defined as

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & j \end{bmatrix} \quad (3)$$

The transformation matrix has the following properties:

$$TT^{*T} = T^{*T}T = I \quad (4)$$

where $(\cdot)^{*T}$ denotes the complex conjugate transpose, and the reverse of the transformation in (2) is given by:

$$\widehat{u} = T^{*T} u. \quad (5)$$

This same transformation can be employed to convert physical forces in the global coordinate system to the transformed SAFE forces. The SAFE formulation ultimately results in the following system of equations (see [8, 9] for details)

$$\left[-\omega^2 M + \kappa^2 K_2 + \kappa K_1 + K_0\right] \widehat{U} = \widehat{F} \quad (6)$$

where the vectors $\widehat{\boldsymbol{U}}(\omega, \kappa)$ and $\widehat{\boldsymbol{F}}(\omega, \kappa)$ represent the transformed nodal displacements and forces respectively. The individual mass and stiffness matrices can be found in Gavric [3].

2.2. Solution of the free and forced vibration problems

The forced response will be used to estimate the frequency dependant stiffness of the waveguide for each degree of freedom of the 3D transducer model that is in contact with the waveguide. The forced response problem was considered by [4, 5, 7]. Equation (6) is cast in linear form as

$$[\mathcal{A} - \kappa \mathcal{B}] \widehat{\mathcal{U}} = \widehat{\mathcal{F}} \quad (7)$$

where:

$$\mathcal{A} = \begin{bmatrix} K_0 - \omega^2 M & 0 \\ 0 & -K_2 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} -K_1 & -K_2 \\ -K_2 & 0 \end{bmatrix} \quad (8)$$

and

$$\widehat{\mathcal{U}}(\omega, \kappa) = \left\{ \begin{array}{c} \widehat{U} \\ \kappa \widehat{U} \end{array} \right\}, \; \widehat{\mathcal{F}}(\omega, \kappa) = \left\{ \begin{array}{c} \widehat{F} \\ 0 \end{array} \right\} \quad (9)$$

The homogeneous form of (7) can be used to compute a set of wavenumbers κ_i and associated mode shapes $\widehat{\boldsymbol{\psi}}_i$ at a fixed frequency ω from the eigenvectors given by

$$\widehat{\Psi}_i = \left\{ \begin{array}{c} \widehat{\psi}_i \\ \kappa \widehat{\psi}_i \end{array} \right\} \quad (10)$$

Treyssede et al. [7] show how the solution of the forced response problem can be written as an expansion of the modes, and also how the inverse Fourier transform of the displacement solution can be computed using the Cauchy residue theorem to yield a response in the transformed SAFE space-time domain. The displacement response is a superposition of the response of each mode, and can be written in terms of the modal amplitudes, denoted α, as:

$$\widehat{U}(z, \omega) = \sum_{r=1}^{3N} \alpha_r e^{-j\kappa_r z} \widehat{\psi}_r, \quad \text{where } \alpha_r = j \frac{\widehat{\Psi}_r^T \widehat{\mathcal{F}}}{\widehat{\Psi}_r^T \mathcal{B} \widehat{\Psi}_r}. \quad (11)$$

To convert these SAFE displacements back to physical displacements in the global coordinate system, a similar transformation to that proposed in (2) is used.

Tressede et al. [7] point out that the summation of modes should be carried out over forward propagating modes as determined from the sign of the energy velocity (or group velocity where applicable). The response in space-time can be computed by taking the inverse temporal Fourier transform of $\boldsymbol{U}(z, \omega)$.

3. Treatment of modes excited close to cut-off frequency

Exciting a guided wave mode at, or close to its cut-off frequency is known to result in resonant-like behaviour, analogous to exciting a mode of vibration in a finite structure at its resonant frequency. In the absence of damping the steady-state response at the cut-off frequency becomes unbounded. This behaviour causes numerical difficulties when converting frequency domain results to the time domain, as may be required when using the SAFE forced response analysis described here. The time domain result at cut-off is dominated by this resonant-like response at the cut-off frequency which manifests as a ringing which wraps-around in the time domain.

These difficulties are effectively controlled by introducing damping into the system [10, 11]. However, realistic values for damping are usually difficult to estimate a priori and are usually iteratively determined through experimental comparison. This can be numerically quite expensive since for each damping value evaluated the SAFE eigenvalue problem needs to be re-solved.

Post-processing or filtering methods, which can be applied without having to resolve the SAFE eigenvalue problem, have also been proposed. Stoyko [12] proposed eliminating the wrap-around effect by adding a homogeneous solution consisting of the response of each mode at cut-off, and using this to enforce the initial displacement and velocity conditions, which are not necessarily satisfied using the inverse Fourier transform. This process is referred to in [12] as enforcing causality. An alternative post-processing method is to simply eliminate the large responses at cut-off by effectively filtering the contribution of the modes in the proximity of the cut-off frequency [13]. High phase velocities or low wavenumber thresholds can be used to identify modes at cut-off.

Post-processing methods and the inclusion of damping are evaluated in this study. Details of the methods evaluated are as follows:

- Causality was enforced as proposed by Stoyko [12]. Only modes which cut-off in the frequency range of interest were used to compute the homogeneous solution. The amplitude and phase of the cut-off modes were computed so as to best enforce the initial conditions (zero initial displacement and velocity at each degree of freedom) in a least-squares sense.
- The filtering method was implemented by simply setting modal amplitudes (11) to zero if the phase velocity associated with the modal amplitude was greater than twice the maximum group velocity in the frequency range of interest. This limit is depicted in Figure 1(b-d), and although it may appear very aggressive it was found to be appropriate if relatively few frequency points are used.
- In order to damp the large response at cut-off, hysteretic damping is employed with complex bulk velocities as defined in [10 11]. Bartoli et al. [11] showed that in the frequency range of interest here, longitudinal and shear bulk wave attenuation of κ_L=0.003 Np/wavelength and κ_T=0.043 Np/wavelength respectively are appropriate. Since these values are not always well know, the effects of over- and under-estimating the damping are simulated by using damping constants 10 times greater and 10 times smaller than those proposed in [11].

4. Abaqus model and mode extraction

In this section, some details regarding the Abaqus/Explicit model used to verify the accuracy of the SAFE forced response results, are presented. Absorbing boundary conditions to prevent reflections from free ends of the 3D FE models and briefly considered. The method used to perform mode extraction from the 3D FE time domain results is also detailed. Figure 2 illustrates the problem under consideration. A transient force is applied at the same position indicated in Figure 1(a), with more detail of the loading presented in Section 5.1.

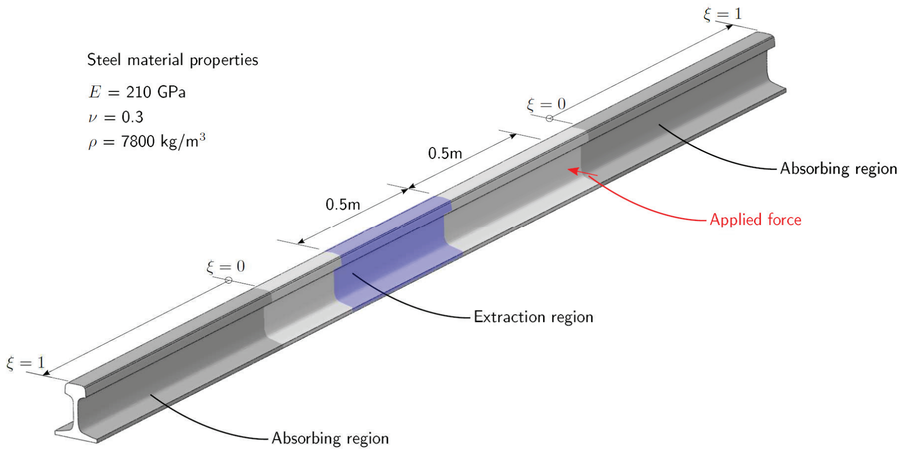


Fig. 2. Abaqus/Explicit model of the rail with a transient point force applied.

4.1. Absorbing boundary conditions

In order to extract modes excited by the transducer in the time domain, relatively long displacement time traces are required. This is especially true when different modes travel at significantly different speeds or when exciting modes close to their cut-off frequency, due to the ringing-like behaviour observed. When considering relatively long simulation times in the time domain, complex mode coupling due to end reflections adds unwanted complexity. It is therefore necessary to be able remove these end reflections in order to simulate an infinitely long rail.

The problem of eliminating end reflections has been studied by various authors. Two methods which can relatively easily be implemented using commercial codes were proposed by Rajagopal et al. [14] (Absorbing Layers using Increasing Damping (ALID)) and Pettit et al. [15] (stiffness reduction method). A hybrid method, combing stiffness reduction and increased damping, was found to work well in this instance for one-dimensional wave propagation.

In the absorbing region, a local coordinate, $0 < \xi \leq 1$ is introduced, with $\xi = 0$ at the start of the absorbing region and $\xi = 1$ at the free end of the waveguide as depicted in Figure 2. A damping factor ($0 \leq d \leq 1$) and a stiffness factor ($0 < s \leq 1$) are then defined as:

$$d(\xi) = \xi^p \tag{12}$$

$$s(\xi) = e^{-d(\xi)\cdot p} - \xi \cdot (e^{-p} - \epsilon) \tag{13}$$

where p is a penalty parameter and the second term in (13) ensures that the stiffness parameter ends with a small value ε. For each element in the absorbing region, the local coordinate of the element centroid is determined and the modified elastic modulus E^* and Rayleigh mass proportional damping constant α^* for the element is computed as

$$E^* = s \cdot E_0 \text{ and } \alpha^* = d \cdot \alpha_{max} \tag{14}$$

where E_0 is the elastic modulus of the waveguide material and α_{max} is the maximum value of Rayleigh mass proportional damping, which is set to the centre circular frequency of the driving signal as suggested in [15].

4.2. Mode extraction from time domain results

In order to compare numerical time-domain results with mode-based results in the frequency domain computed using the SAFE forced response analysis, a method to identify and quantify which modes are excited is required. This is not a trivial task due to the multi-modal and dispersive nature of guided wave propagation [16]. The method employed here decomposes extracted displacements into modal contributions using SAFE information, and has been successfully used to extract modes (and modal amplitudes) from experimental measurements [17]. The displacement response at a specific degree of freedom in the SAFE mesh i at distance z can be written in the physical coordinate system as (see (11)):

$$U_i(z,\omega) = \sum_{r=1}^{3N} \psi_{ir}(\omega)\alpha_r(\omega)e^{-j\kappa_r(\omega)z}, \tag{15}$$

where $\psi_{ir}(\omega)$ is the displacement of dof i of mode shape r and $\kappa_r(\omega)$ is the wavenumber of mode r. The mode shape and wavenumber are computed using a SAFE analysis. We wish to extract the magnitude of each propagating mode $\alpha_r(\omega)$ from the Abaqus time responses $U_i(z,\omega)$. For p time traces of Abaqus/Explicit degrees of freedom corresponding to SAFE degrees of freedom, this can be written in matrix form as

$$\bar{\boldsymbol{\Psi}}(\omega)\boldsymbol{\alpha}(\omega) = \boldsymbol{U}(\omega), \tag{16}$$

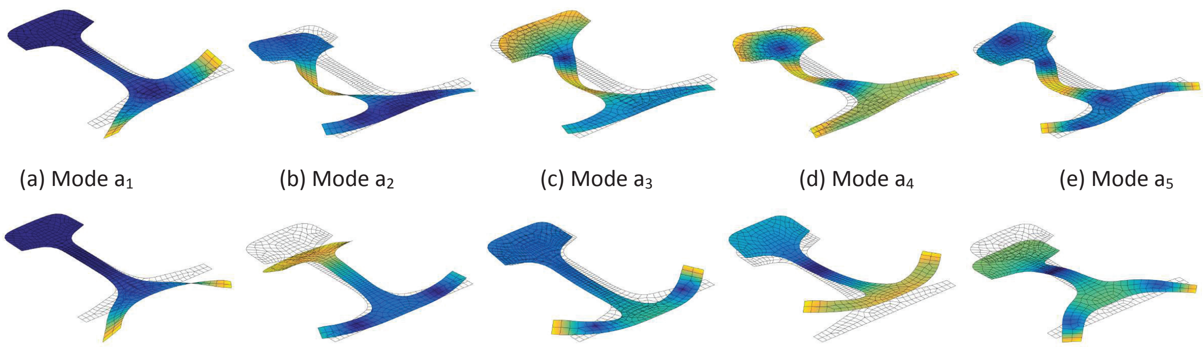
where

$$\begin{bmatrix} \psi_{11}e^{-j\kappa_1 z_1} & \psi_{12}e^{-j\kappa_2 z_1} & \dots & \psi_{1m}e^{-j\kappa_m z_1} \\ \vdots & \vdots & & \vdots \\ \psi_{p1}e^{-j\kappa_1 z_p} & \psi_{p2}e^{-j\kappa_2 z_p} & \dots & \psi_{pm}e^{-j\kappa_m z_p} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{Bmatrix} = \begin{Bmatrix} U_1 \\ \vdots \\ U_p \end{Bmatrix}. \tag{17}$$

The mode shape matrix $\bar{\boldsymbol{\Psi}}$ is assembled from information from the SAFE model while vector $\boldsymbol{U}$ is assembled by performing a FFT on each of the extracted time domain Abaqus/Explicit displacement signals at various propagation distances z. The results in Section 5 are generated using displacement signals extracted from 200 randomly distributed nodes between 0.5m and 1m from the applied force, as illustrated in Figure 2.

5. Results

This section presents the results of the numerical comparison between Abaqus/Explicit and the SAFE forced response solution. The problem under consideration is illustrated in Figure 2, and is depicted for the SAFE case in Figure 1(a). Figures 1(b) to 1(d) depict the dispersion curves for the UIC60 rail with material properties as given in Figure 2. Modes are separated exploiting the fact that for a symmetric waveguide, families of symmetric and antisymmetric modes can cross, but the dispersion curves within a symmetric or antisymmetic family approach and then repel each other and do not cross [18].



(a) Mode a_1 (b) Mode a_2 (c) Mode a_3 (d) Mode a_4 (e) Mode a_5

(f) Mode s_1 (g) Mode s_2 (h) Mode s_3 (i) Mode s_4 (j) Mode s_5

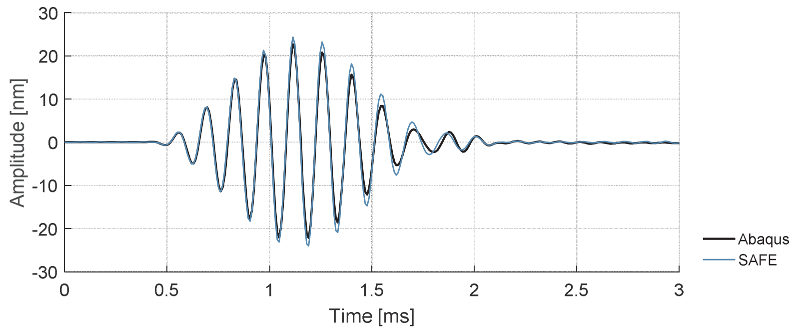
Fig. 3. Antisymmetric modes a_1 to a_5 and symmetric modes s_1 to s_5 computed at 10 kHz.

Figure 3 shows the mode shapes of all the propagating modes at 10kHz. The mode numbering scheme proposed in [18] is used to number the modes with an `a' or `s' to represent antisymmetric and symmetric modes, respectively followed by a number representing the order in which cut-off occurs on the frequency axis. So, for example the 3rd antisymmetric mode to cut-off is numbered a_3.

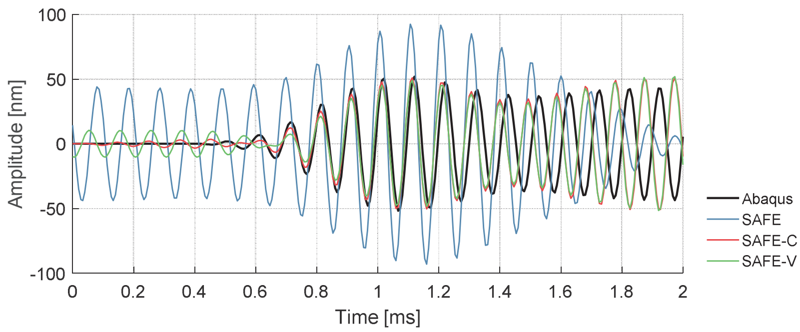
In order to study the effects of cut-off, a frequency range with an isolated cut-off frequency is required. To this end a convenient frequency range was found to be around 10kHz. The two modes which cut-off close to 10kHz are antisymmetric mode a_5 and symmetric mode s_5. Given the mode shapes however, it was noted that a horizontal force applied to the rail would preferentially excite the antisymmetric modes, and generally the symmetric modes have a small influence on the overall response. Frequencies around 7kHz do not have any cut-offs, and this frequency was therefore chosen to compare results with, so that the effects at cut-off can be isolated.

5.1. Time domain comparison

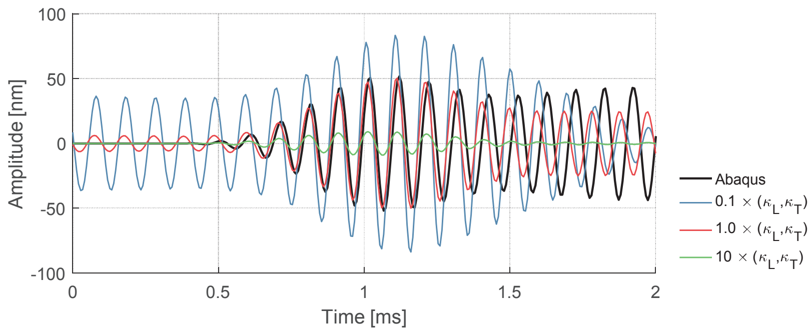
In this section, the time domain results will be compared for the problem illustrated in Figure 2. Since the response is dominated by the displacement in the x −direction, only u_x displacements are presented at a distance of 1 m from the applied force, at the same point on the cross-section as the force. A 100N peak, 10.5 cycle Hanning windowed toneburst transient force is applied. Centre frequencies of 7kHz and 10kHz are considered with various strategies used to eliminate numerical issues at cut-off when considering the SAFE forced response.



(a) 7kHz centre frequency.



(b) 10kHz centre frequency, causality and velocity based filtering.



(c) 10 kHz centre frequency, damped results.

Fig. 4. Displacement u_z at z=1m with forcing function at different frequencies.

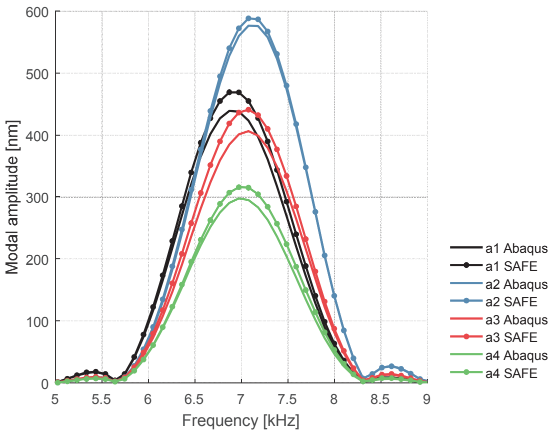
Figure 4(a) depicts the u_x response for the 7kHz case. Good agreement between the Abaqus/Explicit and SAFE results is achieved without any need to alleviate the effects of ringing, since no cut-off is excited in this case. Figure 4(b) and 4(c) show the response due to the 10 kHz excitation. In Figure 4(b), the Abaqus/Explicit response is presented together with the post-processing based methods to eliminate the ringing associated with exciting a mode close to a cut-off frequency. The results without any scheme to treat the effects of cut-off are denoted SAFE in the figure, and in this case it is difficult to determine the first arrival due to the wrap-around effect. The results with causality enforced as detailed in Section 3 are denoted SAFE-C and show a vast improvement over the response with no intervention. Results produced by filtering responses with high phase velocity, as explained in Section 3, are labelled SAFE-V and produced similar results to the case where causality is enforced, except that a slight wrap-around effect is still present making it more difficult to determine the first arrival. A combination of limiting the response based on phase velocity and causality could also be implemented, but these results are not shown for brevity.

Figure 4(c) depicts the results with damping added, instead of using post-processing techniques. Three different damping levels are evaluated as explained in Section 3, i.e. the values proposed by Bartoli et al. [11] denoted \`$1.0 \times (\kappa_L, \kappa_T)$' as well as values 10 times higher \`$10.0 \times (\kappa_L, \kappa_T)$' and ten times lower \`$0.1 \times (\kappa_L, \kappa_T)$'. The results confirm that the material properties suggested in [11] effectively damp the unrealistically large displacements and very good agreement with the Abaqus/Explicit results are achieved. The lower damping values produce results very similar to those where no damping is included, whereas it is clear that over damping the response results in an underestimation of the response. The important point is, however, that for each damping parameter tested the dispersion curves and the modal amplitudes need to be re-calculated and since damping is usually \`tuned' to match experimental measurements, this could be numerically expensive and requires good, quantitative experimental results. On the other hand, post-processing methods such as those evaluated in Figure 4(b), can be tuned without having to recompute dispersion curves or modal amplitudes.

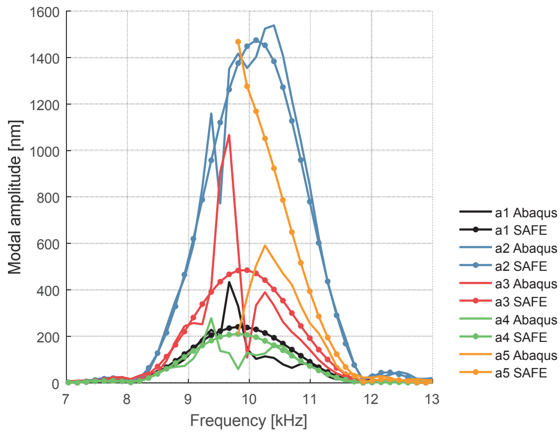
5.2. Frequency domain comparison

Time traces of displacement are useful when comparing simulation and experimental results. Displacement signals on their own are, however, not necessarily a very rich source of information. Instead, what is required for NDT system design is information about how well a certain (targeted) propagating mode is excited. Targeted modes would typically have energy concentrated in the region of the waveguide where discontinuities are sought, and be as non-dispersive as possible [19]. To this end, it is advantageous to present the results in the frequency domain as modal amplitudes.

The SAFE method computes modal amplitudes for each individual mode at different frequencies naturally. Here, a quantitative comparison between SAFE and Abaqus/Explicit is performed for a forced response problem. The problem described in Section 5.1 and Figure 2 is considered again with the same forcing function as in Section 5.1. Since only the antisymmetric modes are strongly excited by the force depicted in Figure 1(a), only results for these modes are presented in Figure 5.



(a) 7kHz centre frequency.



(b) 10kHz centre frequency.

Fig. 5. Modal amplitudes extracted from Abaqus/Explicit and computed using SAFE.

Figure 5(a) depicts modal amplitudes computed using SAFE, and those extracted from Abaqus/Explicit results using the process presented in Section 4.2 for the 7kHz centre frequency case. Excellent agreement is achieved between the modes extracted from the Abaqus/Explicit results and those computed using the SAFE method. Although the excitation has a 7kHz centre frequency, not all of the antisymmetric modes presented have a maximum at 7kHz. This is possibly due to the mode shapes (and the associated modal amplitudes) being frequency dependent.

Results for the 10kHz centre frequency excitation are presented in Figure 5(b). In this case, an additional mode (a_5) is seen to cut-off on the frequency axis between 9 and 10 kHz as shown in Figure 1(b). Considering first the SAFE results, the discrete cut-off of mode a_5 is clear with the modal amplitudes of the other modes being smooth functions of frequency.

The cut-off frequency for the Abaqus/Explicit model is known to be predicted at a slightly lower frequency than the SAFE model. This accounts for the difference in predicted modal amplitudes between the cut-off frequencies predicted using the two models, where essentially the incorrect basis functions (SAFE eigenvectors) are used to extract modes from the Abaqus/Explicit results. Despite these discrepancies similar trends are observed between the two sets of results, even fairly close to the cut-off frequency. At frequencies away from the cut-off frequency (below 9kHz and above 11kHz) better agreement is achieved.

In order to improve the agreement, either mesh refinement strategies could be employed in both the SAFE, but especially in the Abaqus/Explicit model, or the SAFE model could be modified (in terms or material properties or small geometrical modifications) in order to achieve a better agreement in the predicted cut-off frequencies between Abaqus/Explicit and SAFE.

6. Conclusions

The response of an elastic waveguide excited by a time-dependent point force is considered, with particular attention paid to the effects of exciting propagating modes close to their cut off frequency. A time domain comparison between SAFE and Abaqus/Explicit simulations is performed, with absorbing boundary conditions used to prevent end reflections in the Abaqus/Explicit model. Various methods to deal with the large time domain responses predicted using the SAFE method (when a mode is excited close to its cut-off frequency) are evaluated. It is shown that introducing hysteretic damping effectively reduces the response if an appropriate level of damping is introduced. However since realistic damping properties are not always known, some iteration may be required, which could be numerically expensive. Alternatively, post-processing methods were shown to have similar performance and are less numerically expensive. These methods would be recommended when no experimental nor Abaqus/Explicit results are available.

Frequency domain modal amplitudes are extracted from Abaqus/Explicit time domain results using a SAFE-based method, and compared with modal amplitudes computed using SAFE. Excellent agreement is achieved at frequencies where no mode cut-offs occur. However, since the Abaqus/Explicit and SAFE models predict slightly different cut-off frequencies, mode extraction from Abaqus/Explicit is not as accurate close to cut-off frequencies. Despite these differences, there is acceptable agreement between the two methods, demonstrating the accuracy of the SAFE method.

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