KR LITHO PRETORIA

JVK 1993 ATC Plaveiselingenieurswese / Pavement Engineering VOL 4C

JAARLIKSE VERVOERWESEKONVENSIE
ANNUAL TRANSPORTATION CONVENTION

1993

Universiteit van Pretoria / University of Pretoria

JVK ATC

PLAVEISELINGENIEURSWESE/
PAVEMENT ENGINEERING

4C

REFERATE / PAPERS

REVIEWING PERMANENT DEFORMATION OF GRANULAR PAVEMENT LAYERS

H. WOLFF, A.T. VISSER*, M. DE BEER**

Consultant, Division of Roads and Transport Technology, PO Box 395, Pretoria, 0001

*Professor, Department of Civil Engineering, University of Pretoria, Pretoria, 0002

**Division of Roads and Transport Technology, PO Box 395, Pretoria, 0001

SUMMARY

A brief review of the theory of material behaviour is given as background to a discussion on the behaviour of granular materials as determined from accurate measurements of elastic and plastic strains under full-scale Heavy Vehicle Simulator (HVS) testing. Analysis of these measurements indicated that granular materials behave elasto-plastically. Once the behavioural model of granular materials was determined, this model together with elastic and plastic deformation data collected from HVS testing on a number of sites throughout South Africa were used to develop a design model for granular pavement layers. The principle of S-N curves was used in the development of the design model.

OPSOMMING

Die teorie van materiaalgedrag word kortliks beskryf as agtergrond tot 'n bespreking oor die gedrag van granulêre materiale soos bepaal met behulp van akkurate metings van elastiese en plastiese deformasies onder volskaalse toetse met die Swaarvoertuig Nabootser (SVN). Ontleding van hierdie metings het getoon dat granulêre materiale elasto-plasties optree. Hierdie gedragsmodel tesame met elastiese en plastiese deformasiedata verkry uit SVN toetse gedoen op 'n aantal terreine dwarsdeur Suid-Afrika, is gebruik vir die ontwikkeling van 'n ontwerpmodel vir granulêre plaveisel lae. Die beginsel van S-N kurwes is gebruik in die ontwikkeling van die ontwerpmodel.

INTRODUCTION

The characteristics of unbound pavement materials which determine the elastic and plastic deformations in pavements under moving wheel loads are of particular interest to engineers for the following reasons:

- The elastic deformations influence the fatigue life (cracking) of the bound (cemented) and visco-elastic (bituminous) materials on or near the surface of the pavement.
- The plastic or permanent deformation define the rutting and consequently the surface drainage and safety characteristics of the pavement.

This paper concentrates on the plastic or permanent deformation behaviour characteristics of unbound or granular materials.

A literature review (SARB, 1990) indicated that there are broadly three schools of thought on the mechanisms of permanent deformation in granular layers. These schools can be summarised as follows:

School 1: The Mohr-Coulomb theory, where the permanent deformations are caused by failure on the Mohr-Coulomb failure line (Raad et al, 1980).
School 2: Linear elastic combined with Mohr-Coulomb theory, where the permanent deformations can be limited to only "bedding in" deformation, provided that the stresses be kept within a certain "elastic" stress range (Maree, 1978, 1982).
School 3: Elasto-plastic combined with fatigue characteristics, where permanent deformations develop at stresses well below the failure stress of the material and permanent deformation develop with each load cycle (Bullen and Major, 1971; Kenis, 1977; Yandell, 1990).

The first logical step in further research on the development of permanent deformation in granular layers will be to determine which of the above concepts bears the closest resemblance to what is actually happening in the field. In this paper the theory of behaviour of materials is briefly discussed. This theory is then applied to accurate measurements of elastic deflections and permanent deformations of granular layers taken under Heavy Vehicle Simulator (HVS) trafficking to determine the most likely behaviourial model for permanent deformation of granular materials. This is followed by a discussion on the development of a design model for granular materials using this behaviourial model together with elastic and plastic deformation data collected from HVS testing on a number if sites throughout South Africa.

DEFINITION OF GRANULAR MATERIAL

Granular materials referred to in this paper include crushed stone and natural gravel. The TRH 14 (CSRA, 1985) classification system for granular materials is used in this paper. Only G1 to G6 materials are considered.

Granular layered pavements in South Africa typically comprise a thin bituminous surfacing, a base of unstabilised gravel or crushed stone and a granular or cemented subbase supported by the in-situ subgrade. One or two selected layers normally constructed with granular material are included in the pavement structure in cases where the in-situ subgrade is of insufficient strength to support the subbase and base layers.

DEFINITION OF BEHAVIOUR

Behaviour is defined as the way in which a system responds to a stimulus. The response is characterized as follows:

- By the distribution of stresses developed in the system
- By the nature of strains and displacements at all points in the system.

When, for a given stimulus, these two features of structural response are determined, the behaviour of the system is completely defined (Oden, 1967; Harr, 1977).

CONSTITUTIVE EQUATIONS

Components of stress must satisfy equilibrium conditions at all points within a deformable material body, whereas the components of strain must satisfy compatibility conditions at all points. Satisfying these conditions, nine equations and 15 unknowns are obtained. It follows that the resulting system is indeterminate and unique stresses and strains necessary to define the behaviour of the system cannot be obtained (Oden, 1967; Harr 1977).

To overcome this deficiency and to complete the system, without introducing additional unknowns, requires six additional independent expressions. These are the so-called **constitutive equations**. It is this set of equations which accounts for the difference in response of material bodies of like mass and geometry, but of different internal constitution. For any given problem in which the body forces are prescribed within the body and the surface forces or displacements are prescribed at all points on the boundaries, this distribution of stresses, strains and displacements which simultaneously satisfy both the equilibrium conditions and the deformation conditions at all points within the body and at all points on its boundaries and which are also consistent with the constitutive equations of the material, constitute a unique solution to the problem.

STRESS-STRAIN RELATIONSHIPS OF MATERIALS

In practice, materials generally have complex stress-strain relationships that are simplified into the following for purposes of behaviour analysis (Marshall, 1978):

- The single curve shown in Figure 1(a) which is generally known as non-linear elastic behaviour
- Behaviour as a rigid material up to a yield stress σ_y after which it behaves plastically (Figure 1(b))
- A combination of linear elastic and plastic behaviour (Figure 1(c)) which is generally known as elastic-perfectly-plastic behaviour.

On unloading, either one of the following is possible:

- The material returns to its original dimensions along the loading curve and is said to be elastic.
- The material departs from the original loading curve on unloading and is said to be inelastic.

The above stress-strain relationships apply mainly to metals and none of these can really be used to describe the behaviour of granular material in road pavements.

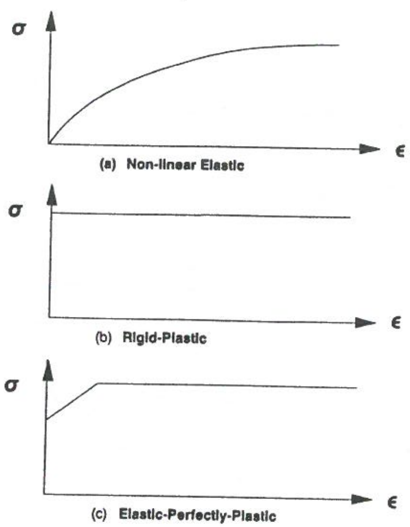


FIGURE 1: SIMPLIFIED STRESS-STRAIN BEHAVIOUR. (MARSHALL, 1978)

The following paragraph attempts to define the stress-strain relationship for granular materials in pavements by observation of the field behaviour of these materials under accelerated simulated traffic loading using the Heavy Vehicle Simulator (HVS).

BEHAVIOUR OF GRANULAR MATERIALS OBTAINED FROM FIELD OBSERVATIONS

In this section data is presented that was obtained from measurements of vertical elastic and plastic strains with the Multidepth Deflectometer (MDD) (De Beer et al, 1988) under HVS loading of full-scale pavement structures incorporating granular layers. The behaviour of the granular material in terms of the applied stresses and the measured strains (induced by the applied stresses), is then explained according to the behaviour observed and the theories of behaviour discussed earlier.

Elastic strains

Figure 2(a) shows elastic deflections in a pavement constructed with a G1 base for a range of wheel loads measured at different depths during HVS testing (Test 339A4, Bultfontein). The dotted line shown in Figure 2(a) indicates linear elastic behaviour in that doubling of the wheel load causes a doubling in elastic deflection. The surface deflection shows that the G1 base does not behave linear elastically, but exhibits non-linear elastic (stress stiffening) behaviour. The deflections deeper down in the pavement show that it behaves almost linear elastically. When the deflections in each layer are examined (Figure 2(b)), the non-linear elastic behaviour of the G1 base is illustrated even better. Figure 2(b) shows that the deflection in the granular base layer actually decreases with an increase in load. From Figure 2 it is concluded that the G1 base behaves non-linear elastically.

Figure 3 shows the effect of an increase in wheel load on the linear elastic backcalculated resilient modulus value for G1 (a) and G6 (b) materials with load repetitions. The load repetitions shown in the figure refer to applications of the standard 40 kN wheel load at 520 kPa tyre pressure. The trafficking was stopped approximately at every 100 000 load repetitions and elastic deflections were then measured for a range of

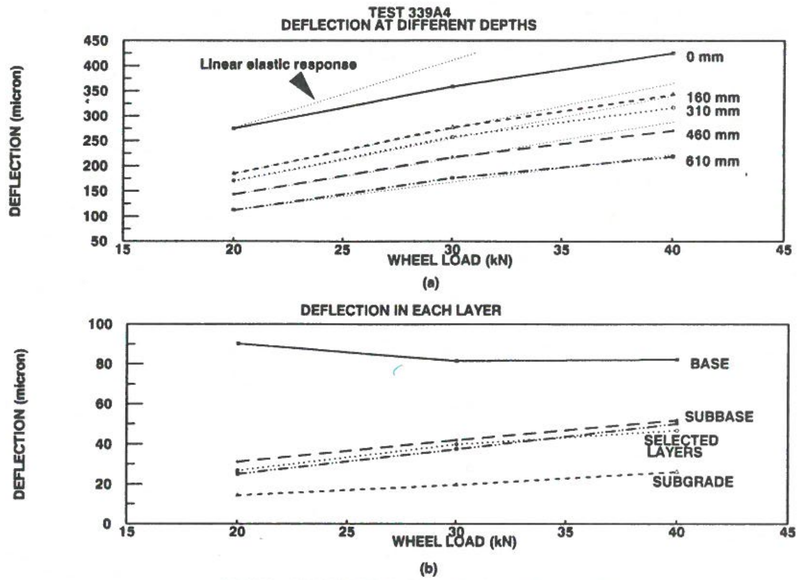
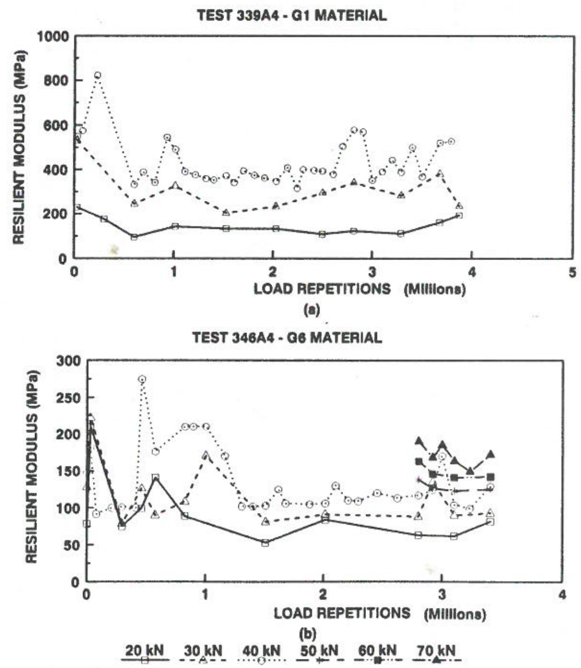


FIGURE 2: NON-LINEAR ELASTIC BEHAVIOUR OF G1 BASE.



Repetitions applied with 40 kN dual wheel load; deflections measured at various stages of trafficking with range of dual wheel loads.

FIGURE 3: NON-LINEAR ELASTIC BEHAVIOUR OF G1 AND G6 MATERIALS.

wheel loads. The measurements were thus done at the same position. Although the measurements show some scatter, it is evident that an increase in wheel load causes an increase in the backcalculated resilient modulus. The resilient moduli of granular materials vary with the applied load (stress) which indicates non-linear behaviour. An increase in stress causes an increase in resilient modulus which is known as stress stiffening behaviour. The observations made from Figure 2 are thus confirmed with Figure 3.

Plastic strains

Figure 4 shows the permanent or plastic strain accumulated under HVS testing of a G1 and G6 material. The HVS tests were run at approximately the same sum of principle stresses θ ($\theta = \sigma_1 + \sigma_2 + \sigma_3$) as calculated with the Michpave finite element computer code (Harichandran and Yeh, 1988). Figure 4(a) shows a phase of relative quick development of permanent strain for the G1 material up to about 1,2 million load repetitions. A decrease in the rate of permanent strain development towards the end of the test is evident from Figure 4(a). The figure also shows that the rate of permanent strain development does not become zero after nearly four million load repetitions. Figure 4(b) shows the same trends for the G6 material, but at much lower load repetitions, as can be expected. Figure 4(c) illustrates the difference in the number of load repetitions in the permanent strain behaviour of the two materials, by superimposing the curves on the same plot.

It follows that permanent strain still accumulates in a high quality crushed stone base (G1 material) at a large number of load (stress) repetitions. Figure 4 illustrates that it is more so for poorer materials (G6 material). It therefore appears that the nature of the material is such that elastic and plastic strains are introduced by one repetition of a stress much smaller than the yield stress of the material as defined by the Mohr-Coulomb failure envelope. The higher rate of permanent strain accumulation at low stress repetitions could be due to densification taking place in the granular layers. The permanent strain taking place at a large number of load (stress) repetitions is dependent on the internal composition of the material and its response to applied stresses.

STRESS-STRAIN RELATIONSHIP OF GRANULAR MATERIALS

It follows from the above paragraphs that granular materials behave non-linearly and inelastically. This indicates that:

- The stress-strain curve of a granular material will not be a straight line with a constant slope, but a curved line with a slope or resilient modulus M_r varying with the applied stress, and
- permanent or plastic strain takes place during each application of a stress much smaller than the yield stress of the material and that these strains accumulate with load or stress repetitions. On the removal of stress, the loading stress-strain curve will not be retraced, but a hysteresis loop will form indicating the permanent deformation that took place during the application of each cycle of stress.

Figure 5 shows the generalised stress-strain curve that best describes the behaviour of granular materials as observed under HVS testing. This type of behaviour is often referred to as elasto-plastic behaviour.

MODELLING ELASTO-PLASTIC BEHAVIOUR

Observation of elastic and plastic strains measured under HVS testing led to the hypothesis that the permanent deformation that unbound granular materials undergo when subjected to repeated loading which

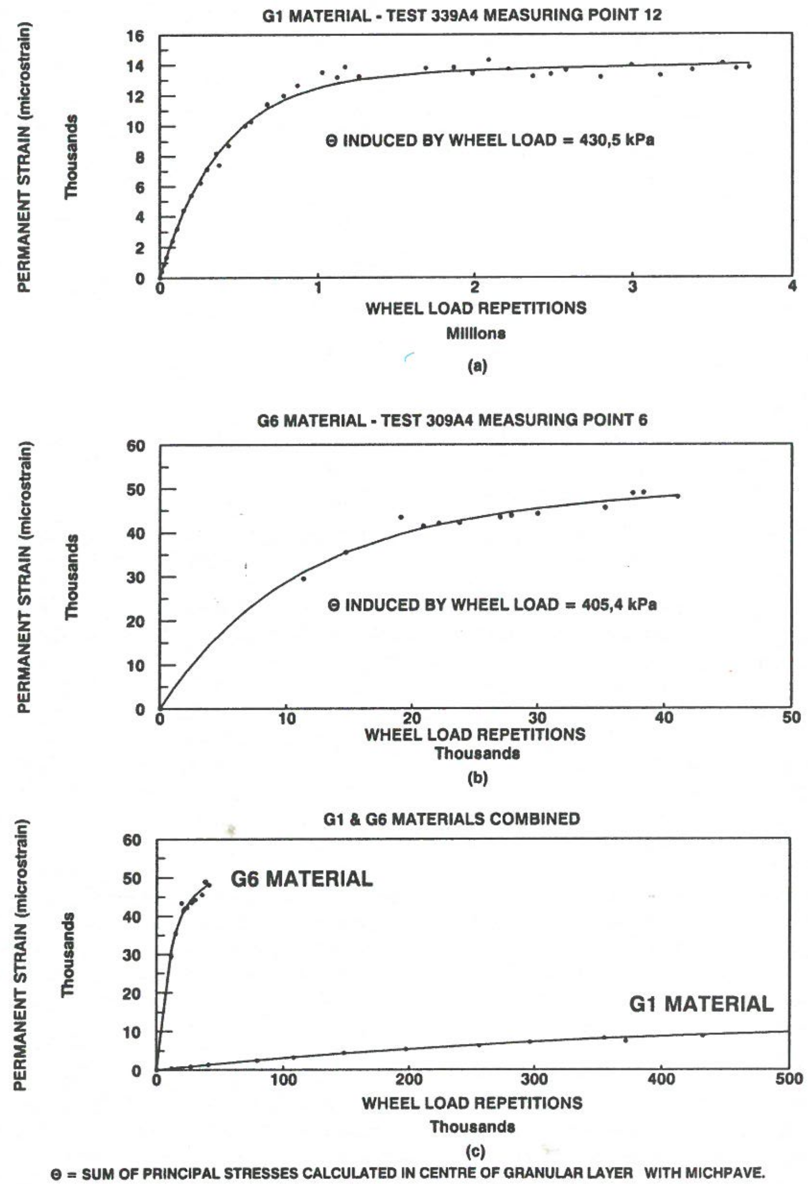


FIGURE 4: PERMANENT STRAIN BEHAVIOUR OF SOME GRANULAR MATERIALS.

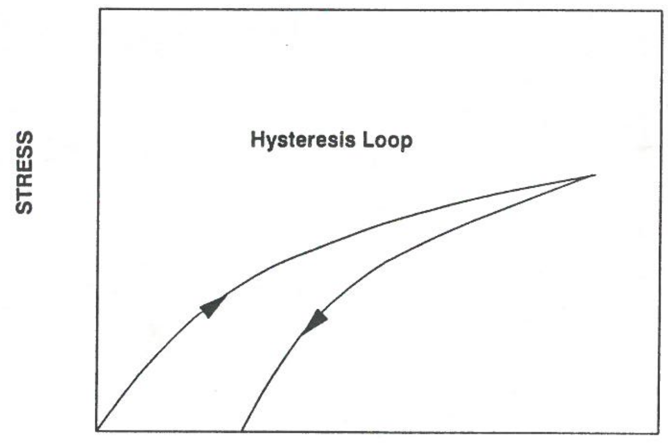


FIGURE 5: ELASTO-PLASTIC MATERIAL BEHAVIOUR.

induces stresses well below the yield stress of the material (Smith and Nair, 1973; Boyce, 1976; Stokoe II, 1990) as defined by the Mohr-Coulomb failure envelope, is caused by the elasto-plastic behaviour of the material and not by plastic deformation that occurs when a material is stressed beyond the Mohr-Coulomb envelope. The plastic deformations that occur when a granular material is stressed beyond the Mohr-Coulomb envelope is analogous to the permanent deformations that occur when a metal is stressed beyond its yield stress. No attempt was therefore made to link the development of permanent strain to the ultimate shearing strength of the material. A method, similar to methods employed by Kenis (1977) and Yandell (1990), which accounts for the permanent strain that takes place during each load cycle, was investigated. It differs from some other methods (Maree, 1978) in that the latter tried to keep the applied stresses within an "elastic" range that does not always exist for unbound granular materials.

The non-linear elasto-plastic behaviour of the unbound granular material is modelled in two phases. Firstly, the stress dependent modulus of elasticity is accommodated by using a non-linear elastic finite element program for backcalculating material parameters from measured maximum deflections. Secondly, the elasto-plasticity of the unbound granular materials is modelled by a transfer function that relates an invariant of backcalculated stress induced by HVS wheel loads to the plastic strain measured under repetitions of the same wheel loads.

It was decided to use the principle of S-N curves in developing a transfer function for rutting. S-N curves were first used by the German railway engineer August Wöhler between 1852 and 1870 and are presently often used by mechanical engineers to describe the fatigue characteristics of metals subjected to repeated loads of different magnitudes (Collins, 1981). The principle is shown schematically in Figure 6. On the vertical axis, one or other indication of the stress level to which the material is subjected, is given. On the horizontal axis, the number of the stress repetitions of the applied stress that the material can withstand before failing, is shown.

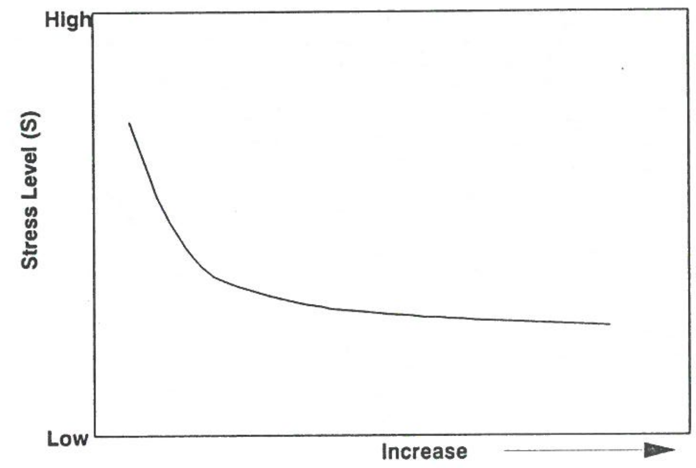


FIGURE 6: S-N CURVE PRINCIPLE.

In order to develop a transfer function for rutting using the S-N curve principle, failure in this case was defined as a specific terminal permanent strain in the pavement layer for which the transfer function is being developed. However, any terminal permanent strain can be considered. Data of permanent strain caused by load (or stress) repetitions of a number of different wheel loads on various pavement types are available from HVS testing. These data are best presented by the following function (Wolff et al, 1992):

$$y = (mx + a)(1 - e^{-bx}) \quad \text{.......(1)}$$

where	y	=	permanent deformation
	x	=	number of load repetitions
	m, a, b	=	constants
	e	=	natural logarithm.

This function can be used to calculate the number of repetitions of a specific wheel load (inducing a certain stress) necessary to obtain the permanent strain corresponding to the chosen terminal rut depth within a layer. The stress induced in the layer by the wheel load can be calculated using the material parameters backcalculated from HVS load/deflection data. One point on a curve resembling an S-N curve can then be obtained by plotting an invariant of the stresses on the vertical axis and the number of load repetitions to failure on the horizontal axis. More points on the curve are obtained by repeating the exercise for other wheel loads.

By repeating the procedure for other HVS tests with different pavement structures and wheel loads but the same material, a distribution of points is obtained. The function that fits the data points best, is referred to as the S-N curve or transfer function for that material.

The parameter to be used on the vertical axis of the S-N curve to represent the state of stress or strain at which the permanent strain was developed, requires careful consideration. The elastic vertical strain (ε_v), for example, is a parameter that is frequently coupled to the development of permanent strain in subgrade layers. The approach followed is that the behaviour of a material at a specific point in the material is determined by the state of stress or strain at that point. Atkinson and Bransby (1978) state that parameters like σ_x, τ_{xz} and ε_v which depend on the orientation of the reference axes as well as on the state of stress or strain, are not entirely suitable on their own as measures of the stress or strain state, because their magnitudes depend on the choice of axes. It would be better to choose stress or strain parameters which are independent of the choice of reference axes and which are unique for a given state of stress or strain. Stress and strain parameters of which the magnitudes are independent of the choice of reference axes are usually known as stress or strain **invariants**; they are invariant in the sense that their magnitudes do not change as the reference axes are varied. In order to test Atkinson and Bransby's statement, the following invariants of stress and strain, together with the elastic vertical strain ε_v and deviator stress $(\sigma_1-\sigma_3)$, were evaluated for use in the S-N type transfer functions (Atkinson and Bransby, 1978):

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \qquad (2)$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \qquad (3)$$

$$\varepsilon_v = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) \qquad (4)$$

$$\varepsilon_s = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2} \qquad (5)$$

where
σ_{oct} = octahedral normal stress
τ_{oct} = octahedral shear stress
ε_v = volumetric strain
ε_s = shear strain
$\sigma_1, \sigma_2, \sigma_3$ = principal stresses
$\varepsilon_1, \varepsilon_2, \varepsilon_3$ = principal strains

The invariants, the elastic vertical strain and the deviator stress were calculated for data collected on G5 materials and plotted against the number of load (or stress/strain) repetitions to a permanent strain of 10 000 $\mu\varepsilon$, as shown in Figure 7. Intuitively, one expects that the octahedral shear stress should best relate the stress state induced by a wheel load in a granular layer to the repetitions of the same load required to accumulate a specific permanent strain in the layer. However, it is evident from Figure 7 that the octahedral normal stress (Equation 2) also gives an acceptable relation between the stress state induced by a wheel load in a granular layer and the number of repetitions of the same load (stress) required to accumulate a specific permanent strain in the layer. There is, in fact, not much to choose between the octahedral shear and octahedral normal stresses as explanatory variable. It was decided to use the octahedral normal stress as the invariant to represent the stress state in the granular material as it appears to give a marginally better relation between the stress state and the number of load repetitions to a specific permanent strain and because it is simpler to calculate. It was also decided to do away with the constant ($^1/_3$) and use the sum of the principal stresses θ as calculated in the centre of the granular layer directly below the wheel as stress parameter on the vertical axis of the S-N curve. The sum of the principal stresses is also often referred to as the bulk stress.

Plots of stress state denoted by θ against the number of that stress repetitions required to generate a permanent strain of 10 000 microstrain ($\mu\varepsilon$) were consequently prepared from the available HVS data for G1, G2, G4, G5 and G6 materials. None of the HVS test sections were constructed with G3 material, hence no data were available for this material type. It was found that the following function fitted the data points best (Wolff, 1992):

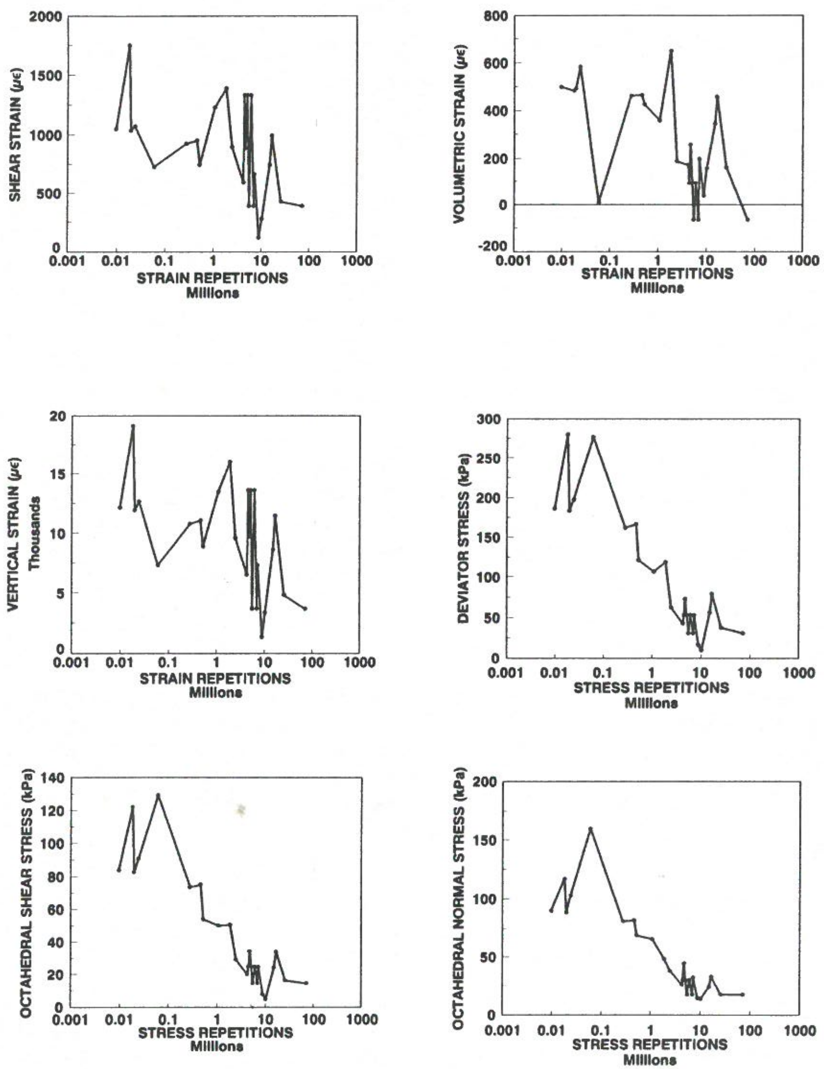


FIGURE 7: INDICATORS OF STRESS/STRAIN STATE PLOTTED AGAINST LOAD REPETITIONS TO 10 000 $\mu\varepsilon$ PERMANENT DEFORMATION. (G5 MATERIAL).

$$\theta = A(1 - e^{-\frac{B}{N^m}}) \quad (6)$$

where θ = sum of principal stresses in centre of layer
N = number of load repetitions
A, B, m = regression constants
e = natural logarithm.

This function was then also fitted for plots of θ against number of load (stress) repetitions required to generate permanent strain of 20, 30, 40 and 50 thousand microstrain ($\mu\varepsilon$) in G1 material; 10, 20, 30, 40, 50 and 100 thousand microstrain ($\mu\varepsilon$) in G2 material and 10, 20, 30, 40, 50, 100 and 200 thousand microstrain ($\mu\varepsilon$) in G4, G5 and G6 material. This relates to 1,5; 3,0; 4,5; 6,0; 7,5; 15 and 30 mm permanent deformation respectively for a layer of 150 mm thickness. The regression constants are shown in Table 1. It follows from Table 1 that no correlation could be found for the relation between θ and load repetitions to produce 10 000 microstrain ($\mu\varepsilon$) in a G1 material. This poor correlation is caused by few data points and wide scatter. Also, the low correlation coefficients obtained for the G2 material relative to the other materials cannot be explained satisfactorily at this stage.

The relationships for a terminal permanent strain of 20 000 microstrain ($\mu\varepsilon$) for the five types of granular materials are shown graphically in Figure 8. The data used for determination of the regression constants in Table 1, are also shown. Similar figures can be prepared for 30, 40, 50, 100 and 200 thousand microstrain ($\mu\varepsilon$) for the five material types. However, such figures were not prepared as the correlation coefficient, significance level of the correlation coefficient and standard error of each fit are given in Table 1. Figure 9 is an example of S-N curves developed for G6 material for different permanent strains and at a 95 per cent confidence level (Wolff, 1992).

CONCLUSIONS AND RECOMMENDATIONS

A theoretical study on the behaviour of systems and observation of elastic and plastic deformations in granular pavement layers under HVS testing lead to the conclusion that granular materials behave elasto-plastically. This means that the resilient modulus is a function of the applied stress and that permanent deformations develop with each load cycle, even when the applied stresses are smaller than the Mohr-Coulomb failure stress. Observation of the relationship between permanent strain development and number of stress repetitions further indicated that the curve relating the two could be divided into two distinct phases namely:

- A first phase where rapid development of permanent strain takes place with load repetitions and the rate of permanent strain development is not a constant, but continuously diminishing. This phase of permanent deformation could be due to densification taking place in the granular layer.
- A second phase where the development of permanent strain is relatively slow and approaches a constant value. This phase could be could be seen as similar to fatigue that takes place in metals when subjected to cyclic stresses that are smaller than the yield stress of the material.

Elasto-plastic behaviour therefore also implies that a design method for granular materials should be able to take cognisance of the permanent deformation that takes place with each load cycle. Such a design method was developed by using the principle of S-N curves and elastic and plastic deformation data collected from HVS testing of a number of sites throughout South Africa.

TABLE 1: Regression constants and correlation coefficients for S-N curves for G1 to G6 materials and 10 000 to 200 000 $\mu\varepsilon$ permanent strain.

		PERMANENT STRAIN (microstrain)						
		10 000	20 000	30 000	40 000	50 000	100 000	200 000
G1	A		1390	1500	1570	1600		
	B		15	15,1	15,1	15,4		
	m		0,18	0,18	0,18	0,18		
	r^2		0,44	0,45	0,44	0,43		
	SL*		50	50	50	50		
	SE**		188	187	187	190		
G2	A	700	680	710	750	760	790	
	B	20,2	25,4	25,3	24,2	24,8	26,3	
	m	0,20	0,20	0,20	0,20	0,20	0,20	
	r^2	0,28	0,23	0,22	0,23	0,23	0,20	
	SL	95	95	95	95	95	95	
	SE	83	87	87	86	86	89	
G4	A	1300	1610	2010	2410	3090	6240	7980
	B	3,1	2,9	2,5	2,2	1,78	1,0	0,9
	m	0,20	0,20	0,20	0,20	0,20	0,20	0,20
	r^2	0,62	0,46	0,42	0,41	0,40	0,38	0,33
	SL	90	85	85	85	85	85	75
	SE	65	76	80	82	82	84	91
G5	A	1380	1590	1920	2530	3110	6580	7820
	B	2,0	2,1	1,89	1,51	1,30	0,7	0,7
	m	0,20	0,20	0,20	0,20	0,20	0,20	0,20
	r^2	0,77	0,80	0,81	0,81	0,82	0,79	0,51
	SL	99	99	99	99	99	99	99
	SE	57	54	52	52	51	54	84
G6	A	1710	2180	2690	4170	6710	12400	14830
	B	1,11	0,99	0,89	0,61	0,40	0,30	0,30
	m	0,20	0,20	0,20	0,20	0,20	0,20	0,20
	r^2	0,76	0,67	0,59	0,55	0,53	0,57	0,56
	SL	99	99	99	99	99	99	99
	SE	73	85	94	98	101	96	98

* Significance level of r^2 (%)
** Standard error of estimate

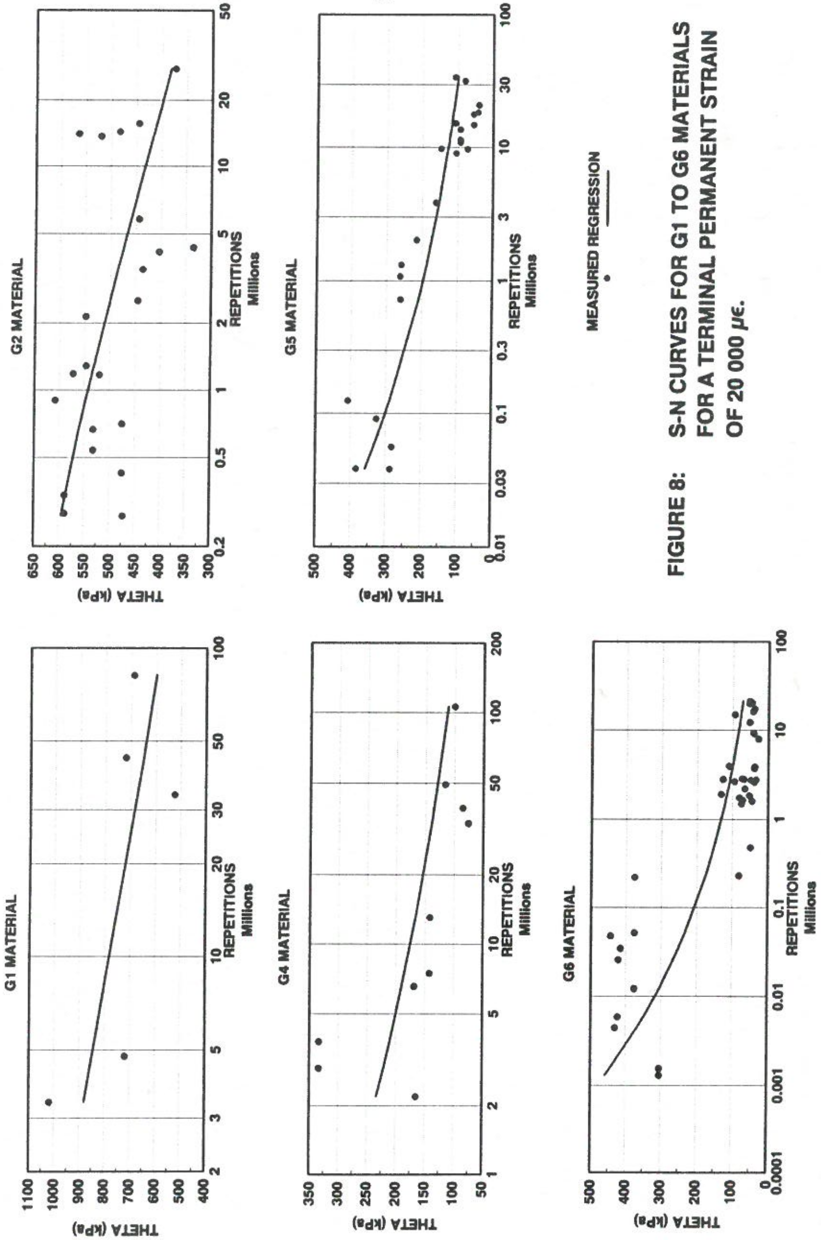


FIGURE 8: S-N CURVES FOR G1 TO G6 MATERIALS FOR A TERMINAL PERMANENT STRAIN OF 20 000 $\mu\epsilon$.

H. WOLFF

G6 MATERIAL

95 % LOWER CONFIDENCE LIMIT

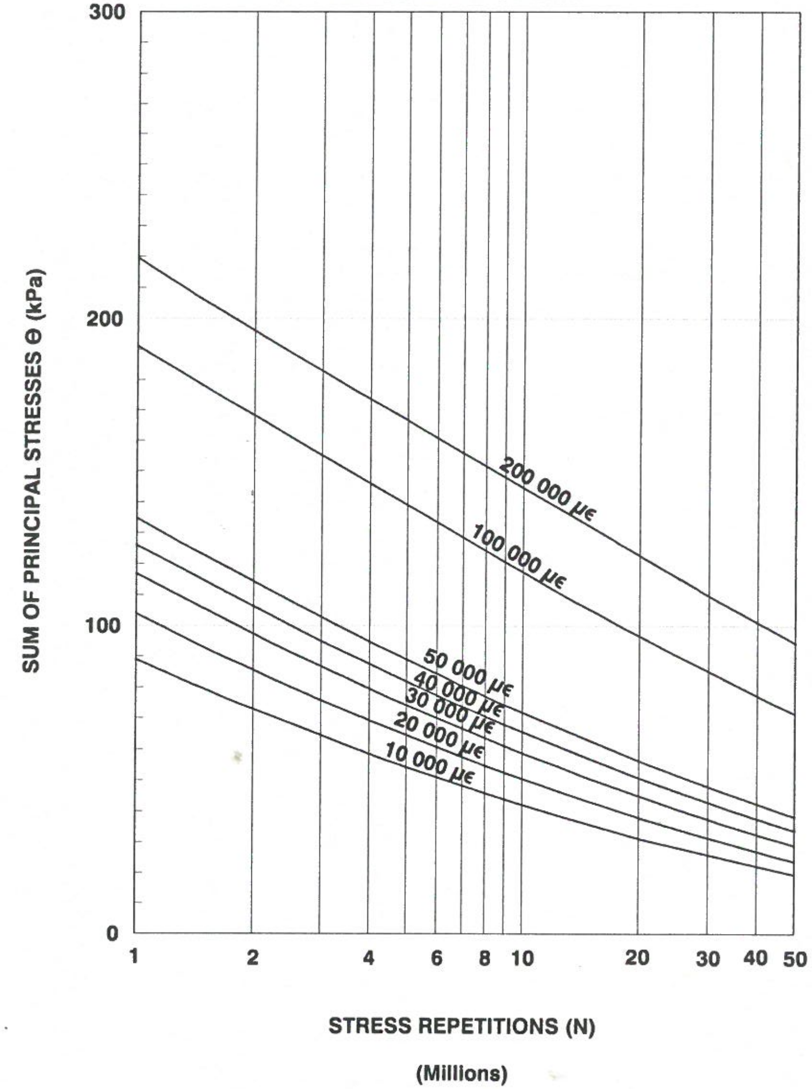


FIGURE 9: S-N CURVES AT DIFFERENT PERMANENT STRAIN LEVELS FOR G6 MATERAIL AT A 95 PER CENT LOWER CONFIDENCE LIMIT.

H. WOLFF

ACKNOWLEDGEMENTS

Support for this research by the South African Roads Board is gratefully acknowledged. The permission of the Director of the Division of Roads and Transport Technology of the CSIR to publish this paper , is acknowledged with thanks.

REFERENCES

ATKINSON, J H AND BRANSBY, P L. (1978) The mechanics of soils; An introduction to critical state soil mechanics. McGraw-Hill, London.

BOYCE, J R. (1976) The behaviour of a granular material under repeated loading. PhD dissertation, Department of Civil Engineering, University of Nottingham, Nottingham, United Kingdom.

BULLEN, R O, AND MAJOR, N G. (1971) Granular base layers - What do we really know. Proceedings of the Roading Symposium, Road Research Unit, National Roads Board, New Zealand.

COLLINS, J A. (1981) Failure of materials in mechanical design; analysis, prediction, prevention. John Wiley and Sons Inc., New York.

COMMITTEE OF STATE ROAD AUTHORITIES (CSRA). (1985) TRH 14: Guidelines for road construction materials. Council for Scientific and Industrial Research, Pretoria, South Africa.

DE BEER, M, HORAK, E AND VISSER, A T. (1988) The multi-depth deflectometer (MDD) system for determining the effective elastic moduli of pavement layers. First International Symposium on non-destructive testing of pavements and back calculation of moduli, ASTM STP 1026, Baltimore.

HARRICHANDRAN, R S AND YEH, M S. (1988) Flexible boundary in finite-element analysis of pavements. Transportation Research Record 1207, Transportation Research Board, National Research Council, Washington, D.C.

HARR, M E. (1977) Mechanics of particulate media, a probabilistic approach. McGraw-Hill Book Company, New York.

KENIS, W J. (1977) A design method for flexible pavements using the VESYS structural subsystem. Proceedings of the Fourth International Conference on Structural Design of Asphalt Pavements, Ann Arbor, Michigan.

MAREE, J H. (1978) Design parameters for crushed stone in pavements (in Afrikaans). M.Sc (Eng.) thesis, Department of Civil Engineering, University of Pretoria, Pretoria, South Africa.

MAREE, J H. (1982) Aspects of the design and behaviour of road pavements with granular material base layers (in Afrikaans). PhD dissertation, Department of Civil Engineering, University of Pretoria, Pretoria, South Africa.

MARSHALL, W T AND NELSON, H M. (1978) Structures. Pitman Publishing Limited, London.

ODEN, J T. (1967) Mechanics of elastic structures. McGraw-Hill Book Company, New York.

RAAD, L AND FIGUEROA, J L. (1980) Load response of transportation support systems. Transportation Engineering Journal, ASCE.

SMITH, W S AND NAIR, K. (1973) Development of procedures for characterization of untreated granular base course and asphalt-treated base course materials. Report no. FHWA-RD-74-61, Federal Highway Administration, Offices of Research & Development, Washington, D.C.

SOUTH AFRICAN ROADS BOARD (SARB). (1990) Behaviour of unbound granular material under repeated loading - a literature review. Unpublished Interim Report No. IR 88/021/8, Research and Development Advisory Committee, Department of Transport, Pretoria, South Africa.

STOKOE II, K H. (1990) Dynamic behaviour of pavement materials. Lecture notes, Department of Civil Engineering, University of Stellenbosch, Stellenbosch, South Africa.

WOLFF, H. (1992) The elasto-plastic behaviour of granular pavement layers in South Africa. PhD dissertation, Faculty of Engineering, University of Pretoria, Pretoria, South Africa.

WOLFF, H, VISSER, A T, KLEYN, E G AND DE BEER, M. (1992) The elasto-plastic behaviour of crushed stone bases in flexible pavement structures. Proceedings of the Seventh International Conference on Asphalt Pavements, Nottingham, United Kingdom.

YANDELL, W O. (1990) Mechano-lattice lecture notes. LGI Short Course, Department of Civil Engineering, University of Pretoria, Pretoria, South Africa.