State Estimation in Water Distribution Network: A Review

Kgaogelo S. Tshehla
Department of Electrical Engineering
Tshwane University of Technology
Pretoria, 0001
South Africa
Email: tshehla6@gmail.com

Yskandar Hamam
Department of Electrical Engineering
Tshwane University of Technology
Pretoria, 0001
South Africa
Email: HamamA@tut.ac.za

Adnan M. Abu-Mahfouz
Department of Electrical Engineering
Tshwane University of Technology;
and
Meraka Institute, CSIR
Pretoria, South Africa
Email: A.Abumahfouz@ieee.org

***Abstract*—Monitoring of a Water Distribution Network (WDN) requires information about the present state of the network. Since all variables are usually not measurable directly, State estimation is employed. State estimation is a process of determining the unknown variables of a system based on the measurements and mathematical network model. Measurements are often noisy, but state estimation procedure makes use of a set of redundant measurements in order to filter out such errors and find an optimal estimate. This paper presents a literature review of static state estimation problem pertaining to Water distribution Networks.**

I. Introduction

A water distribution network (WDN) is a system of interconnected pipes designed to deliver potable water from treatment works to various residential and business consumers, at sufficient pressure and flow [1]. Typical components that make up the network include pipes, valves, reservoirs/tanks, and pumping stations. This complex network of pipelines is usually buried underground and relatively inaccessible, which provides a significant challenge for operational monitoring. It is vital to maintain the integrity of the network to provide water to consumers without any disruption in service. The monitoring of WDNs also allows detection of peak demands, leakages [2], [3] and excessive water flow (e.g. pipe bursts). Moreover, it enables a better control mechanism such as real-time pressure control systems [4]–[8] that will reduce the water loss in the WDN.

Most water companies use telemetry systems not only to measure flows [9] and pressure in the network, but also obtain information on the present state of the network for control and operation purposes. The calculation of all flows and pressures in a WDN may be obtained by formulating and solving the mass and/or energy conservation equations depending on the measurements in the system. However, the WDNs are very large and thus not practical to measure all variables of interest due to financial constraints relating to cost of telemetry devices. Using a minimal set of measurements only, often consisting of pressure and flow measurements at pump stations, reservoirs and key nodes, pressure and flow for the whole system is problematic. The measurements are prone to errors from transducers and may be missing due to failure in communication systems, thus preventing the analytical solution to the equations, that results in an incomplete representation of the network state. A method that overcomes these drawbacks is known as state estimation (SE). State estimation is the process of determining the unknown variables of a system based on the measurements and mathematical network model, to gain a global system view [10]. Its strength lays in processing all available measurements and formulating the problem in terms of redundant equations. This redundancy is essential for the successful performance of SE procedure since it enables the erroneous information to be filtered out. In water systems the degree of redundancy is achieved by combining the measurement information with the pseudo-measurements, to improve both the reliability and accuracy of SE. Pseudo-measurements are predicted water consumptions at the network nodes, determined from population densities and past records.

This paper focuses on the static state estimation problem applied in WDNs. Section II outlines the current literature in WDN state estimation. Section III describes the SE problem formulation and the solution methods applied are discussed in section IV. In section V, the mathematical modelling of the network is highlighted. The paper is concluded in section VI.

II. State Estimation Methods

The study of state estimation and its techniques in power system is well developed, with relatively high measurement redundancy and quality, and is widely used. However, the application of SE algorithms to water distribution networks (WDNs) is still an ongoing research, especially for real-time estimation, due to low measurement redundancy. Smart Water Management systems make use of SE as an integral part of a collection of applications designed to monitor and control the water distribution network [11]. The increase in complexity of modern water networks makes SE an important tool in assisting operators in their operational decision-making. The state estimators are required to be both capable of real time data processing and be relatively immune to numerical convergence problems that might be caused by incomplete or inaccurate data. The application and development of Information and communications technologies (ICT) in the management of

WDNs provides an opportunity for real-time (or near real-time) monitoring. Communications technologies include cellular, radio, telephone, satellite and Ethernet [3].

In general, static state estimation can be categorised according to the minimisation cost function used. This is formulated as either a quadratic or a non-quadratic function. According to Arsene and Gabrys [12] the criterion used in the SE procedure can be divided into the following three major groups:

- Least Square (LS) criterion where the sum of the squared differences between the measured and estimated values is minimised,
- Least Absolute Value (LAV) criterion where the sum of the absolute differences between the measured and estimated values is minimised and
- Min-Max criterion where the maximum difference between the measured and estimated values is minimised.

The LS criterion is popular but known to be sensitive to bad data and large errors, while LAV and Min-max tend to have non-differentiable objective functions which may cause analytical problems and require more computational time. The choice of the estimation criterion depends on the type of errors that are likely to occur in the system, as a result the LS and LAV and their variations have been used in WDN state estimation problem. The most common method of estimation of the state vector from an over-determined set of measurements is the LS criterion, which gives a minimum-variance estimate provided that the measurements are affected solely by Gaussian noise [13]. Unfortunately, this is rarely the case in on-line computer control systems, where the measurement inaccuracies are far from a Gaussian distribution but, in fact, contain gross errors derived from telemetry or instrumentation malfunction. The LAV criterion performs better for an error with Cauchy distribution and the min-max criterion is a better when the measurements are relatively free from outliers. Although a standard LS method may be poor for a non-Gaussian error distribution, the method of weighted least squares (WLS) can used when the variance in the errors is not constant (outliers). The LAV criterion may be also preferable when very little is known about the distribution of errors, and weights can also be introduced to form weighted least absolute value (WLAV). In order to reduce the influence of outliers, a more robust iteratively re-weighted LS or LAV estimator can be used. The method for weighted least-absolute-values (WLAV) estimation, based on the least sum of absolute values (a non-quadratic criterion) was implemented by Sterling and Bargiela [13] for water distribution networks.

This method was formulated as a linear program and it is sometimes considered more robust in the presence of bad data because it selects a number of measurements equal to the number of state variables and ignores the remaining measurements. The comparison of WLAV and WLS was shown Bargiela [14]. A modification to the standard WLS approach [15] showed that a WLAV cost function could be approached from a WLS cost function by a process known as re-weighting. Powell et al. [16] further developed this idea showing that the advantages of both the WLS and the WLAV approaches could be obtained from one algorithm. Bargiela and Hainsworth [17] introduced the idea of incorporating measurement bounds with the aim of increasing the robustness of SE under uncertainty. Carpentier and Cohen [18] used a graph-theoretic approach for classifying variables and parameters as redundant, calculable and as observable/unobservable, given a set of measurements and topology of a WDN. Based on this classification, the redundant measured variables and calculable variables are used to derive the estimated values. Kumar et al. [19] argue that, even though the procedure leads to significant reduction in the problem size for an under-determined system, the reduction is not significant. An implicit formulation of the standard WLS state estimation technique for leak detection for an idealized grid network under steady conditions was presented in [20]. The formulation is based on the loop equations and the state variables are the unknown nodal demands. The minimisation problem is solved using a Lagrangian approach. Andersen et al. [21] proposed a constrained WLS technique to investigate the effect of measurement bounds. The assumption so far was that demands were measured or estimated from knowledge of water consumers' characteristics.

Kumar et al. [19] proposed a SE method using graph-theoretic approach for well instrumented networks. They applied the method to a realistic urban water networks assuming sufficient measurements, such as pipe flow rates, nodal pressures, and demands are available. The chord flows were used as independent variables and a constrained optimization problem was solved using a Successive Quadratic Programming (SQP) technique which implied the calculation of the Jacobian and the Hessian matrices of the objective function and the Jacobian of the reduced constraints. This case of a well instrumented WDN is not feasible yet. Cheng et al. [10] presented a real-time WDN hydraulic model from SCADA measurements. A WLS scheme based recursive state estimator and local linear matrix transform algorithm are applied to estimate demand. Piller et al. [22] presented a least squares problem with bound constraints is formulated to adjust the demand class coefficient to best fit the observed values at a given time. The criterion is a Huber function to limit the influence of outliers. A Tikhonov regularization is introduced for consideration of prior information in the parameter vector. Then the Levenberg-Marquardt algorithm is applied that uses derivative information for limiting the number of iterations. A summary of the methods and algorithms used to minimise the SE problem are shown in table I. Different algorithms such as linear and non-linear programming, and optimisation have been used to solve the non-linear SE problem. The iterative solution methods are derivatives of Newton's method, and the Gauss-Newton is one the common algorithm used to solve non-linear least squares problems. The least squares method is indeed popular in the literature and will be the focus of the next sections.

III. State Estimation Formulation

The state estimation process is based on a mathematical network model of the water distribution system. It involves

TABLE I

A SUMMARY OF THE DIFFERENT METHODS USED

Algorithm	Solution Method	Category
Revised Simplex [13]	Newton's method	LAV
Augmented matrix [14]	Gauss-Newton	LS
Interval Linear program Sensitivity Matrix [17]	Newton's method	LS
Lagrangian approach [18]	Gauss-Newton	LS
Lagrangian approach [20]	Newton-Raphson	LS
Quadratic programming [21]	Gauss-Newton	LS
SQP and BFGS [19]	Gradient Method	LS
Golden section search [10]	Direct Search	LS
Levenberg-Marquard and Tikhonov regularisation [22]	Gradient method	LS

solving an over-determined set of equations describing mass-balances in network nodes and pressure-flow relationships, including pseudo-measurements. The measurement errors result in differences between measured and theoretical values, which lead to the measurement equation:

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} g_1(x) \\ \vdots \\ g_m(x) \end{bmatrix} + \begin{bmatrix} e_1 \\ \vdots \\ e_m \end{bmatrix} = g(x) + e \tag{1}$$

Where x is a vector of state variables, z is a measurement vector which consists of real measurement values and pseudo-measurements; $g(x)$ are non-linear functions of the system relating state vector to the measurements; e is a vector of measurement errors and m is the number of measurements. Then the error vector is

$$e(x) = z - g(x) \tag{2}$$

Given the measurement vector z, the objective of the state estimator is to estimate the state vector x such that some norm of the error vector e(x) is minimised. For simplicity, the error vector e is assumed to be standard Gaussian, with zero mean and independent covariance. Hence:

$$Cov(e) = E(e \cdot e^T) = R = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_m^2 \end{bmatrix} \tag{3}$$

where R is the measurement covariance matrix, σ^2 is the variance of the measurement. Therefore, SE can be expressed as a problem of minimisation of discrepancies between the actual measurements and the values calculated from the mathematical model. The classical WLS state-estimation criteria is to minimise the objective function $\boldsymbol{F}$, which is the sum of the squared normalised residuals. It may be expressed as:

$$\begin{aligned} \min_x F(x) &= \sum_{i=1}^{m} \frac{[z_i - g_i(x)]^2}{\sigma_i^2} \\ &= [z - g(x)]^T R^{-1} [z - g(x)] \end{aligned} \tag{4}$$

To minimise equation (4), the first-order optimality conditions will have to be satisfied by equating the gradient of $F(x)$ to zero;

$$\frac{\partial F(x)}{\partial x} = -\boldsymbol{J^T(x) R^{-1}}[\boldsymbol{z} - \boldsymbol{g(x)}] = 0 = \boldsymbol{G(x)} \tag{5}$$

where $\boldsymbol{G(x)}$ is the gradient and $\boldsymbol{J(x)} = \frac{\partial g(x)}{\partial x}$ is the Jacobian matrix of $\boldsymbol{g(x)}$.

The second-order optimality condition distinguishes between a minimum and a maximum, and is given by the second derivative;

$$\frac{\partial^2 F(x)}{\partial x^2} = \boldsymbol{J^T R^{-1} J} = \boldsymbol{H(x)} \tag{6}$$

where $\boldsymbol{H(x)}$ is called the Hessian matrix.

Solving for $\boldsymbol{x}$ using Newton's method, and taking into account non-linear $\boldsymbol{g(x)}$, leads to a linear WLS problem to be solved;

$$\boldsymbol{H}\Delta\boldsymbol{x} = -\boldsymbol{G} \tag{7}$$

$$[\boldsymbol{J^T R^{-1} J}]\Delta\boldsymbol{x} = \boldsymbol{J^T R^{-1}}[\boldsymbol{z} - \boldsymbol{f}(\boldsymbol{\hat{x}^{(k)}})] \tag{8}$$

$$\Delta x = [\boldsymbol{J^T R^{-1} J}]^{-1} J^T R^{-1} [z - g(\hat{x}^{(k)})] \tag{9}$$

Weight is introduced to emphasise the trusted measurement while de-emphasise the less trusted ones, and therefore the measurement weight matrix is given by $\boldsymbol{W} = \boldsymbol{R^{-1}} = diag[\sigma_1^{-2}, \sigma_2^{-2}, ..., \sigma_m^{-2}]$. Equation (9) can then be written as follows;

$$\Delta x = \boldsymbol{H^{-1} J^T W}[\boldsymbol{z} - \boldsymbol{g}(\boldsymbol{\hat{x}^{(k)}})] \tag{10}$$

The estimate of the state vector is obtained from the iterative relationship, with the correction vector computed by solving equation (10);

$$\hat{x}^{(k+1)} = \hat{x}^{(k)} + \Delta x \tag{11}$$

IV. STATE ESTIMATION SOLUTION METHODS

The main challenge for state estimation problem in finding the solution, is the term $\boldsymbol{J^T R^{-1} J}$ or $\boldsymbol{J^T W J}$ and it's inverse, while including sparsity and optimal ordering techniques.

A. Gauss-Newton method

The solution to equation (9) using this method, is achieved by Cholesky's factorisation. The result is quadratic convergence, but requires computational effort. The method tends to fail if divergence occurs and according to Bargiela [14], it also suffers tendency to ill-conditioning of the Jacobian matrix due to the low measurement redundancy, and the strong non-linearity of the network equations requires several iterations where round-off error may be considerably amplified. If divergence occurs, shift-cutting may be employed, but since the direction of the shift vector remains unchanged, which, makes it not very effective.

Bargiela [14] presented a solution via the augmented matrix approach. Equation (8) is written as;

$$[\boldsymbol{J^T R^{-1} J}] \cdot \Delta\boldsymbol{x} = \boldsymbol{J^T R^{-1} \Delta z} \tag{12}$$

The normal matrix was written as a system of three simultaneous equations;

$$r = \Delta z - J\Delta\hat{x} \tag{13a}$$

$$\lambda = R^{-1}r \tag{13b}$$

$$J^T\lambda = 0 \tag{13c}$$

where r and λ are the auxiliary vectors which do not have to be calculated explicitly. These equations are assembled into a supermatrix structure:

$$\begin{bmatrix} 0 & I & J \\ -I & R^{-1} & 0 \\ J^T & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \lambda \\ r \\ \Delta\hat{x} \end{bmatrix} = \begin{bmatrix} \Delta z \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

Although the dimension of the augmented matrix is now way bigger than the normal matrix, the equation (14) can be solved very efficiently using a sparse linear equation solution technique. This procedure requires less computational effort and is capable of significantly reducing the ill-conditioning.

Carpentier and Cohen's [18] approach to the solution of a least-square minimization problem, incorporates all measurement equations in the cost function, solving iteratively the linear quadratic problem to find (x^k, z^k), then the next solution is obtained from;

$$\min_{x,z} \frac{1}{2}[(z-z^*)^T M(z-z^*) + \frac{\epsilon}{2}||x-x^k||^2] \tag{15}$$

subject to

$$g(x^k, z^k) + \frac{\partial g}{\partial x}(x^k, z^k)(x-x^k) + \frac{\partial g}{\partial z}(x^k, z^k)(z-z^k) = 0$$

where ϵ is the tolerance value and M is the diagonal matrix with weighting coefficients. Replacing $u^k = (x^k, z^k)$, and with λ as the Lagrange multiplier of the constraint, the optimality conditions of the previous linear quadratic problem in matrix form are:

$$\begin{bmatrix} \epsilon I & 0 & (\frac{\partial g}{\partial x}(u^k))^T \\ 0 & M & (\frac{\partial g}{\partial z}(u^k))^T \\ \frac{\partial g}{\partial x}(u^k) & \frac{\partial g}{\partial z}(u^k) & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ \lambda \end{bmatrix} = \begin{bmatrix} \epsilon x^k \\ M z^* \\ \frac{\partial g}{\partial x}(u^k)x^k + \frac{\partial g}{\partial z}(u^k)z^k - g(u^k) \end{bmatrix} \tag{16}$$

The Zollenkopf bi-factorization was used to invert the large sparse matrix

B. Levenberg-Marquardt

The Levenberg-Marquardt algorithm is commonly used instead of shift-cutting. Equation (8) is modified to give;

$$[J^T W J + \lambda I]\Delta x = J^T W[z - g(\hat{x}^{(k)})] \tag{17}$$

where λ is the Marquardt parameter and I is an identity matrix. Marquardt parameter can change the direction and length of the shift vector, when it is increased. This results in the shift vector being rotated towards the direction of steepest descent. Piller et al. [22] A constrained Least-squares problem is solved with a projected Levenberg-Marquardt method. The criterion consists of two terms, a Hubert function of the residuals and a Tikhonov regularization term, for convexification of the problem.

C. Gradient method

Kumar et al. [19] is solved the least square problem using successive quadratic programming (SQP) technique. SQP requires the gradient of the objective function and the gradient of the reduced constraints. Analytical expressions for the derivatives with respect to the chord flow variables are obtained using the chain rule. The Hessian of the objective function for the reduced SQP problem is obtained using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm.

D. Direct search methods

Direct search methods depend on evaluations of the objective function at a variety of parameter values and do not use derivatives at all. They offer alternatives to the use of numerical derivatives in the Gauss-Newton method and gradient methods. Cheng et al. [10] solved the least square estimation problem using optimal search vector. One dimension search method, i.e., golden section search, is applied along with this vector in the optimization process as well as using what they termed, the local linearisation matrix transformation.

V. State Estimation Mathematical Modelling

State estimation problem needs a model of the water system, including all hydraulic elements of the network as well as consumer parameters. A water distribution network is usually modelled as a collection of arcs connected to nodes. The arcs (edges) represent pipes, pumps, and control valves. The nodes represent junctions, tanks, and reservoirs. Junctions are points in the network where links join together and where water enters or leaves the network. Any sequence of pipes within the network that begin and end at the same node forms a Loop. There are two variables associated with nodes of the network; heads (pressure) and flows as a result of consumption. Also, there are two variables associated with arcs; head losses (pressure drop at the end nodes of the arcs) and flows in the arc. Steady-state hydraulic relationships in WDN can be described by conservation principles [23], which leads to two sets:

$$H_i - H_j - \Phi(Q_k) = 0 \tag{18}$$

where i and j are nodes at the extremes of pipe k

H_i, H_j are the heads at node i and j

$\Phi(Q_k)$ head loss due to friction as a function of flow rate.

$$\sum_{k=1}^{n_i} Q_{k_{i,j}} = q_i \tag{19}$$

where $Q_{k_{i;j}}$ is flow in the generic pipe $k_{i;j}$ connected to node i;

n_i is number of pipes connected to node i; and

q_i is known demand at node i.

The frictional head loss function is typically expressed as a power function of Q_k,

$$\Phi = r_k |Q_k|^{\alpha-1} Q_k \tag{20}$$

where r_k is the resistance coefficient of pipe k
α is 2 when using the Darcy-Weisbach equation or 1.852 for Hazen-Williams equation.

Cross [24] proposed the first mathematical method for calculating flows in complex networks, which dealt with one equation at a time. This is a manual, iterative procedure that was implemented in a number of network analysis software and used throughout the water industry for decades. Recent approaches that replaced the Hardy Cross method solves simultaneous equations of large systems with improved convergence characteristics, using either the Newton-Raphson (NR) linearisation approach or the linear theory (LT) successive approximation [25] approach to accommodate the systems non-linear equations. The method for network analysis are usually based on either the loop, nodal or the hybrid node-loop formulation.
Hamam and Brameller [26] pioneered the "Hybrid Method" to take advantage of the loop formulation for its best convergence characteristics and nodal formulation for its inherent maximum sparsity. There are similar approaches later as described by Osiadacz [27]termed the new loop-node method, Todini and Pilati [28] and salgado et al. [29], called it the "Gradient Method". Rossman et al. [30] in developing the algorithm for EPANET, used Todini and Pilatis [28] approach,to solve the flow continuity and head-loss equations.
The system of equations for the hybrid method as described by Hamam and Brameller [26] is derived from conservation principles, a modified version of equation (19) by eliminating the equation of the reference node and written as follows:

$$q_i = \sum_{k=1}^{\beta} C_{ik} Q_k \tag{21}$$

where Q_k is the flow in branch k
β is the number of branches
n is the number of nodes excluding the reference
q are the known nodal demands at node i
and

$$C_{ik} = \begin{cases} -1, & \text{if the flow in branch k leaves node i} \\ 0, & \text{if branch k is not connected to node i} \\ +1, & \text{if the flow in branch k enters node i} \end{cases} \tag{22}$$

Equation (21) can be written to relate the flow in branches of a network to the load at the various nodes. The matrix form is represented as;

$$\boldsymbol{q_n} = \boldsymbol{C_{n\beta} Q_\beta} \tag{23}$$

where $\boldsymbol{q_n}$ is the load vector $(n \times 1)$
$\boldsymbol{Q_\beta}$ is the vector of the flows in the branches $(\beta \times 1)$
$\boldsymbol{C_{n\beta}}$ is the topological node incidence matrix $(n \times \beta)$
The equation relating the pressure drop across the branches to the nodal pressures is derived in the same manner as above, and taking the pressure at the reference node to be zero. It is represented as;

$$\boldsymbol{C_{\beta n}^T H_n} = \boldsymbol{\Delta H_\beta} \tag{24}$$

where $\boldsymbol{H_n}$ is the nodal pressure vector
$\boldsymbol{\Delta H_\beta}$ is the vector of pressure drop across the branches
$\boldsymbol{C_{\beta n}^T}$ is the transpose of $\boldsymbol{C_{n\beta}}$
Another critical set of equations needed are the branch-flow equations, relating the pressure drop across a branch to the flow in that branch. In vector form:

$$\boldsymbol{\Delta H_\beta} = \boldsymbol{\phi_\beta(Q_\beta)} \tag{25}$$

The converse is also true, where $\boldsymbol{Q_\beta}$ can be expressed as a function of $\boldsymbol{\Delta H_\beta}$, which results in;

$$\boldsymbol{Q_\beta} = \boldsymbol{\phi'_\beta(\Delta H_\beta)} \tag{26}$$

If equations (21) and (24) are substituted into equation (26), the resultant nodal analysis equation is given by;

$$\boldsymbol{q_n} = \boldsymbol{C_{n\beta} \phi'_\beta(C_{\beta n}^T H_n)} \tag{27}$$

Loop analysis equations are derived from equation (18), generally written as

$$\sum_{j=1}^{\beta} D_{ij} Q_j = 0 \tag{28}$$

where

$$D_{ij} = \begin{cases} -1, & \text{if the branch j is in the opposite direction as loop i} \\ 0, & \text{if branch j is not in loop i} \\ +1, & \text{if the branch j is in the same direction as loop i} \end{cases} \tag{29}$$

In matrix form;

$$\boldsymbol{D_{\gamma\beta} \Delta H_\beta} = \boldsymbol{D_{\gamma\beta} \phi_\beta(Q_\beta)} = \boldsymbol{0_\gamma} \tag{30}$$

where $\gamma = \beta - n$, the number of loops in the network.
The loop analysis equation of the solution is derived by forming a tree and co-tree for the system. A tree is defined as a minimum number of branches connecting all the nodes, and a co-tree is the loop forming branches. Equation (23) is then partitioned into;

$$\boldsymbol{q_n} = \boldsymbol{C_{nn} Q_n} + \boldsymbol{C_{n\gamma} Q_\gamma} \tag{31}$$

or

$$\boldsymbol{Q_n} = \boldsymbol{C_{nn}^{-1} q_n} - \boldsymbol{C_{nn}^{-1} C_{n\gamma} Q_\gamma} \tag{32}$$

where $\boldsymbol{C_{nn}}$ is the nodal-incidence matrix for the tree branch and $\boldsymbol{C_{n\gamma}}$ is the nodal-incidence matrix for the cotree. The loop analysis equation was derived by combining equations (30) and (32), resulting in;

$$\boldsymbol{D_{\gamma\beta} \phi_\beta(Q_\beta^0} + \boldsymbol{D_{\beta\gamma}^T Q_\gamma}) = 0 \tag{33}$$

where

$$\boldsymbol{Q_\beta^0} = \begin{bmatrix} -\boldsymbol{C_{nn}^{-1} q_n} \\ \cdots \\ \boldsymbol{0_\gamma} \end{bmatrix} \tag{34}$$

and

$$D^T_{\beta\gamma} = \begin{bmatrix} -C^{-1}_{nn}C_{n\gamma} \\ \dots \\ U_{\gamma\gamma} \end{bmatrix} \tag{35}$$

VI. Conclusion

State estimation studies in water networks have witnessed several techniques in calculating the estimates of the system. Least square method is still widely applied with various iterative techniques to solve the non-linear equations. The LS method is mostly preferred from the LAV, because the LS solution tends to be stable and unique. The LAV however is very robust, in that it is resistant to outliers in the data.The system of equations that describes the water distribution network is non-linear and thus for a large system, non-linearity introduces complexity to the derivation and implementation of the state estimation problem.

Non-linear estimators are not as mature, cohesive, or well understood as linear ones. Generally, there is no optimal estimator for non-linear problem, but techniques are based on sub-optimal approaches. Graph theory, tree and co-tree decomposition, significantly has been shown to reduce the size of the optimization problem. Advances in ICT provides near real-time estimation for real-time decision support. In the area of the solution algorithm, the numerical optimization method needs to be explored to increase the accuracy of the estimates and reduced computational time.

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