Mode structure analysis of a Bessel–Gauss resonator

Igor A. Litvin[#]

National Laser Centre (CSIR) PO Box 395, Pretoria, South Africa, ph: +27-12-8412368 fax: +27-12-8413152

B.I. Stepanov Institute of Physics of the National Academy of Sciences of Belarus, F. Scaryna Av.,68, Minsk 220072, Belarus

ABSTRACT

In this work the mode structure of a resonator with Bessel – Gauss transverse mode distribution was investigated. Under some configuration of like resonators, we have found that diffraction losses in high mode number are less when compared with losses of low number of modes.

Key words: Bessel – Gauss resonator, Bessel – Gauss beam, Fox-Li method.

1. INTRODUCTION

Bessel light beams (BLB) have been well researched with both linear and non-linear properties investigated in detail[1-8]. One of the main properties of BLB is the reconstruction of their amplitude–phase profile behind non-transparent and partially transparent particles[9-15]. The mathematical formulation and physics approximation of zero order Bessel–Gauss beams (BGB)[16] and higher orders of BGB[17,18] were presented in [1, 2]. BGB are an interesting class of wave equation solutions which allow very good description of real BLB, which are formed by real Gauss beams. A number of schemes for the forming of BGB with limit aperture in resonators were offered early[19,20]. In some works another resonator systems which form BGB were investigated in detail[21,22].

In this work the mode structure of a resonator, which is similar to a resonator considered in the work [23] (instead of mirrors, which was used in this work, in [23] was used Durnin scheme of Bessel beam generation based on creating of Bessel beam with an annual slit followed by a lens) was examined. Some interesting regularities of mode behavior inherent to these types of resonators were found.

2. BESSEL–GAUSS RESONATOR

A scheme of resonator investigated in this work is shown on fig. 1. Two equal round concave mirrors L1 and L0 with equal radiuses R0 are located at distance F apart from each other (F – focal length of mirrors).
The core zone of mirror L1 is blinded by diaphragm with radius R1. As a whole this scheme is similar to scheme of resonator which was investigated by work [23].

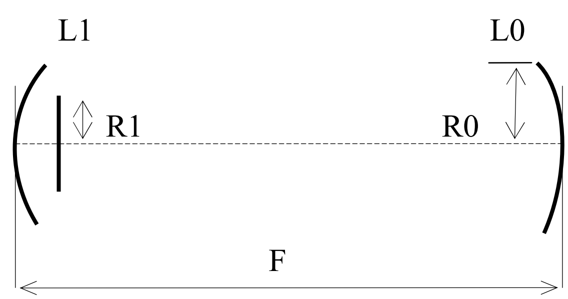

Figure 1: Illustration of the Bessel – Gauss resonator.

[#] ilitvin@csir.co.za; phone +27 12 841 2368; fax +27 12 841 3152

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When light passes from mirror L0 to mirror L1 it suffers transformation which is like a Fourier – Bessel[24].

$$F_n(q,\varphi)=\int_0^{R_0} f(r,\varphi)J_n(qr)rdr \tag{1}$$

Where $f(r,\varphi)$ – the field on mirror L0, R_0 – radius of mirror, $F(q,\varphi)$ – the field on mirror L1, $q=k_0 r_1/f$, $k_0=2\pi/\lambda$.
As was shown in [23], distributions of light field on mirror L0 when $R_0 \rightarrow \infty$ is a Bessel – Gauss distribution.

$$f(r,\varphi)=AJ_n(qr)\exp(-r^2/w^2)\exp(in\varphi)\,. \tag{2}$$

In the sequel we take that field on mirror L0 is approximately Bessel – Gauss. This approximation is very good to allow the assessing of mode behavior. Because of the field on mirror L1 has a view like displaced Gauss (circle with gauss field distribution of cross section) for all mode numbers[23], then increasing or decreasing of diaphragm R1 or mirror L1 itself, we can expect an increasing or decreasing losses for each modes simultaneous only. We have another situation with mirror L0, because of field structure on this mirror has Bessel – Gauss view and asymptotic approximation, corresponding to $R_0\rightarrow\infty$, for modes with odd and even numbers has either sinusoidal for odd modes and cosine view for even modes, this approximation works enough good after third maximum of Bessel beam. Consequently, when we increase or decrease the radius of mirror L0 then diffraction losses of odd or even modes on this mirror will have oscillating character, moreover the decreasing of losses for even modes implies increasing of losses for odd one and vice versa. For mathematical description of the given mode behavior let's insert the view of field on mirror L0 by inserting equation (2) into equation (1) which describes field transformation passing from L0 to L1. After inserting let's take the next equation for field view of n – mode on mirror L1.

$$F_n(q,\varphi)=\exp(in\varphi)A\int_0^{R0} J_n(qr)^2\exp(-\frac{r^2}{w^2})rdr\,. \tag{3}$$

The graphs of maximums of displaced Gauss for different modes on mirror L1 of resonator depending on mirror radius L0 (eq. 3) are shown on (fig. 2).

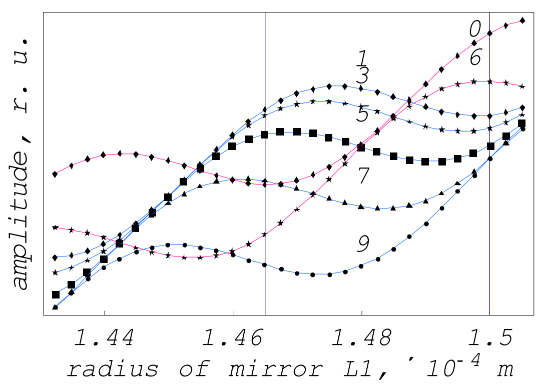

Figure 2: The dependence of maximums of displaced Gauss intensity for different modes on radius R_0. (f = 0.7 m, R1=0.95 R0, $\lambda=0.53\ 10^{-6} m$)

As we see from fig. 2 diffraction losses of zero mode under some value of mirror radius L0 can be more than losses 1, 3, 5 and 7 modes. This, as we mentioned above, is a consequence of different asymptotic behavior of odd and even modes. For clearness, let's illustrate a graph of dependence of diffraction loses on mode number under fix resonator parameters which was taken by analogous way.

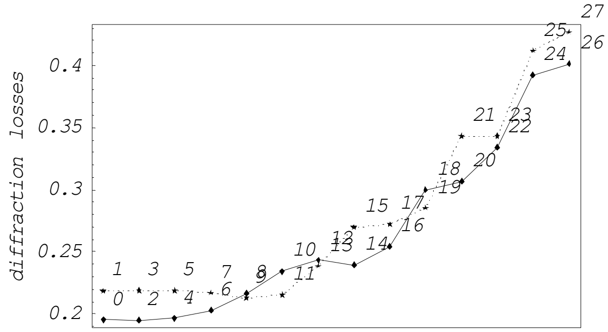

Figure 3: The dependence of diffraction losses under passing from mirror L0 to mirror L1 on mode number. (f = 0.7 *m*, R0=1.5 10^{-4}*m*, R1=0.9 R0, $\lambda=0.53\ 10^{-6}m$)

Fig.3 corresponds to right vertical straight line represented on fig. 2. (R0=1.5 10^{-4}m).
As we see from fig.2, under given radius R0 of mirror border approximately coincides with odd mode maximums and even mode minimums.
Consequently, even modes must have lesser losses than odd. However, as we see from figure 2, this behavior can be absence. A reason of absence is inapplicability of asymptotic approximation under given mirror radius for higher order modes.
Similarly, let's plot a graph of diffraction losses which corresponds to left straight line (R0=1.465 10^{-4}м) on fig. 2 (see fig. 4). As we see from fig. 2 border of mirror coincides approximately with even mode maximums and minimums of odd modes under given radius R0.

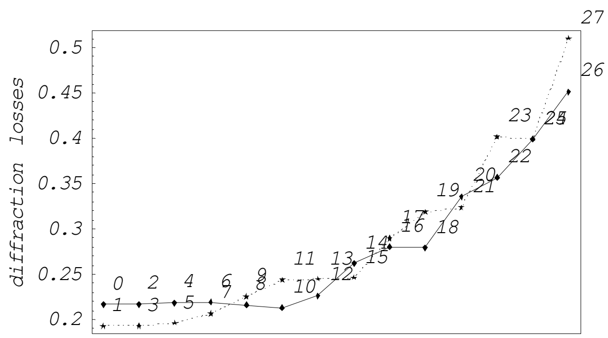

Figure 4: The dependence of diffraction losses per pass under expansion from L0 to L1 on mode number. . (f = 0.7 *м*, R0=1.465 10^{-4}*м*, R1=0.9 R0, $\lambda=0.53\ 10^{-6}м$)

As we expected, even modes, for which asymptotic approximation is correct, have more losses than odd modes under given mirror radius L0. For confirmation of taken result this task was solved by Fox – Li method [25]. Let's plot the graph of diffraction losses per pass which corresponds to right straight line on fig. 2, of stationary field of our resonator (see appendix).

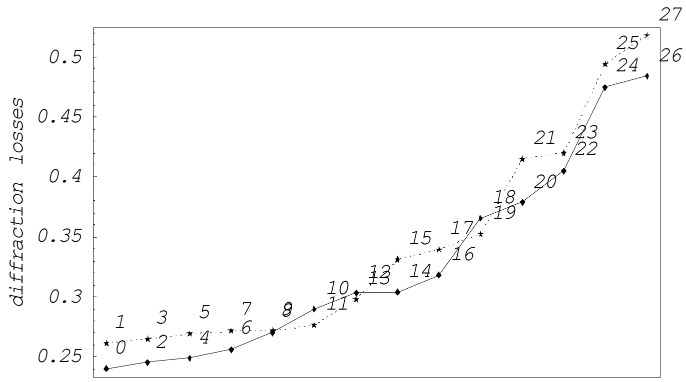

Figure 5: The dependence of diffraction losses per one passes on mode number given by Fox –Li method. (f = 0.7 m, R0=1.5 10 ^{-4}m, R1=0.9 R0, $\lambda=0.53\ 10^{-6}m$)

Obvious that approximation, which was taken above, is adequate to describe the behaviors of modes in the given resonator depending on mirror radius, because of fig. 5 coincides completely with fig. 3 in workmanlike manner. These graphs have some differences because we take into account on fig 5. diffraction losses which suffer the field under expansion from mirror L1 to mirror L0. But, as mentioned above, the taking into account given losses leads to simultaneous increasing or decreasing of losses of all modes that don't show qualitative behaviors of modes in the given resonator.

3. CONCLUSION

In this work mode structure of resonator (see. fig. 1) was investigated. The specific modes behavior of resonators with Bessel field structure on one of the mirror was determined. Due to given field structure we observe unusual, for Fabry – Pero type resonators, behavior of modes which determines by difference of diffraction losses for odd and even modes. Given difference we can easy understand if we know the difference of asymptotic behaviors of Bessel type field for odd and even numbers of modes. Thanks to given property of Bessel type fields in like resonators we can expect, under defined value of mirror radius, that diffraction losses of zero mode will be more than losses of some odd number modes, that we can see on fig. 2. Due to specific mode behavior of resonators with Bessel field structure on one of mirror we can conclude that calculation and analysis of like type resonators with assumption zero mode has minimum of losses is not correct ever. The Fox-Li algorithm was changed for increasing of calculation speed. (see appendix)

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APPENDIX

The Fox-Li algorithm was changed for increasing of calculation speed. For this I have divided two mirrors into many parts and if parts enough small we can consider complex amplitude on any part is constant. Take into account mentioned above we can divide Fresnel integral into sum of integrals on each part of division and because of on each parts amplitude is constant we can take out ones from integrals:

$$A_1(x_1) = a_0 \frac{\exp(ikL)\exp(i\frac{k}{2L}x_1^{\ 2})}{i\lambda L} \int_{-R_0}^{R_0} A_0(x)\exp(i\frac{k}{2L}x^2)\exp(i\frac{k}{L}x_1 x)dx =$$

$$= a_0 \frac{\exp(ikL)\exp(i\frac{k}{2L}x_1^{\ 2})}{i\lambda L} A_i \sum_i \int_{-R_i}^{-R_i+i\Delta r} \exp(i\frac{k}{2L}x^2)\exp(i\frac{k}{L}x_1 x)dx$$

Because of the integrals do not depend on amplitude and don't changed with passes then we can present ones in the form of matrix which does not change from one pass of field in resonator to another and present all field change in resonator as multiplication of amplitude matrix on Fresnel integrals matrix.

$$\begin{pmatrix} A^1{}_1 \\ A^1{}_2 \\ . \end{pmatrix} = \begin{pmatrix} A^0{}_1 \\ A^0{}_2 \\ . \end{pmatrix} \begin{pmatrix} \int_{-R_{0i}}^{-R_i+\Delta r} \exp(i\frac{k}{2L}x^2)\exp(i\frac{k}{L}x_1 x)dx & \int_{-R_i}^{-R_i+\Delta r} \exp(i\frac{k}{2L}x^2)\exp(i\frac{k}{L}x_2 x)dx & \ldots \\ \int_{-R_i+\Delta r_i}^{-R_i+2\Delta r} \exp(i\frac{k}{2L}x^2)\exp(i\frac{k}{L}x_1 x)dx & \int_{-R_i+\Delta r}^{-R_i+2\Delta r} \exp(i\frac{k}{2L}x^2)\exp(i\frac{k}{L}x_2 x)dx & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix}$$

$$A_{i+1} = A_i B$$

All integration of Fresnel integral during calculation of Fox-Li method we have presented as multiplication of two matrixes.

Using method mention above the time of calculation of field distribution inside resonator decreases and comes to time of calculation Fresnel integral per pass approximately.